



SET MEMBERSHIP IDENTIFICATION OF A LIGHTLY DAMPED FLEXIBLE BEAM FOR ROBUST CONTROL

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Abstract

The aim of this paper is to identify the nominal model and associated uncertainty bound of a lightly damped flexible beam in order to be utilized in robust controller design methods. Our approach is based on Set Membership theory where the system's uncertainties assumed to be unknown but bounded (UBB). Both parametric and non-parametric uncertainties have been accounted in the robust identification problem. The problem has been solved using Ellipsoidal and Parallelotopic approximation methods.

INTRODUCTION

Robust control theory plays an important role in the application of control theory in practical problems. The main concept is to consider a physical system as an uncertain model which may be represented as a family of mathematical models. Using robust control techniques, all models in this family will be stabilized in an appropriate manner. This family is described by a nominal model and a bounded uncertainty. Thus it is customary to identify not only a nominal model, but also an uncertainty bound associated to this nominal model. Identification methods producing a nominal model and its associated uncertainty are known as "*Robust Identification*" or "*(Robust) Control-Oriented Identification*" methods. Because of the outspread use of robust control techniques in practical problems, robust identification is an area which has received a growing interest of researchers since beginning of 1990's due to the weakness of classical identification methods to produce suitable models for robust control theory. Robust identification algorithms use *a priori information* on system and its input-output data (posteriori information) to produce a nominal model and its associated uncertainty.

Two main philosophies for description of model's uncertainties have been used. The first one is based on statistical assumptions and produces so-called "soft bound" on model's uncertainty. Second approach is based on deterministic hypothesizes and gives "hard bound" on uncertainty. Indeed in this approach, uncertainties are assumed to be "Unknown but Bounded" (UBB) [1]. Deterministic hypothesis on model's uncertainties, leads to set membership identification methodologies.

In all system identification problems, perturbation are potentially arise from two main sources: a variance error due to the measurement noises and a bias term due to effect of unmodeled dynamics (dynamics that have not been included by nominal estimated model- also known as model error). The nature of these two error types is quite different. Variance error generally uncorrelated with the input signal (in open loop data collection case), but bias error is strongly depends on nominal model's structure and identification experiment input signal [1].

Three main approaches for robust identification have been addressed in the literature, namely:

1. Stochastic Embedding (SE)
2. Model Error Modeling (MEM)
3. Set Membership (SM)

Set Membership technique is a time/frequency domain method, based on deterministic assumptions on system's perturbations. In fact uncertainties deem to be unknown but bounded by a suitable norm. In the first works the idea is used for state estimation [3, 4]. Later, SM theory is employed for the aim of system identification [5, 6]. Because of its deterministic framework, this approach to robust identification is more popular than SE and other statistical based approaches. Both parametric and non-parametric uncertainties can be accounted in SM identification problem. In [5], [6], [7] and [8] just parametric uncertainties are considered while [1], [9], [10], [11], [12] and [13] deal with both parametric and non-parametric uncertainties.

In this paper the SM approach has been employed in order to robust identification of a lightly damped flexible beam. Fundamentally, lightly damped flexible structures are distributed parameter systems and thus have infinite dimensional analytic models. In order to design a controller one has to have a finite dimensional model. Using truncated or reduced order model, "*spill over effect*" is a possible phenomenon. Spill over effect is called to the degradation of controller's performance due to excitation of unmodeled dynamics [14]. To fulfill this problem, robust controller is a beneficial tool. So, robust identification of lightly damped flexible structures is an evident necessity.

The remainder of this paper is organized as follow: in the next section the SM robust identification problem and its solutions will be briefly introduced. Then this approach will be employed in order to identification of the nominal model and uncertainty bound of a lightly damped flexible beam. The last section concludes the paper.

SET MEMBERSHIP ROBUST IDENTIFICATION PROBLEM FORMULATION

Suppose that N samples of input-output data, which have been generated by real system $G(q)$, are available:

$$y_m(k) = G(q)u_m(k) + v(k) \quad (1)$$

where $v(k)$ is the measurement noise and is bounded by a suitable norm:

$$\|v(k)\|_b \leq d(k) \quad (2)$$

It is possible to represent the real system as follow:

$$G(q) = G(q, q) + \Delta G(q) \quad (3)$$

where $G(q, q)$ is the parameterized nominal model and $\Delta G(q)$ stands for possible unmodeled dynamics and is also bounded by suitable norm in the space of transfer functions. For our identification problem we choose ∞ -norm. Regarding (3), the input-output relationship (1) can be presented as:

$$y_m(k) = \Delta G(q)u_m(k) + G(q, q)u_m(k) + v(k) \quad (4)$$

Considering $L\infty$ and $H\infty$ norms for noise and unmodeled dynamics:

$$y_m(k) - G(q, q)u_m(k) = \Delta G(q)u_m(k) + v(k) \quad (5)$$

$$\|y_m(k) - G(q, q)u_m(k)\|_\infty \leq \|\Delta G(q)u_m(k)\|_\infty + \|v(k)\|_\infty \quad (6)$$

$$\|y_m(k) - G(q, q)u_m(k)\|_\infty \leq \|\Delta G(q)\|_\infty \|u(k)\|_\infty + \|v(k)\|_\infty \quad (7)$$

where $\|\Delta G(q)\|_\infty$ and $\|v(k)\|_\infty$ are nonparametric and parametric perturbation bounds respectively and come from a priori information on system to be identified. Let:

$$\|\Delta G(q)\|_\infty \leq g ; \quad \|v(k)\|_\infty \leq v_k ; \quad \|u(k)\|_\infty = u_k \Rightarrow w_k = gu_k + v_k$$

Our approach in determining the nonparametric uncertainty bound (g) is similar to what have been addressed in [2], [10] and [11].

Now (7) can be expressed as:

$$\|y_m(k) - G(q, q)u(k)\|_\infty \leq w_k \quad (8)$$

It is customary to use a constant upper bound instead of variable bound in (8). In order to do this, one can choose the maximum value of the variable perturbation bound over all N samples and consider it in (8) for all data samples. Although considering constant upper bound on system's perturbations increases the conservativeness of identification algorithm, but it reduces the computational complexity of the algorithm.

In order to complete the set membership inequality in (8), the structure of $G(q, q)$ must be chosen. This model structure must have some general characteristics such as: low dimensional and linear in parameter. Considering this, the output-error (OE) and ARX structures may be used in the identification problem. Among them, OE structure is a popular model structure. To avoid high computational complexity due to nonlinear optimization in the process of parameter estimation and to obtain linear in model structure, we use the linear combination of orthonormal basis functions for OE model structure. This choice has an another advantage in the way that much more a priori information can be imported in to the identification algorithm by proper choice of basis functions. In other words by selecting basis functions whose dynamics are close to the dynamics of the real system, it will be conceivable to estimate the nominal model by minimum number of parameters [15, 16]. Because of resonant nature of our system, we use so-called "Kautz" or two-parameter basis functions [17]:

$$G(q, q) = \sum_{i=1}^n q_i y_i(q) \quad (9)$$

$$\begin{aligned} y_{2k-1}(q, b, c) &= \frac{\sqrt{1-c^2}(q-b)}{q^2 + b(c-1)q - c} \left[\frac{-cq^2 + b(c-1)q + 1}{q^2 + b(c-1)q - c} \right]^{k-1} \\ y_{2k}(q, b, c) &= \frac{\sqrt{(1-c^2)(1-b^2)}}{q^2 + b(c-1)q - c} \left[\frac{-cq^2 + b(c-1)q + 1}{q^2 + b(c-1)q - c} \right]^{k-1} \\ -1 < b < 1 \quad -1 < c < 1 \end{aligned} \quad (10)$$

where n is the order of nominal model and $y_i(q)$ is Kautz basis function. Now by (8) and (9):

$$\left\| y_m(k) - \sum_{i=1}^n q_i y_i(q) u_m(k) \right\|_{\infty} \leq w_k \quad (11)$$

Or equivalently:

$$\left\| y_m(k) - \mathbf{q}^T \mathbf{x}_m(q, k) \right\|_{\infty} \leq w_k \quad (12)$$

where $q = [q_1 \ q_2 \ \dots \ q_n]^T$ is the vector of parameters and $\mathbf{x}_m(q, k)$ is the regression(information) vector and computed as:

$$\mathbf{x}_m(q, k) = [y_1(q)u_m(k) \ y_2(q)u_m(k) \ \dots \ y_n(q)u_m(k)]^T \quad (13)$$

For each time stamp ($k=1, 2, \dots, N$), (12) produces a so-called “strip” in the space of parameters. By intersecting these strips, “Feasible Parameter Set” (FPS) will be obtained as follow:

$$\Theta = \{q : \bigcap_{k=1}^N \|y_m(k) - q^T \mathbf{x}_m(q, k)\|_{\infty} \leq w_k\} \quad (14)$$

In fact, Θ is the set of all parameters compatible with input-output data, a priori information on system and the uncertainty bounds. For the case that inequalities are linear in parameters, as (14), the FPS is a convex polytope in the space of nominal model’s parameters. The aim of set membership robust identification problem is to compute the FPS and determine an optimal point in FPS (in some sense) as the nominal model’s parameters. Exact computation of FPS and nominal model’s parameters is a laborious task and requires high amount of numerical computations and is not conceivable in practical situations [18, 19, 20]. An alternative is to outbound the FPS by simple geometrical shapes like “Ellipsoid” and “Parallelotope” (fig.1) and consider their center as the parameters of nominal model [5, 6, 7, 11]. Because of the greater DOF of the parallelotopes, they can outbound the FPS tighter than ellipsoids. Although this can be observed explicitly in fig.1 but it will be verified with respect to identification results in the next section.

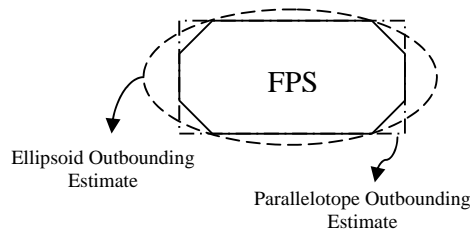


Figure 1 – Ellipsoidal and Parallelotopic estimation of FPS

SIMULATION RESULTS

This section presents the robust identification results of a lightly damped flexible beam with simply-supported boundary condition (fig. 2).

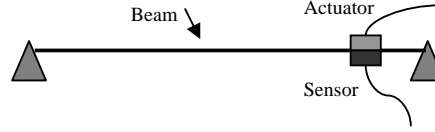


Figure 2 – The under study simply-supported beam

The flexible beam which is considered in this work is assumed to be out of steel which its exact specifications are presented in table1. The identification experiment has been simulated using a “*Finite Element*” model of the beam.

Table 1. The under study beam properties

Length	500 mm
Width	20 mm
Thickness	1 mm
Modulus of Young	2.07e+11
Density	7800 Kg/m ³
Damping	5e-3

The input and output time domain signals for identification are force and displacement, respectively (fig. 3). The input signal is the combination of 260 sinusoidal with proper frequencies that have been picked according to the a priori information of system which in this case is the beam’s FRF (fig. 4). Distribution of these frequencies is a key point in the experiment design. The output signal has been corrupted by a normally distributed Gaussian random signal with the SNR of 1% in order to simulate the measurement noise.

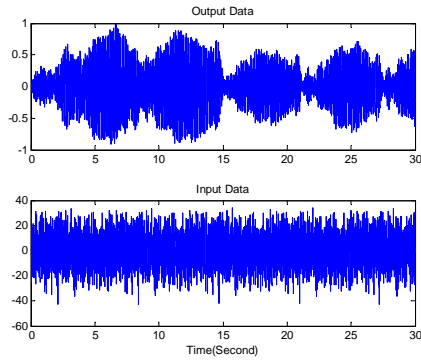


Figure 3 – Input-Output identification data

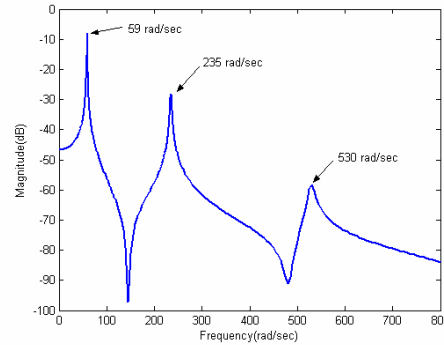


Figure 4 – FRF of the beam under study

The aim of identification problem is to consider the first two modes in the nominal model and the third mode as unmodeled dynamic or non-parametric uncertainty. We use Kautz basis functions for this purpose, which their parameters (b, c) have been

tuned with respect to the system's FRF. We also use two different outbounding algorithms for approximation of FPS: the ellipsoidal and the parallelotopic, which are shown in fig. 5 and fig. 6, respectively. In comparison with fig. 5, it can be seen that the tightness of parallelotopic approximation is better. As it is stated earlier, this is because of greater DOF of the parallelotopes. So they can enclose FPS better than ellipsoids.

From the robust controller design view, the parallelotopic approximated model has the advantage over the ellipsoidal approximated model in the way that it can achieve a suitable compromise between a good performance (which is depends on the tightness of uncertainty bound) and an appropriate stability (which is depends on the coverage of the real system with the uncertainty bound).

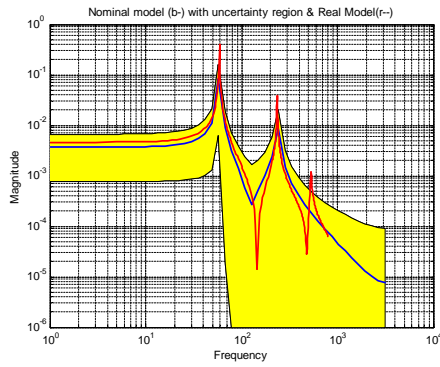


Figure 5 – Nominal identified model and its associated uncertainty (ellipsoidal approximation)

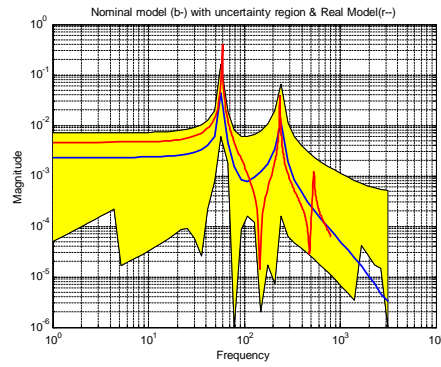


Figure 6 – Nominal identified model and its associated uncertainty (parallelotopic approximation)

CONCLUSION

In order to design robust controllers one has to have a suitable model which consists of the nominal model and some measure of its uncertainties. Robust identification methods provide such models that are indicate the real uncertainties of the system. SM method is one of these techniques that is based on deterministic assumptions on uncertainties. This type of uncertainty representation is greatly adopted by various robust control methods. In this paper this method is used for the purpose of robust identification of lightly damped flexible beams. It is shown that using the parallelotopic approximation method has better quality over the ellipsoidal method in the way that it can guaranty performance and stability of the designed controller.

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