ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

DYNAMIC RELIABILITY ANALYSIS OF A STOCHASTIC STRUCTURE WITH RESPONSE SURFACE METHOD

Chen Ying*, Wang Dongsheng, Zhu Changchun

Institute of Structural Mechanics，China Academy of Engineering Physics
Mailbox 919-402, Mianyang, Sichuan, China
cheny75@sohu.com

Abstract

Response Surface Method（RSM）for structure reliability analysis is simple, efficient and practical, which has been widely paid attention to by experts. However, it has been mainly used for static reliability analysis before. In this paper, it is introduced in the dynamic reliability analysis of a stochastic structure. Firstly, based on First Excursion Failure Criterion of random vibration, the performance function of dynamic reliability of a stochastic structure is established. Then RSM is used for fitting of the performance function. And finally the reliability index is obtained with JC method. An example demonstrates the method to have good calculation precision and high efficiency. It has especially wide prospect to be applied to the dynamic reliability analysis of complicated engineering structures.

INTRODUCTION

Response Surface Method（RSM） is a numerical simulation method well developed in the past two decades for structure reliability analysis. It has the characteristics of simplicity of calculation and universal application. In 1984, RSM was firstly used by Wong to calculate the reliability of soil slope stability[1], which was immediately paid attention to in the field of reliability and has drawn many researchers to make efforts in its envelopment and application. As to the algorithm, Bucherd and Bourgun suggested to use linear interpolation to select sampling points in the process of approaching the real surface[2]. Schueller etc. discussed the precision of the quadratic multinomial RSM[3]. Rajashekhar and Ellingwod proposed the adaptive iterative process of RSM with interpolation technique [4]. Kim suggested vector projection method to determine the design checking points, thus to raise the calculation efficiency [5]. As the algorithm become more and more mature, it is more widely used in engineering. For example, it was used by Liu etc. in the reliability analysis of aircraft structural systems[6], by Soares

Eds.: J. Eberhardsteiner, H.A. Mang, H. Waubke

in the reliability analysis of non-linear reinforced concrete frames[7] and by Vipman in the reliability analysis of laterally loaded piles[8].

Though many achievements have been made in the application of RSM, it has been used mainly for the static reliability analysis. In this paper, RSM is introduced in the dynamic analysis. The method for dynamics reliability analysis of a stochastic structure is developed based on RSM.

PRINCIPLE OF THE RESPONSE SURFACE METHOD

Assume the structure has n random variables of parameters $(x_1, x_2, \cdots, x_n)$, which mean values and variances are respectively μ_{x_i} and σ_{x_i}. The performance function of the structure is usually the implicit function regarding to those random variables:

$$Z = g(x_1, x_2, \cdots x_n) \tag{1}$$

One key technique in RSM is to select appropriate form of the fitting function. If the form of the function is too simple, the fitting precision would be poor, while the form is too complicated, the sampling and the calculation of the undetermined coefficients would be a tremendous work. Currently, the quadratic multinomial without cross term[2], which was developed by Bucher in 1990, is usually used as the response surface function:

$$Z' = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \tag{2}$$

where a, b_i and c_i are undetermined coefficients.

The other key in RSM is to select appropriate sampling points to determine the undetermined coefficients of the function. Since there are totally 2n+1 undetermined coefficients in expression (2), 2n+1 groups of sampling points are needed for the estimation. Sampling points are usually selected within $\left(\mu_x - f\sigma_x, \mu_x + f\sigma_x\right)$ and the value of f is taken between 1～3. 2n+1 groups of sampling points are obtained as: $\left(x_1, x_2, \cdots x_n\right)$ and $\left(x_1, \cdots, x_i \pm f\sigma_i, \cdots x_n\right)$. Based on those sampling points, analytical method or finite element analysis method can be used to calculate the value of Z. The undetermined coefficients can be obtained by substituting the results into expression (2).

After the explicit function of the performance function is obtained, the reliability index β and the design checking point x^* can be calculated by using the JC Method[9]. To the normal random variables, the calculation formulae are:

$$\beta = \frac{u_{z'}}{\sigma_{z'}} = \frac{\sum_{i=1}^{n} \left(\mu_{x_i} - x_i^*\right) \frac{\partial Z'}{\partial x_i}\bigg|_{x^*}}{\sqrt{\sum_{i=1}^{n} \left(\sigma_{x_i} \frac{\partial Z'}{\partial x_i}\bigg|_{x^*} \right)^2}} \tag{3}$$

$$x_i^* = u_{x_i} + \alpha_i \beta \sigma_{x_i} \tag{4}$$

where α_i is the sensitivity coefficient and calculated with

$$\alpha_i = -\sigma_{x_i} \frac{\partial Z'}{\partial x_i}\bigg|_{x_i^*} \bigg/ \sqrt{\sum_{i=1}^{n} \left(\frac{\partial Z'}{\partial x_i}\bigg|_{x_i^*} \sigma_{x_i} \right)^2} \tag{5}$$

To the other non-normal random variables, they can be equivalently normalized firstly. Then β and x^* are calculated with expression(3)~(5) [9].

DYNAMIC RELIABILITY ANALYSIS OF A STOCHASTIC STRUCTURE WITH RESPONSE SURFACE METHOD

The commonly used performance function in static reliability analysis of a stochastic structure is:

$$Z = L - Y \tag{6}$$

where L is limit value, Y is some kind of response. To calculate the reliability is to calculate the probability of $\mathrm{Z} < 0$.

Let that mode be extended into the dynamic analysis. Under random loads, the response of the structure is a random process varying with time; therefore, the performance function can be regarded as a function random process [10].

$$Z(t) = L - Y(t) \tag{7}$$

In a certain time range $[0, \mathrm{T}]$, following expression is obtained through minimum transformation of expression (7):

$$Z_{\min}(\mathrm{t}) = L - \underset{t \in [0,T]}{y_{\max}}(\mathrm{t}) \tag{8}$$

where $\underset{t\in[0,T]}{y_{\max}}(t)$ is the maximal value of the structure response in the time range $[0,\mathrm{T}]$. The calculation result with (8) is the minimal probability of the reliability of the structure in $[0,\mathrm{T}]$.

In this paper, assume dynamic load to be stationary random process. According to First Excursion Failure Criterion [10] (the structure is considered to be damaged when the response exceeds for the first time the upper and lower limit value specified), $\underset{t\in[0,T]}{y_{\max}}(t)$ can be expressed as:

$$\underset{t\in[0,T]}{y_{\max}}(t) = p\sigma_y \tag{9}$$

where p is the random variable. Its mean value, variance and distribution function are detailedly listed in literature[11].

σ_y is the total mean square root of the dynamic response. For a deterministic structure, σ_y is determined value. For a stochastic structure, such parameters as the elastic modulus, Poisson's ratio, density and damping ratio are random variables. A total mean square root is corresponded to a group of sampling points of these variables. So, σ_y should be described as a random variable that varies with the structure parameters.

Substitute (9) into (8). The performance function of the dynamic reliability of the stochastic structure can be obtained as:

$$Z = L - p\sigma_y \tag{10}$$

The key to the solution of the reliability is to obtain the rule of σ_y that varies with the structure parameters. σ_y is a complicated implicit function related to the structure parameters, so RSM is introduced to fit its explicit function. The fitting expression of the performance function is:

$$Z' = L - p\sigma_y' = L - p\left(a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2\right) \tag{11}$$

As above described, 2n+1 groups of sampling points are selected to estimate the undetermined coefficients. Then, the reliability index β and design checking point x^* can be obtained with JC Method.

In order to make the response surface function approach more possibly to the real function, reconstitution of the function is needed. At the first fitting, the center point of sampling is mean point μ_{xi}. After the reliability index and the design checking point have been obtained, the center point of sampling x_m for the next round should be adjusted as design checking point x^*. Then, new fitting and calculation of the next round are made as before. This procedure is repeated until the calculation precision meets the specification.

In the finite element modeling and the response analysis of the structure, such current finite element analysis software as ANSYS, NASTRAN can be used to greatly simplify the calculation.

For double limits problem（limit is ±L）, the probability that the response crosses +L is equal to that crosses -L in a stationary random process, therefore the reliability is:

$$P_f = 1 - 2 \times \left[1 - \Phi(\beta)\right] \tag{12}$$

To sum up, the procedure for the dynamic reliability analysis of a stochastic structure is shown in figure 1:

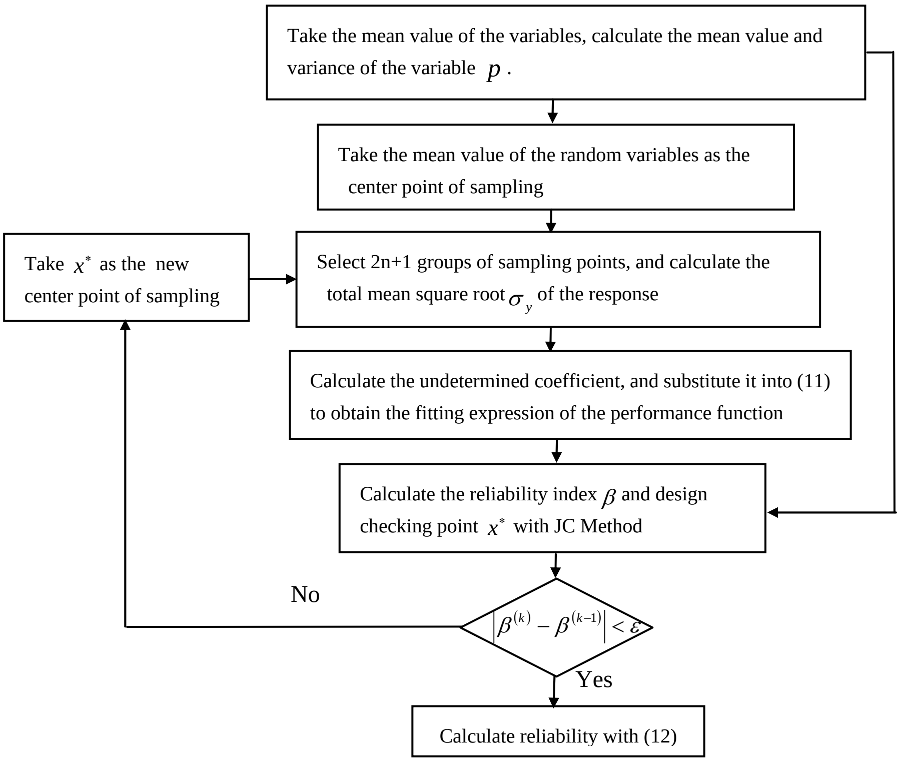

Fig 1 — procedure of calculation

ANALYSIS ON AN EXAMPLE

A cylindrical structure (see figure 2) is taken as an example. The finite element model of the structure established with ANSYS software is shown in figure 3. The constraint condition is the fixed constraint on the bottom side. The random variables of the structure parameters and their statistical characteristics are listed in table 1. Random loads in Y direction are imposed on the constrained side and the loading condition is shown in figure 4.

The key point of the structure is point A. Take the limit value of the acceleration response $L = \pm 200$g and the time $T = 10$s. Analyze the dynamic reliability of the structure with the method given in this paper.

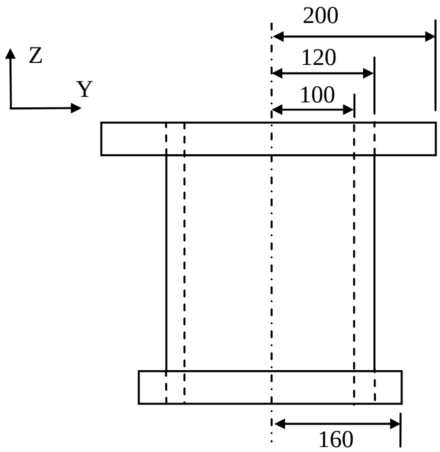

Fig 2 — structure of the example

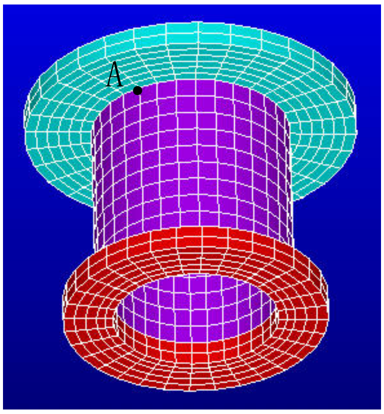

Fig3 — finite element

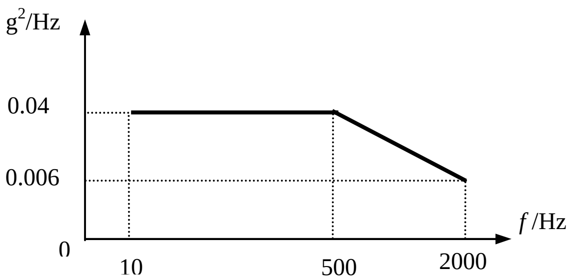

Fig4 — Loading condition

Table 1 Random variables of the structure parameters

	Elastic modulus E (Mpa)	Poisson ratio ν	Density ρ (kg/m^3)	Damping ratio ξ
Bottom	Mean value: 6.5×10^4 Mean variance: 6.5×10^3 Normal distribution	0.3	2.7×10^3	Mean value:0.01 Mean variance: 0.0015 Normal distribution
Middle	Mean value:8.43×10^3 Mean variance:1.2645×10^3 Normal distribution	Mean value:0.3 Mean variance:0.03 Normal distribution	Mean value:1.82×10^3 Mean variance:3.64×10^2 Logarithmic normal distribution	
Top	Mean value:4.3×10^4 Mean variance:5.16×10^3 Normal distribution	0.32	Mean value:1.458×10^4 Mean variance:2.187×10^3 Logarithmic normal distribution	

Following the procedure shown in figure 1, the convergence solution for the reliability index can be obtained after 4 iterations (see table 2). Since this example has 7 variables, 60 times of dynamic response calculation has been made with ANSYS in seeking the solution.

To validate the method in this paper, 20000 groups of sampling points are taken with Monte-Calo method and their dynamic responses have been made with ANSYS to statistically obtain the reliability index. From the results comparison shown in table 2, it is seen that the method in this paper has good precision and efficiency.

Table 2 Comparison of the calculation results with the two different methods

	Time of iteration	Reliability index β	Design check point						
			E_1	E_2	ν_2	ρ_2	E_3	ρ_3	ξ
Method in this paper	1	2.386	6.51610^4	8.947×10^3	0.352	1.828×10^3	4.302×10^4	1.443×10^4	0.00725
	2	2.525	6.520×10^4	9.043×10^3	0.350	1.837×10^3	4.302×10^4	1.443×10^4	0.00697
	3	2.530	6.520×10^4	9.043×10^3	0.350	1.860×10^3	4.301×10^4	1.443×10^4	0.00695
	4	2.530	6.520×10^4	9.038×10^3	0.350	1.847×10^3	4.301×10^4	1.443×10^4	0.00694
MC Method （20000 sampling）		2.676							
Error		5.46%							

The dynamic reliability for double limits is:

$$P_f = 1 - 2 \times \left[1 - \Phi(\beta)\right] = 98.86\%$$

CONCLUSION

RSM is introduced in the field of dynamic reliability analysis in this paper, based on which a method for the dynamic reliability analysis of a random structure has been developed. In this method, the total mean square root of the dynamic response of the structure is considered as a variable, and the performance function of the dynamic reliability of the stochastic structure is established based on the First Excursion Failure Criterion. Quadratic multinomial is used to approximately fit the function. JC Method is used to calculate the reliability index. The calculation precision is improved through sequential reconstitution of the fitting function. This method has the advantage of simplicity, generality, high precision and efficiency, and can be combined with the existing commercial software. It has wide prospect of application especially in the dynamic reliability analysis of the complicated stochastic structure.

REFERENCES

[1| Wong F.S., “Slope reliability and response surface method”, Journal of Geotechnical Engineering, **111**, 32-53 (1985).

[2| Bucher C.G., Bourgund U., “A fast and efficient response surface approach for structural reliability problems”, Structural Safety, **7**, 57-66 (1990).

[3| Schueller G.I., Bucher C.G., etc., “On efficient computation scheme to calculate structural failure probability”, Probability Engineering Mechanic, **4**, 10-18 (1989).

[4| Rajashekhar M.R., Ellingwood B.R., “A new look at the surface approach for reliability analysis”, Structural Safety, **12**, 205-20 (1993).

[5| Kim S.H., Na S.W., “Response surface method using vector projected sampling point”, Structural Safety, **19**, 3-19 (1997).

[6| Liu Y.W., Moses F., “A sequential response surface method and its application the reliability analysis of aircraft structural systems”, Structural Safety, **16**, 39-46 (1994).

[7| Soares R.C., Mohamed A., etc., “Reliability analysis of non-linear reinforced concrete frames using the response surface method”, Reliability Engineering and System Safety, **75**, 1-16 (2002).

[8| Vipman Tandjiria, Cee Ing The, etc., “Reliability analysis of laterally loaded piles using response surface methods”, Structural Safety, **22**, 335-355 (2000).

[9] Wu Shiwei, *Analysis of Structural Reliability*. (People’s Transportation Press, Beijing, 1990).

[10] He Shuiqing, Wang Shan, *Analysis and Design of Structural reliability*. (Defense Industry Press, Beijing, 1993).

[11] Zhu Weiqiu, *Random Vibration*. (Science Press, Beijing, 1998).