ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

MODELING OF NUTATION GYROPTIC DAMPERS

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Abstract

The tracking sensors acting in a tracking loop as stabilizer are mounted on a two degree of freedom gyros. The gyro must align its rotor axis with the line of sight in order to remove tracking errors. The tracking precision and sensitivity are functions of the gyros performance. One of the main factors in reducing the precision and producing instabilities is nutation vibrations. This fluctuating motion, which is a dynamical inherent property of the system, is related to the gyro lateral moment of inertia, the length of gyro and its rotating speed. In order to investigate the capabilities of nutation dampers, two models are presented: a simple viscoelastic cantilever beam with a tip mass, another model of viscose liquid damper containing mercury or oil in a ring. In each model presented here, the behaviour of the damper and its subsystems are taking into account. The equation of motions for the dynamical motion of Gyros, are obtained using Lagrangian approach, taking into account the friction of the dampers and the interaction of the liquid with the system equations of motion.

INTRODUCTION

A set of optical components placed on the rotor of gyroscope. The optical set has the duty of filtering, concentrating, separation and finally signal processing of received waves. The mechanical set of Gyro consists of inner and outer gimbals, which can sensitively rotate about two orthogonal axes. The magnetic rotor of gyro surrounds these two axes. For increasing the accuracy, all the accuracy stealing factors should be removed. One of the main factors which reduces the accuracy and even makes the gyro unstable is the dynamical inherent property of the system called nutation.When a moment-free inertially symmetric spinning body is subjected to an impulsive torque, i.e., a suddenly applied torque with brief duration, it will result in coning (or precession) motion of the spin axis about the angular momentum vector (which is fixed in space in the absence of subsequent external torques). For removing this nutation vibration, this paper investigates two dampers as: Voscoelastic and viscose dampers. The ring damper, partially filled with fluid and mounted on the spinning body (or the rotor of a gyroscope), has the effect in reducing the cone angle (or nutation angle), so it has been extensively used in satellites to keep their orientation and in gyroscopic seeker to confirm precise tracking. In present paper, the nutation of a gyroscopic seeker, which carries a ring damper partially filled with fluid and spins at a high speed of 60 Hz, is analyzed. When the optical detector inside the rotor detects the deviation of the target, the rotor is driven immediately to lock it by an impulsive torque generated by the coil surrounding the rotor. From some experimental observation, the shape of fluid in the ring looks like a crescent, so the fluid in the ring is modelled as a rigid slug in our analysis.

VICOSE DAMPER, DYNAMIC ANALYSIS

The idealized rotor and fluid-filled ring are shown in Fig.1. Let H denote the height of the ring damper to the point O which is center of mass of the gyroscope, ΔR and D the width and depth of the rectangular cross section of the ring, respectively, γ the angle of fill of the fluid in the ring, and R the mean radius of the ring. The angular momentum vector of the gyroscope $\boldsymbol{h}$ about the point O is fixed in space since the gyroscope is free after application of an impulsive torque. Let the Cartesian coordinate system X, Y, Z be the inertial reference frame with origin coinciding with the mass center of the gyroscope and the Z axis parallel to $\boldsymbol{h}$. the Cartesian coordinate system x,y,z is fixed on the rotor with origin coinciding with the center of the ring o' and xy plane lying on the plane the ring damper. The system u,v,z is fixed on the slug with the u axis passing through its center of mass, the angle β measured from the x axis to the u axis is the angular displacement of the slug relative to the rotor.

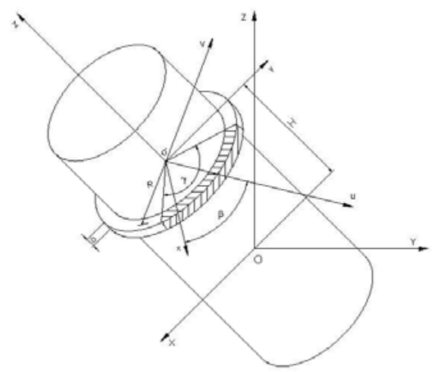

Figure 1 –Idealized rotor and Fluid-Filled ring

The Euler's parameters $p=(e_0,e_1,e_2,e_3)^T$, which are a quaternion, are introduced here instead of Eulerian angles to describe the orientation of the rotor with respect to the system X,Y,Z. the reason is that quaternion have no inherent geometrical similarities and no singularities in kinematic differential equations, but Eulerian angles have. In term of Euler's parameters, the rotational transformation matrix A from system X,Y,Z to system x,y,z can be expressed as the product of two 3×4 matrices as:

$$\mathbf{A} = \mathbf{E}\mathbf{G}^{\mathbf{T}} = \begin{bmatrix} e_0^2 + e_1^2 - e_2^2 - e_3^2 & 2(e_1e_2 - e_0e_3) & 2(e_0e_2 + e_1e_3) \\ 2(e_1e_2 + e_0e_3) & e_0^2 - e_1^2 + e_2^2 - e_3^2 & 2(e_2e_3 - e_0e_1) \\ 2(e_1e_3 - e_0e_2) & 2(e_0e_1 + e_2e_3) & e_0^2 - e_1^2 - e_2^2 + e_3^2 \end{bmatrix} \quad (1)$$

Where

$$\boldsymbol{E} = \begin{bmatrix} -e_1 & e_0 & -e_3 & e_2 \\ -e_2 & e_3 & e_0 & -e_1 \\ -e_3 & -e_2 & e_1 & e_0 \end{bmatrix} \quad \& \quad \boldsymbol{G} = \begin{bmatrix} -e_1 & e_0 & e_3 & -e_2 \\ -e_2 & -e_3 & e_0 & e_1 \\ -e_3 & e_2 & -e_1 & e_0 \end{bmatrix} \quad (2)$$

These Four Euler's parameters are not independent, and they satisfy the following constraint equation:

$$\mathbf{p}.\mathbf{p}^T = 1 \quad (3)$$

Let $\boldsymbol{\omega} = (\omega_X, \omega_Y, \omega_Z)^T$ and $\boldsymbol{\omega}' = (\omega_x, \omega_y, \omega_z)^T$ denote the angular velocity of the rotor in the *X*,*Y*,*Z* system and *x*,*y*,*z* system, respectively, they satisfy the following kinematic equations:

$$\boldsymbol{\omega} = \mathbf{A}\boldsymbol{\omega}' = 2\mathbf{E}\dot{\mathbf{p}} = -2\dot{\mathbf{E}}\mathbf{p} \quad (4)$$

$$\boldsymbol{\omega}' = 2\mathbf{G}\dot{\mathbf{p}} = -2\dot{\mathbf{G}}\mathbf{p} \quad (5)$$

Multiplying both sides of Eq.(5) by GT , and using Eq. (3) and the following identity,

$$\mathbf{G}^{\mathbf{T}}\mathbf{G} = -\mathbf{p}\mathbf{p}^{\mathbf{T}} + \mathbf{I}_{(4\times 4)} \quad (6)$$

One obtains:

$$\dot{\mathbf{p}} = \frac{1}{2}\mathbf{G}^{\mathbf{T}}\boldsymbol{\omega}' \quad (7)$$

EQUATION OF MOTION

The inertia matrix $\boldsymbol{I_g}$ of the rotor about the point *O* in the *x*,*y*,*z* system is defined as $\mathbf{I}_g = diag(J, J, J_3)$, Where J_1, J_2 and J_3 are the moments of inertia of the rotor about the *x*,*y*,*z* axes. The inertia matrix $\boldsymbol{I_m}$ of the mercury about the point *o* in the *u*,*v*,*z* system is:

$$\boldsymbol{I_m} = \begin{bmatrix} I_1 & 0 & -I_4 \\ 0 & I_2 & 0 \\ -I_4 & 0 & I_3 \end{bmatrix}$$

Where the values of I_1, I_2, I_3 and I_4 are:

$$I_1 = I_{uu} = \int (v^2 + z^2)dm = 2\int_0^{\frac{\gamma}{2}} (R^2 Sin^2\theta + H^2)\frac{m}{\gamma} d\theta = m\left[H^2 + \frac{R^2}{2}(1-K)\right]$$

$$I_2 = I_{vv} = \int (u^2 + z^2)dm = 2\int_0^{\frac{\gamma}{2}} (R^2 Cos^2\theta + H^2)\frac{m}{\gamma} d\theta = m\left[H^2 + \frac{R^2}{2}(1+K)\right]$$

$$I_3 = I_{zz} = \int (u^2 + v^2)dm = 2\int_0^{\frac{\gamma}{2}} R^2 (Sin^2\theta + Cos^2\theta)\frac{m}{\gamma} d\theta = mR^2$$

$$I_4 = I_{uz} = \int uz dm = 2\int_0^{\frac{\gamma}{2}} \frac{mHR}{\gamma} Cos\theta\, d\theta = mRHK'$$

$$K=\frac{Sin\gamma}{\gamma},K'=\frac{Sin\gamma/2}{\gamma/2}, \qquad m=\gamma RD\rho\Delta R$$

The kinetic energy $\boldsymbol{T_g}$ of the rotor is:

$$T_g=\frac{1}{2}\boldsymbol{\omega}'^T\boldsymbol{I}_g\boldsymbol{\omega}'=\frac{1}{2}\left[J(\omega_x^2+\omega_y^2)+J_3\omega_z^2\right]$$

The kinetic energy $\boldsymbol{T_m}$ of the fluid is:

$$T_m=\frac{1}{2}\boldsymbol{\omega}_s^T\boldsymbol{B}^T\boldsymbol{I}_m\boldsymbol{B}\boldsymbol{\omega}_s=\frac{1}{2}\{(I_1Cos^2\beta+I_2Sin^2\beta)\omega_x^2+(I_2Cos^2\beta+I_1Sin^2\beta)\omega_y^2+I_3\omega_z^2+$$
$$I_3\dot{\beta}^2+(I_1-I_2)Sin2\beta\omega_x\omega_y-2I_4Cos\beta\omega_x\omega_z-2I_4Sin\beta\omega_y\omega_z-2I_4Cos\beta\dot{\beta}\omega_x-2I_4Sin\beta\dot{\beta}\omega_y+2I_3\dot{\beta}\omega_z$$

Where $\boldsymbol{B}$ is the transformation matrix from system x,y,z to system u,v,z.
$\mathbf{B}=\begin{bmatrix}Cos\beta & Sin\beta & 0\\ -Sin\beta & Cos\beta & 0\\ 0 & 0 & 1\end{bmatrix}$, and $\boldsymbol{\omega}_s=(\omega_x,\omega_y,\omega_z+\dot{\beta})^T$

The gravitational potential $\boldsymbol{V_m}$ of the fluid is:

$$V_m=-m\boldsymbol{g}^T\boldsymbol{A}\boldsymbol{B}^T(RK',0,H)^T$$
$$=m\{gH(e_0^2-e_1^2-e_2^2+e_3^2)-RK'[2gCos\beta(e_0e_2-e_1e_3)-2gSin\beta(e_0e_1-e_2e_3)]\}$$

Where $\mathbf{g}=(0,0,g)^T$ is the gravity with components parallel to X,Y,Z. Using the Lagrange multiplier method, the equations of motion of the gyroscope are:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{p}}}\right)-\frac{\partial L}{\partial \mathbf{p}}=Q_p+\lambda\mathbf{p} \tag{8}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\beta}}\right)-\frac{\partial L}{\partial \beta}=Q_s \tag{9}$$

where $L=Tg+Tm-Vm$ is the Lagrangian of the gyroscope, λ is the Lagrange multiplier due to the constraint Eq.(3), Q_P is the generalized force resulting from the gravitational force of the fluid, and Q_β is the generalized force due to the frictional force between the fluid and the wall of the ring. The above equations of motion are five nonlinear second-order ordinary differential equations and one nonlinear algebraic equation for six unknowns, i.e., p, β, and λ,. In order to avoid solving the λ and the constraint equation, as well as to reduce the number of governing equations, the arguments $\dot{\mathbf{p}}$ in L are replaced by the quasi-coordinates $\boldsymbol{\omega}'$. Thus, $L(\mathbf{p},\dot{\mathbf{p}},\beta,\dot{\beta})$ becomes $\overline{L}(\mathbf{p},\boldsymbol{\omega},\beta,\dot{\beta})$. Using the chain rule, one has:

$$\frac{\partial L}{\partial \dot{p}_i}=\frac{\partial \overline{L}}{\partial \omega'_j}\frac{\partial \omega'_j}{\partial \dot{p}_i} \tag{10}$$

$$\frac{\partial L}{\partial p_i}=\frac{\partial \overline{L}}{\partial \omega'_j}\frac{\partial \omega'_j}{\partial p_i}+\frac{\partial \overline{L}}{\partial p_i} \tag{11}$$

From Eq.(5) and the following identity

$$\tilde{\boldsymbol{\omega}}' = 2\mathbf{G}\dot{\mathbf{G}}^{\mathrm{T}} = 2\dot{\mathbf{G}}\mathbf{G}^{\mathrm{T}} \tag{12}$$

Where

$$\tilde{\boldsymbol{\omega}}' = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

One obtains:

$$\frac{\partial L}{\partial \dot{\mathbf{p}}} = 2\mathbf{G}^{\mathrm{T}} \frac{\partial \overline{L}}{\partial \boldsymbol{\omega}'} \tag{13}$$

$$\frac{\partial L}{\partial \mathbf{p}} = -2\dot{\mathbf{G}}^{\mathrm{T}} \frac{\partial \overline{L}}{\partial \boldsymbol{\omega}'} + \frac{\partial \overline{L}}{\partial \mathbf{p}} \tag{14}$$

From Eqs.(13) and (14), Eq.(8) can be rewritten as:

$$2\mathbf{G}^{\mathrm{T}} \frac{d}{dt}\left(\frac{\partial \overline{L}}{\partial \boldsymbol{\omega}'}\right) + 4\dot{\mathbf{G}}^{\mathrm{T}} \frac{\partial \overline{L}}{\partial \boldsymbol{\omega}} - \frac{\partial \overline{L}}{\partial \mathbf{p}} = \lambda \mathbf{p} \tag{15}$$

Premultiplication of Eq.(15) by ***G*** and using two identities, i.e., $\boldsymbol{GG^T=I}$ And $\boldsymbol{Gp=0}$, one has:

$$\frac{d}{dt}\left(\frac{\partial \overline{L}}{\partial \boldsymbol{\omega}'}\right) + \tilde{\omega}' \frac{\partial \overline{L}}{\partial \boldsymbol{\omega}} - \frac{1}{2}\mathbf{G}\frac{\partial \overline{L}}{\partial \mathbf{p}} = 0 \tag{16}$$

Then the equations of motion can be recast in the following form:

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial \overline{L}}{\partial \boldsymbol{\omega}'}\right) + \tilde{\omega}' \dfrac{\partial \overline{L}}{\partial \boldsymbol{\omega}} - \dfrac{1}{2}\mathbf{G}\dfrac{\partial \overline{L}}{\partial \mathbf{p}} = 0 \\ \dfrac{d}{dt}\left(\dfrac{\partial \overline{L}}{\partial \dot{\beta}}\right) - \dfrac{\partial \overline{L}}{\partial \beta} = Q_s \\ \dot{\mathbf{p}} = \dfrac{1}{2}\mathbf{G}^{\mathrm{T}}\boldsymbol{\omega}' \end{cases} \tag{17}$$

Frictional Force

Since the shear stress is a linear function of the gradients of velocities with respect to spatial coordinates from the viewpoint of viscous fluid mechanics, we assumed that the frictional force $\boldsymbol{F_D}$ between the mercury and the wall of the ring is proportional od virtual work, one can obtain the generalized force resulting from $\boldsymbol{F_D}$ as:

$$Q_s = -C_d R^2 \dot{\beta} \tag{18}$$

The equivalent Reynolds number of a straight rectangular pipe is:

$$R_e = \frac{U_m D_h}{\nu} = \frac{R\dot{\beta}}{\nu} \frac{2D\,\Delta R}{(D + \Delta R)}$$

Where ν is the kinematic velocity. Since the cross-sectional area of the ring is small and the spin rate of the rotor is high in our case, the magnitude of the Reynolds number id of order 10^4 on an average. So the evaluation of shear stress from turbulent flow must be considered. For the turbulent flow, the shear stress τ_0 on the wall of a straight pipe with circular cross section is:

$$\tau_0 = 0.0791 R_e^{-1/4} (1/2\, \rho U_m^2) \tag{19}$$

where ρ is the density of fluid and U_m is the average flow velocity ($U_m \cong R\dot{\beta}$). Considering the effect of the curved pipe with circular cross section, Eq.(19) is modifies as:

$$\frac{\tau}{\tau_0} = 1 + 0.075 R_e^{1/4} (D_h / 2R)^{1/2} \tag{20}$$

where D_h is diameter of the circular cross section. The D_h in Eq.(20) is replaced by $\frac{2D\Delta R}{(D+\Delta R)}$ for the curved pipe with rectangular cross section. Equating the frictional force $\boldsymbol{F_D}$ to the shearing force which is obtained by multiplying the shear stress by the common contact area of the fluid slug and the ring, i.e.,

$$F_D = C_d R\dot{\beta} = 2(D+\Delta R) R\gamma\tau \tag{21}$$

we can evaluate the damping coefficient c_d. Substituting Eqs.(19) and (20) into (21) and using $\dot{\beta} \cong (\sigma - 1)\omega_z$, one has

$$C_d = 6.65\times 10^{-2} \rho (D+\Delta R) \left[\frac{(D+\Delta R)\upsilon}{D\Delta R}\right]^{1/4} R\gamma \left[(\sigma-1)R\omega_z\right]^{3/4} + 5.93\times 10^{-3} \rho \left[(D+\Delta R) D\Delta R\right]^{1/2} R^{3/2} \gamma (\sigma - 1)\omega_z \tag{22}$$

Where $\sigma = J_3 / J$.

State Equations

Considering state vector **x** as $\mathbf{X} = (e_0, e_1, e_2, e_3, \omega_x, \omega_y, \omega_z, \dot{\beta}, \beta)^T$ and using Eqs.(17) and (22) we can write the state form of dynamic equations in the form of $\dot{\mathbf{X}} = \mathbf{g}(\mathbf{X})$ where:

$$\mathbf{g} = \begin{bmatrix} \frac{1}{2}\mathbf{G}^{\mathrm{T}} \begin{bmatrix} X_5 \\ X_6 \\ X_7 \end{bmatrix} \\ \mathbf{M}^{-1}\mathbf{f} \end{bmatrix} \tag{23}$$

Hence,

$$\mathbf{M} = \begin{bmatrix} J + I_1 Cos^2 X_9 + I_2 Sin^2 X_9 & \frac{I_1 - I_2}{2} Sin2X_9 & -I_4 CosX_9 & -I_4 CosX_9 & 0 \\ \frac{I_1 - I_2}{2} Sin2X_9 & J + I_2 Cos^2 X_9 + I_1 Sin^2 X_9 & -I_4 SinX_9 & -I_4 SinX_9 & 0 \\ -I_4 CosX_9 & -I_4 SinX_9 & I_3 + J_3 & I_3 & 0 \\ -I_4 CosX_9 & -I_4 SinX_9 & I_3 & I_3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

and,

$$\mathbf{f} = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 \end{bmatrix}^T \tag{25}$$

Where,

$$f_1 = (I_4 CosX_9)X_5X_6 + (J - I_3 - J_3 + I_1 Sin^2 X_9 + I_2 Cos^2 X_9)X_6X_7 + (\frac{I_1 - I_2}{2} Sin2X_9)X_5X_7$$
$$+ (I_4 SinX_9)X_6^2 - (I_4 SinX_9)X_7^2 + [(I_1 - I_2)Sin2X_9]X_5X_8 + [(I_2 - I_1)Cos2X_9 - I_3]X_6X_8$$
$$-(2I_4 SinX_9)X_7X_8 - (I_4 SinX_9)X_8^2 + 2mgH(X_1X_2 + X_3X_4) + mgK'R(-X_1^2 + X_2^2 + X_3^2 - X_4^2)SinX_9$$

$$f_2 = (-I_4 SinX_9)X_5X_6 + (I_3 - J + J_3 - I_2 Sin^2 X_9 - I_1 Cos^2 X_9)X_5X_7 + (\frac{I_2 - I_1}{2} Sin2X_9)X_6X_7$$
$$-(I_4 CosX_9)X_5^2 + (I_4 CosX_9)X_7^2 + [(I_2 - I_1)Sin2X_9]X_6X_8 + [I_3 + (I_2 - I_1)Cos2X_9]X_5X_8$$
$$+(2I_4 CosX_9)X_7X_8 + (I_4 CosX_9)X_8^2 + 2mgH(X_1X_3 - X_2X_4) + mgK'R(X_1^2 - X_2^2 - X_3^2 + X_4^2)CosX_9$$

$$f_3 = [(I_1 - I_2)Cos2X_9]X_5X_6 + (I_4 SinX_9)X_5X_7 - (I_4 CosX_9)X_6X_7 + (\frac{I_1 - I_2}{2} Sin2X_9)(X_6^2 - X_5^2)$$
$$-2mgK'R(X_1X_2 + X_3X_4)CosX_9 + 2mgK'R(X_2X_4 - X_1X_3)SinX_9$$

$$f_4 = [(I_1 - I_2)Cos2X_9]X_5X_6 - (I_4 CosX_9)X_6X_7 + (I_4 SinX_9)X_5X_7 + (\frac{I_1 - I_2}{2} Sin2X_9)(X_6^2 - X_5^2)$$
$$-2mgK'R(X_1X_2 + X_3X_4)CosX_9 + 2mgK'R(X_2X_4 - X_1X_3)SinX_9 - C_d R^2 \dot{\beta}$$

NUMERICAL RESULTS

Implementing above equations for two type of fluid such as mercury and oil with following values of parameters, nutation angle variation versus time is displayed in Figs.2 and 3. As shown in these figures, mercury-filled ring damper has better performance to reducing nutation angle than oil-filled ring damper.

$$R = 16.71\,mm, \Delta R = 1.4mm, H = 30.39mm, D = 0.78mm, \gamma = 1.33Rad, J = 400g - cm^2, J_3 = 600g - cm^2$$
$$\rho_{Mercury} = 13.6g/cm^3, \nu_{Mercury} = 0.00117cm^2/s, \rho_{Oil} = 0.912g/cm^3, \nu_{Oil} = 4.2cm^2/s$$
$$\mathbf{X_0} = [1 \quad 0 \quad 0 \quad 0 \quad 100 \quad 0 \quad 250\pi \quad 0 \quad 0]^T$$

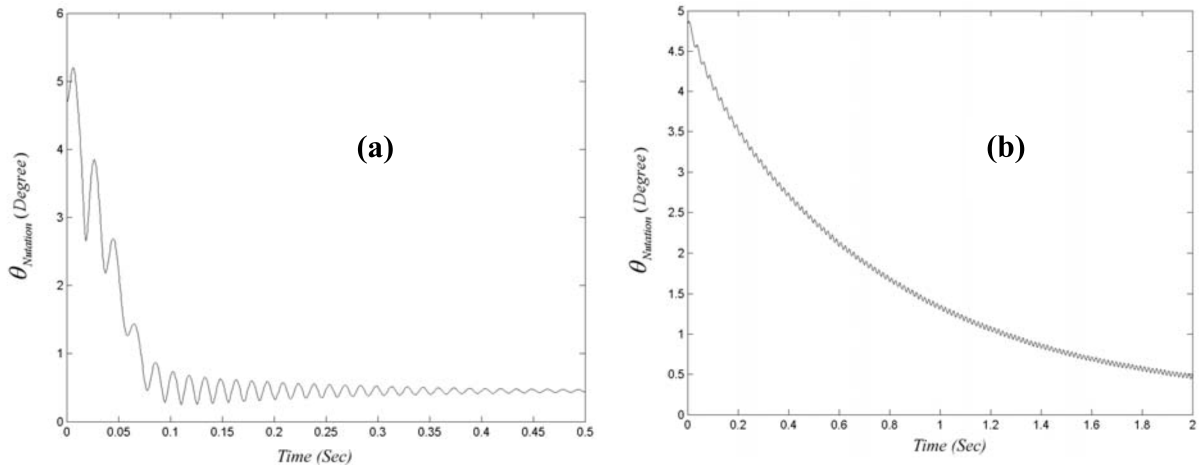

Figure 2 – Nutation angle variation for a) Mercury-Filled Damper & b) oil-Filled Damper

VISCOELASTIC DAMPER

The simplest form of viscoelastisity is a combination of Hookean Solid and Newtonian Liquid. The linear viscoelastic behaviour is modelled by Mechanical models, which are composed of set of springs and dampers. One of them is Kelvin model that is composed of a linear spring and a damper, which are jointed in parallel. Generally these kinds of dampers are used in different forms on rolling axis gyroptics. Here, the Kelvin's model is used. Martin [7] showed that an elastomer damper was better than a fluid viscose damper from the point of stability.

This elastomer damper is made of a viscoelastic cantilever beam and a rigid body at the end. For simplifying effects of gimbals are neglected. This gyro consists of a main mirror and a secondary mirror. Origin of the inertial fixed coordinates of oxyz is assumed to be on center of gravity of the system. The new system of $ox_p y_p z_p$ is made by rotating the inertial system about y axis in amount of α_1.axis of y and y_p coincide on axes of ball bearing of the outer gimbal. By rotating system of $ox_p y_p z_p$ about x_p in amount of α_2, system of $ox_n y_n z_n$ is generated. x_n axes is the axes of inner ball bearing gimbal. Spinning rate of the main rotor is shown by $\dot{\psi}$. For increasing and raising the vibration absorption, a tip body is applied in the end of damper. Figure 3 shows modeling of this damper.

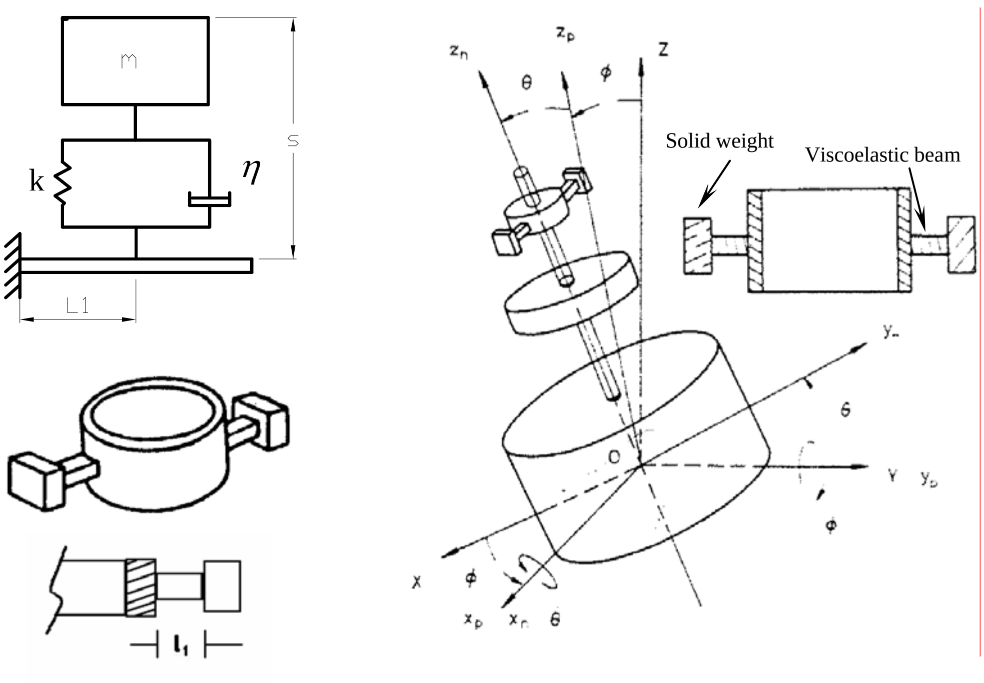

Figure 3 – Ideal Model of elastomer damper

State space form equations are extracted from lograngian equations as:

$$\alpha_1 = X_1 \;,\; \dot{\alpha}_1 = X_2 \;,\; \alpha_2 = X_3 \;,\; \dot{\alpha}_2 = X_4 \;,\; \Psi = X_5 \;,\; \dot{\Psi} = X_6 \;,\; S = X_7 \;,\; \dot{S} = X_8$$

$$V = 2\mathrm{b}x_8x_4 Sin X_3 + b_1 x_7 x_4^2 Cos X_3 - b_1 Cos X_3 F \;,\; F = (\mathrm{b} + x_7)(X_2^2 Cos^2 x_3 + X_4^2) - (K/M)x_7 - (\eta / M)x_8$$

We would have:

$$k_{11} = \dot{k}_1 \;,\; \dot{X}_1 = X_2 \;,\; \dot{X}_2 = \frac{\left[-mv + mRy/z + B\Psi X_4 Cos X_3 - X_2(-2c_2 X_4 Sin X_3 Cos X_3 + k_{11})\right]}{\left[-c_2 Sin^2 X_3 + k_1 - m^2 R^2 / z - mb_1^2 Cos^2 X_3\right]} \;,\; \dot{X}_3 = X_4 \;,$$

$$\dot{X}_4 = (-y - mRX_2)/z \;,\; \dot{X}_5 = X_6 \;,\; \dot{X}_6 = X_2 Sin X_3 + X_2 X_4 Cos X_3 \;,\; \dot{X}_7 = X_8 \;,\; \dot{X}_8 = b_1 X_2 Cos X_3 + F$$

A simulation program is written and figures 4 & 5 show the results for a practical example. The parameters are: $I_p = diag(AP, AP, CP)$: Inertial matrix of the main rotor, $I_s = diag(AS, AS, CS)$: Inertial matrix of the secondary rotor, *K*: stiffness coefficient, *n*: damping coefficient, L_1 :length of the place where model of damper locates, *S*: translation of the mass on the head of damper, *m*: mass effect of beam and damper, $ae_1 + be_3$:end of cantilever beam location vector, $r_E = (a + \mathrm{L}_1)e_1 + (b + s)e_3$:location vector of the mass and:

$$b_1 = a + L_1 \;,\; A = Ap + As + mb^2 \;,\; k_1 = (Ap + As)\cos^2\theta + (C_p + C_s)\sin^2\theta + m\left[(2b^2 + 2bs + s^2)\cos^2\theta + 2b_1^2\right]$$

Here, as a practical example, a system studied which the rolling speed of the main and the secondary mirror were the same, so $\alpha = 1$ and as a result, $C_2 = C_1$. For simulation of this system and computer programming, equations are altered to State space form equations of $X = f(x, u)$. The parameters have the value of: $b = 0.08m$, $\mathrm{m} = 0.01\,\mathrm{kg}$, $\mathrm{a} = 0.01\mathrm{m}$, $C_p = 0.00016 kgm^2$, $A_p = 0.000086666 kgm^2$

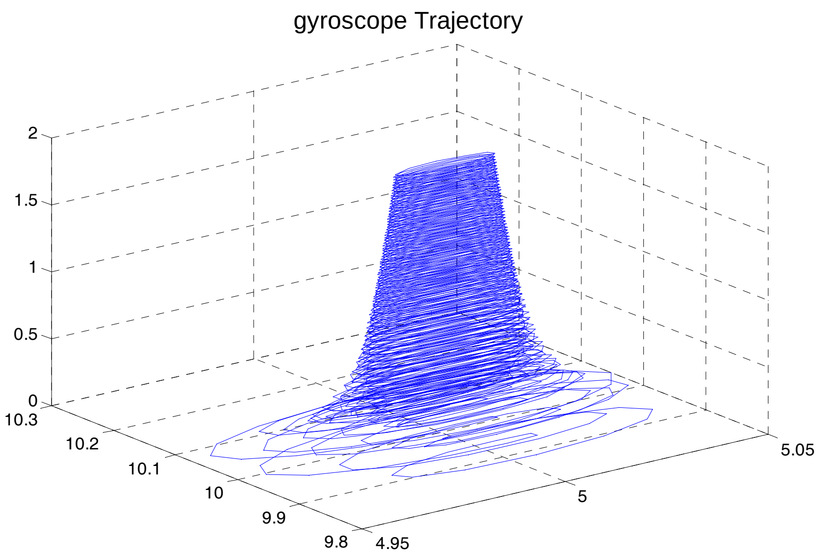

Figure 4– Gyroptic motion path

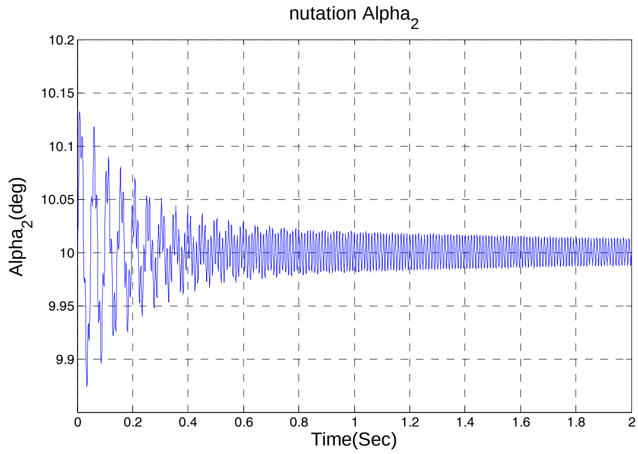

Figure 5– Nutation angle variation for Viscoelastic damper

SUMMARY

On this paper the effect of parameters of two kinds of dampers on the decay of wobble motion of the rotor is analyzed. Complete nonlinear equations of motion are adopted here on the parameter analysis, since the use of simplified equations of motion may lose something important. Coupled equations of motion are derived in terms of quasi-coordinates in order to reduce the number of equations of motion. The shearing force between rotor and the fluid is obtained by assuming a steady turbulent flow over a straight pipe. Finally, after comparing two different fluid types such as mercury and oil on viscose dampers, Viscoelastic dampers are studied.

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