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FUNDAMENTAL CONSIDERATION ON INDEPENDENT COMPONENT ANALYSIS FOR 3-DIMENSIONAL COMPLEX SIGNALS DUE TO 3-DIMENSIONAL UNITARY TRANSFORMATION –MAXIMIZATION OF EVALUATION FUNCTION BASED ON COMPLEX HERMITE MOMENT–

Noboru Nakasako*[1], Hisanao Ogura[1], Shunsuke Nakamura[1], and Daisuke Yamamoto[1]

[1] School of Biology-Oriented Science and Technology, Kinki University
Nishi-Mitani 930, Kinokawa, Wakayama 649–6493, Japan.
nakasako@info.waka.kindai.ac.jp

Abstract
In this paper, we expand the previous Independent Component Analysis (ICA) method to the case when 3 source signals are mixed with time-delay or convolution. More concretely, a practical scheme for 3-dimensional (3D) blind separation is formulated based on the cost function in terms of the complex Hermite moment, combined with the use of the group properties of 3-D unitary group SU(3). To confirm the validity and the effectiveness of our method, it is applied to speech data, introducing Principal Component Analysis as a preprocessing method.

1 Introduction

There have been many works[1, 2, 3, 4], related to the Blind Source Separation (BSS) which separates the observations into the original signals. This is because the BSS itself is a very attractive problem and has a wide potential in applications. In particular, a solution based on the statistical independency between the original signals is very popular and is called Independent Component Analysis (ICA)[5].

Authors have also proposed a scheme[6, 7, 8] for separating blindly N-dimensional signals based on the cost function in terms of the cumulants or the Hermite moments of low orders $n = 3, 4, 5, 6$, combined with the use of the group properties of N-D rotational group SO(N). The optimal rotation that transforms N-D standardized mixed signals into a possible of N independent components is searched for by the gradient method.

When the signals are observed with time-delayed or convoluted mixture, it is convenient to treat the BSS problem in frequency region. We developed Independent Component Analysis (ICA) for narrowband complex signals with time-delayed or convoluted mixture by means of complex Hermite moments and unitary transformations (SU(2)) for 2-dimensional (2D) case[9].

In this paper, we expand the previous method to the case when 3 source signals are mixed with time-delay or convolution. More concretely, a practical scheme for 3-dimensional (3D) blind separation is formulated based on the cost function in terms of the complex Hermite moment, combined with the use of the group properties of 3-D unitary group SU(3). The cost function is defined as a square norm of the complex Hermite moments with respect to separated signals and the optimal solution is found by maximizing the cost function. The optimal unitary matrix that transforms 3-D standardized mixed signals into a possible of 3 independent components is searched for by the gradient method, introducing parameterized matrices for SU(3). To confirm the validity and the effectiveness of our method, it is applied to speech data, introducing Principal Component Analysis as a preprocessing method.

2 THEORETICAL CONSIDERATION

2.1 Independent Components and Observation Process

Let us consider the actual situation where 3 original signals $s_i(t)(i = 1, 2, 3)$ affected by the transmission paths are observed. Letting $\mathbf{S}(t) = (s_1(t), s_2(t), s_3(t))^{\mathrm{T}}$ be source signal vector, and the observation vector $\mathbf{X}(t) = (x_1(t), x_2(t), x_3(t))^{\mathrm{T}}$ is given as:

$$\mathbf{X}(t) = \mathbf{A}(z)\mathbf{S}(t) \text{ with } \mathbf{A}(z) = \begin{pmatrix} a_{11}(z) & a_{12}(z) & a_{13}(z) \\ a_{21}(z) & a_{22}(z) & a_{23}(z) \\ a_{31}(z) & a_{32}(z) & a_{33}(z) \end{pmatrix} \tag{1}$$

where $\mathbf{A}(z)$ denotes the transfer matrix. The mixture $\mathbf{X}$ will be blindly separated into the independent components, utilizing the statistical independency between the source signals $s_i(t)$ and $s_j(t)$ $(i, j = 1, 2, 3; i \neq j)$.

Figure 1 a flow diagram of the blind source separation for complex signals. The complex filter first makes real observations narrow-banded into $\mathbf{X}_\nu$, where ν denotes the ν-th center frequency. Then, the narrow-banded complex observations are ortho-normalized with use of the eigenvalue matrix $\boldsymbol{\sigma}$ of observations and corresponding eigenvector $\mathbf{U}$. Finally, they are transformed by a unitary matrix $\mathbf{U}(g)$.

(i)Mixed observations(Real signals) $\mathbf{X}(t)$

$\downarrow$

(ii)Narrow-banded complex signals $\mathbf{X}_\nu(t)$

$\downarrow$

(iii)Standardization $\mathbf{Z}_\nu = \sigma^{-1}\mathbf{U}^*\mathbf{X}_\nu$

$\downarrow$

(iv)Separated signals $\mathbf{Y} = \mathbf{U}(g)\mathbf{Z}_\nu = \mathbf{U}(g)\sigma^{-1}\mathbf{U}^*\mathbf{X}_\nu$

$\downarrow$

(v)Complex Hermite moment $\langle H_{22}(Y_i, \overline{Y_i})\rangle$

$\downarrow$

(vi)Cost function $Q_{22} = \sum_{i=1}^{3}\langle H_{22}(Y_i, \overline{Y_i})\rangle^2 \to \max$

Figure 1: A flow of blind source separation for convolved signals.

2.2 Complex Signal and Complex Hermite Polynomial

2.2.1 Complex signal

In this paper a complex signal is meant to be a complex-valued random signal with an isotropic probability distribution. A normalized complex signal Z (mean 0 and variance 1) is a complex random variable with its 2nd-order statistical property,

$$z = x + iy, \ \ \overline{z} = x - iy, \tag{2}$$

$$\langle x^2\rangle = \langle y^2\rangle = \frac{1}{2}, \ \ \langle xy\rangle = 0, \ \ \langle \overline{z}z\rangle \equiv \langle |z|^2\rangle = 1, \ \ \langle z^2\rangle = \langle \overline{z}^2\rangle = 0 \tag{3}$$

with $\langle \ \rangle$ denoting the ensemble average. The probability density $p(x, y)$ of (X, Y) is isotropic;

$$P_{XY}(x, y)\mathrm{d}x\mathrm{d}y \equiv P_R(r)\mathrm{d}r(1/2\pi)\mathrm{d}\theta \tag{4}$$

where $p_R(r)$ denotes the probability density for $R = \sqrt{X^2 + Y^2}$. The random variables (X, Y), which are orthogonal but not necessarily independent of each other, can not be transformed to independent pair by rotation unless they are Gaussian. The moments of the complex signals $Z, \overline{Z}$ satisfy the relation

$$\langle \overline{Z}^m Z^n \rangle = \langle R^{n+m} \mathrm{e}^{\mathrm{i}(n-m)\Theta} \rangle = \delta_{mn} \langle R^{2m} \rangle, \ \ \langle R^{2m} \rangle = \int_0^\infty r^{2m} p_R(r) \mathrm{d}r \quad (=1, \ \ m=1) \tag{5}$$

2.2.2 Unitary transformation group SU(N)

Let a unitary matrix with determinant 1 be

$$\mathbf{U}\mathbf{U}^* = \mathbf{U}^*\mathbf{U} = \mathbf{I}, \ \ |\mathbf{U}| = 1 \tag{6}$$

The totality SU(N) of such transformations forms a group, which we denote by G for short. Let $g \in G$ be an element of G, and let the matrices corresponding to g, e (e:identity) be denoted by $\mathbf{U} = \mathbf{U}(g), \mathbf{U}(e) = \mathbf{I}$. We note the following correspondence for the multiplication and the inversion, $g_1 g_2 \to \mathbf{U}(g_1)\mathbf{U}(g_2), g^{-1} \to \mathbf{U}^{-1}(g) = \mathbf{U}^*(g)$.

2.2.3 Parametrization of SU(2)

We represent the matrix $\mathbf{U}(g)$ in terms of the three parameters (α, β, γ), which correspond to the Euler angles of 3D rotation [10]. Hence, we also represent the matrix and g by $\mathbf{U}(g) = \mathbf{U}(\alpha, \beta, \gamma)$ $g = (\alpha, \beta, \gamma)$.

$$\mathbf{U}(\alpha,\beta,\gamma) = \begin{pmatrix} \cos\frac{\beta}{2}\mathrm{e}^{i\frac{\gamma+\alpha}{2}} & i\sin\frac{\beta}{2}\mathrm{e}^{i\frac{\gamma-\alpha}{2}} \\ i\sin\frac{\beta}{2}\mathrm{e}^{i\frac{\alpha-\gamma}{2}} & \cos\frac{\beta}{2}\mathrm{e}^{-i\frac{\gamma+\alpha}{2}} \end{pmatrix} \tag{7}$$

$$= \begin{pmatrix} \mathrm{e}^{i\frac{\gamma}{2}} & 0 \\ 0 & \mathrm{e}^{-i\frac{\gamma}{2}} \end{pmatrix} \begin{pmatrix} \cos\frac{\beta}{2} & i\sin\frac{\beta}{2} \\ i\sin\frac{\beta}{2} & \cos\frac{\beta}{2} \end{pmatrix} \begin{pmatrix} \mathrm{e}^{i\frac{\alpha}{2}} & 0 \\ 0 & \mathrm{e}^{-i\frac{\alpha}{2}} \end{pmatrix} \tag{8}$$

$$\equiv \mathbf{A}(\gamma)\mathbf{B}(\beta)\mathbf{A}(\alpha) \ \ 0 \le \alpha < 2\pi, \ \ 0 \le \beta < \pi, \quad 0 \le \gamma < 2\pi \tag{9}$$

A unitary matrix can be factorized as in Eq.(9), where $\mathbf{A}(\alpha), \mathbf{A}(\gamma)$ are diagonal with unity moduli, so that the absolute values of the entries $|u_{\nu\mu}(g)|$ depends only on β. Since γ may take arbitrary values, we can set γ to be 0.

2.2.4 Unitary transformation of 2D complex signals

By a unitary matrix $\mathbf{U}(g), g \in SU(2)$, the ortho-normalized complex vector $\mathbf{Z} = (z_1, z_2)^T$ is transformed as

$$\mathbf{Y}(g) = \mathbf{U}(g)\mathbf{Z} \tag{10}$$

where $\mathbf{Y}(g)$ designates explicitly that it is a function of g. A unitary transformation conserves the vector norm as well as the correlation matrix of $\mathbf{Y}(g)$:

$$\langle \mathbf{Y}\mathbf{Y}^* \rangle = \langle \mathbf{U}(g)\mathbf{Z}(\mathbf{U}(g)\mathbf{Z})^* \rangle = \mathbf{U}(g)\langle \mathbf{Z}\mathbf{Z}^* \rangle \mathbf{U}(g)^* = \mathbf{I} \tag{11}$$

A complex conjugate variable $\mathbf{Z}^* = (\overline{z}_1, \overline{z}_2)^T$ is subject to the transformation by complex conjugate unitary matrix as a contravariant spinor.

2.2.5 Parametrization of SU(3)

As for 3D unitary transformation group SU(3), we represent the matrix $\mathbf{U}(g)$ in terms of three kinds of Eq.(9). Thus, the 3D unitary matrix for the ortho-normalized complex vector $\mathbf{Z} = (z_1, z_2, z_3)^T$ can be parameterized as follows:

$$\mathbf{U}(g) = \mathbf{U}_{zx}(g_3)\mathbf{U}_{yz}(g_2)\mathbf{U}_{xy}(g_1) \tag{12}$$

Here, the suffix i and j of $\mathbf{U}_{ij}$ denote ij coordinates respectively, i.e., $(i, j = x, y, z, i \ne j)$, and $g_i = (\alpha_i, \beta_i, \gamma_i)$.

Equation (12) can be also expressed in another form as:

$$\mathbf{U}(g) = \mathbf{U}_{xy}(g_3)\mathbf{U}_{yz}(g_2)\mathbf{U}_{xy}(g_1) \tag{13}$$

The matrix in Eq.(13) can be represented by angle parameters as follows:

$$\begin{aligned}\mathbf{U}(g) = & \begin{pmatrix} \mathrm{e}^{i\frac{\gamma_3}{2}} & 0 & 0 \\ 0 & \mathrm{e}^{-i\frac{\gamma_3}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\frac{\beta_3}{2} & i\sin\frac{\beta_3}{2} & 0 \\ i\sin\frac{\beta_3}{2} & \cos\frac{\beta_3}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{e}^{i\frac{\alpha_3}{2}} & 0 & 0 \\ 0 & \mathrm{e}^{-i\frac{\alpha_3}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ & \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{e}^{i\frac{\gamma_2}{2}} & 0 \\ 0 & 0 & \mathrm{e}^{-i\frac{\gamma_2}{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\beta_2}{2} & i\sin\frac{\beta_2}{2} \\ 0 & i\sin\frac{\beta_2}{2} & \cos\frac{\beta_2}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{e}^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & \mathrm{e}^{-i\frac{\alpha_2}{2}} \end{pmatrix} \\ & \cdot \begin{pmatrix} \mathrm{e}^{i\frac{\gamma_1}{2}} & 0 & 0 \\ 0 & \mathrm{e}^{-i\frac{\gamma_1}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\frac{\beta_1}{2} & i\sin\frac{\beta_1}{2} & 0 \\ i\sin\frac{\beta_1}{2} & \cos\frac{\beta_1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{e}^{i\frac{\alpha_1}{2}} & 0 & 0 \\ 0 & \mathrm{e}^{-i\frac{\alpha_1}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}\end{aligned} \tag{14}$$

Similarly to 2D case, we can set γ_3 to be 0, because γ may take arbitrary values.

$$\mathbf{U}(g) = \mathbf{U}_{xy}(\alpha_3, \beta_3, 0)\mathbf{U}_{yz}(\alpha_2, \beta_2, \gamma_2)\mathbf{U}_{xy}(\alpha_1, \beta_1, \gamma_1) \tag{15}$$

2.3 ICA for Observed Complex-valued Signals

The unitary matrix transforms the complex-valued signals keeping the non-correlation among them. For the purpose of searching for the optimal angles which make the complex-valued signals independent of each other, the evaluation function based on the complex Hermite moment is introduced.

2.3.1 Complex Hermite polynomials

As a bi-variate polynomial of $z = x + \mathrm{i}y,\ \ \overline{z} = x - \mathrm{i}y$, a complex Hermite polynomial $H_{mn}(z, \overline{z})$ is defined by the formula [11, 12],

$$H_{mn}(z, \overline{z}) = G(z, 0, 1)^{-1} \left(-\frac{\partial}{\partial \overline{z}}\right)^m \left(-\frac{\partial}{\partial z}\right)^n G(z, 0, 1) \tag{16}$$

where $G(z; 1)$ denotes the complex Gaussian probability density with variance 1. A general expression of complex Hermite polynomial is given by

$$H_{mn}(z, \overline{z}) = \sum_{j=0}^{\min(m,n)} (-1)^j \frac{m!n!}{j!(m-j)!(n-j)!} z^{m-j}\overline{z}^{n-j} \qquad m, n = 0, 1, 2, \cdots \tag{17}$$

In what follows we mainly deal with $H_{nn}(z, \overline{z})$;

$$H_{00}(z, \overline{z}) = 1 \tag{18}$$

$$H_{11}(z, \overline{z}) = |z|^2 - 1 \tag{19}$$

$$H_{22}(z, \overline{z}) = |z|^4 - 4|z|^2 + 2 \tag{20}$$

$$H_{33}(z, \overline{z}) = |z|^6 - 9|z|^4 + 18|z|^2 - 6 \tag{21}$$

$$H_{44}(z, \overline{z}) = |z|^8 - 16|z|^6 + 72|z|^4 - 96|z|^2 + 24 \tag{22}$$

The orthogonality of the complex Hermite polynomial is written as follows,

$$\int_{-\infty}^{\infty} G(z; 1)H_{mn}(z, \overline{z})H_{m'n'}(z, \overline{z})\mathrm{d}z = \delta_{mm'}\delta_{nn'}m!n! \tag{23}$$

2.3.2 3D complex Hermite polynomial and tensorial transformation

Generally, a multivariate complex Hermite polynomial is transformed as a mixed tensor of order (m, n) [12]. We write down the transformation rule for bi-variate complex Hermite polynomial $H_{mn}(z_\nu, \overline{z}_\nu)$,$(N = 3, \nu = 1, 2, 3)$. For a 3D unitary transformation, SU(3),

$$\begin{cases} y_1 = u_{11}z_1 + u_{12}z_2 + u_{13}z_3 \\ y_2 = u_{21}z_1 + u_{22}z_2 + u_{23}z_3 \\ y_3 = u_{31}z_1 + u_{32}z_2 + u_{33}z_3 \end{cases} \tag{24}$$

we have the formula

$$\begin{aligned} H_{mn}(y_\nu, \overline{y}_\nu) = & \sum_{m_1+m_2+m_3=m,} \sum_{n_1+n_2+n_3=n} \frac{m!}{m_1!m_2!m_3!} \frac{n!}{n_1!n_2!n_3!} \\ & \cdot u_{\nu1}^{m_1} u_{\nu2}^{m_2} u_{\nu3}^{m_3} \overline{u}_{\nu1}^{n_1} \overline{u}_{\nu2}^{n_2} \overline{u}_{\nu3}^{n_3} H_{m_1n_1}(z_1, \overline{z}_1) H_{m_2n_2}(z_2, \overline{z}_2) H_{m_3n_3}(z_3, \overline{z}_3) \end{aligned} \tag{25}$$

We mainly deal with $H_{nn}(z_\nu, \overline{z}_\nu)$, $(n = 1, 2, 3, \cdots)$ in what follows. We show here the formula only for $n = 2$:

$$\begin{aligned} H_{22}(y_\nu, \overline{y}_\nu) = & |u_{\nu1}|^4(|z_\nu|^4 - 4|z_\nu|^2 + 2) + |u_{\nu2}|^4(|z_\nu|^4 - 4|z_\nu|^2 + 2) \\ & + |u_{\nu3}|^4(|z_\nu|^4 - 4|z_\nu|^2 + 2) + u_{\nu1}^2\overline{u}_{\nu2}^2 z_1^2\overline{z}_2^2 + \overline{u}_{\nu1}^2 u_{\nu2}^2 z_1^2\overline{z}_2^2 \\ & + u_{\nu1}^2\overline{u}_{\nu3}^2 z_1^2\overline{z}_3^2 + \overline{u}_{\nu1}^2 u_{\nu3}^2 z_1^2\overline{z}_3^2 + u_{\nu2}^2\overline{u}_{\nu3}^2 z_2^2\overline{z}_3^2 + \overline{u}_{\nu2}^2 u_{\nu3}^2 z_2^2\overline{z}_3^2 \\ & + 2u_{\nu1}^2\overline{u}_{\nu2}\overline{u}_{\nu3} z_1^2\overline{z}_2\overline{z}_3 + 2\overline{u}_{\nu1}^2 u_{\nu2}u_{\nu3}\overline{z}_1^2 z_2 z_3 \\ & + 2\overline{u}_{\nu1}u_{\nu2}^2\overline{u}_{\nu3}\overline{z}_1 z_2^2\overline{z}_3 + 2u_{\nu1}\overline{u}_{\nu2}^2 u_{\nu3} z_1\overline{z}_2^2 z_3 \\ & + 2\overline{u}_{\nu1}\overline{u}_{\nu2}u_{\nu3}^2\overline{z}_1\overline{z}_2 z_3^2 + 2u_{\nu1}u_{\nu2}\overline{u}_{\nu3}^2 z_1 z_2\overline{z}_3^2 \\ & + 2u_{\nu1}^2\overline{u}_{\nu1}\overline{u}_{\nu2}(|z_1|^2 - 1)z_1\overline{z}_2 + 2\overline{u}_{\nu1}^2 u_{\nu1}u_{\nu2}(|z_1|^2 - 1)\overline{z}_1 z_2 \\ & + 2u_{\nu1}^2\overline{u}_{\nu1}\overline{u}_{\nu3}(|z_1|^2 - 1)z_1\overline{z}_3 + 2\overline{u}_{\nu1}^2 u_{\nu1}u_{\nu3}(|z_1|^2 - 1)\overline{z}_1 z_3 \\ & + 2u_{\nu2}^2\overline{u}_{\nu2}\overline{u}_{\nu1}(|z_2|^2 - 1)z_2\overline{z}_1 + 2\overline{u}_{\nu2}^2 u_{\nu2}u_{\nu1}(|z_2|^2 - 1)\overline{z}_2 z_1 \\ & + 2u_{\nu2}^2\overline{u}_{\nu2}\overline{u}_{\nu3}(|z_2|^2 - 1)z_2\overline{z}_3 + 2\overline{u}_{\nu2}^2 u_{\nu2}u_{\nu3}(|z_2|^2 - 1)\overline{z}_2 z_3 \\ & + 2u_{\nu3}^2\overline{u}_{\nu3}\overline{u}_{\nu1}(|z_3|^2 - 1)z_3\overline{z}_1 + 2\overline{u}_{\nu3}^2 u_{\nu3}u_{\nu1}(|z_3|^2 - 1)\overline{z}_3 z_1 \\ & + 2u_{\nu3}^2\overline{u}_{\nu3}\overline{u}_{\nu2}(|z_3|^2 - 1)z_3\overline{z}_2 + 2\overline{u}_{\nu3}^2 u_{\nu3}u_{\nu2}(|z_3|^2 - 1)\overline{z}_3 z_2 \\ & + 4|u_{\nu1}|^2|u_{\nu2}|^2(|z_1|^2 - 1)(|z_2|^2 - 1) \\ & + 4|u_{\nu1}|^2|u_{\nu3}|^2(|z_1|^2 - 1)(|z_3|^2 - 1) \\ & + 4|u_{\nu2}|^2|u_{\nu3}|^2(|z_2|^2 - 1)(|z_3|^2 - 1) \\ & + 4|u_{\nu1}|^2 u_{\nu2}\overline{u}_{\nu3}(|z_1|^2 - 1)z_2\overline{z}_3 + 4|u_{\nu1}|^2\overline{u}_{\nu2}u_{\nu3}(|z_1|^2 - 1)\overline{z}_2 z_3 \\ & + 4|u_{\nu2}|^2 u_{\nu3}\overline{u}_{\nu1}(|z_2|^2 - 1)z_3\overline{z}_1 + 4|u_{\nu2}|^2\overline{u}_{\nu3}u_{\nu1}(|z_2|^2 - 1)\overline{z}_3 z_1 \\ & + 4|u_{\nu3}|^2 u_{\nu1}\overline{u}_{\nu2}(|z_3|^2 - 1)z_1\overline{z}_2 + 4|u_{\nu3}|^2\overline{u}_{\nu1}u_{\nu2}(|z_3|^2 - 1)\overline{z}_1 z_2 \end{aligned} \tag{26}$$

2.3.3 Evaluation Function in Terms of Complex Hermite moment

By a unitary matrix $\mathbf{U}(g), g \in$ SU(3), $\mathbf{Z}$ is transformed as

$$\mathbf{Y} \equiv \mathbf{Y}(g) = \mathbf{U}(g)\mathbf{Z} \tag{27}$$

where $\mathbf{Y}(g)$ designates explicitly that it is a function of g. $\mathbf{Z}$ denotes 3D ortho-normalized observations and $\mathbf{U}$ is 3D unitary matrix. To make 3 components of $\mathbf{Y}$ statistically independent of each other, we may search for the optimal rotation angles $(\alpha_i, \beta_i, \gamma_i)(i = 1, 2, 3)$ which are entries of SU(3).

Now, quite analogous to the cost function for the blind separation of real signals given in terms of the vector norm of Hermite moments, we can introduce here the cost function in terms of the norm of complex Hermite moments:

$$Q(g) = \sum_{\nu=1}^{3} E[H_{22}(y_\nu, \overline{y}_\nu)]^2 \to \max \tag{28}$$

The lefthand notation implicates that it is a function of $g \in$ SU(3). This is analogous to the cost function $Q_4(g)$ for real signal separation [7].

Thus, the evaluation function to achieve the statistical independency among 3D signals is given as follows:

$$Q(g) = \sum_{\nu=1}^{3} \langle H_{22}(y_\nu, \overline{y}_\nu)\rangle^2 = \sum_{\nu=1}^{3} \langle |y_\nu|^4 - 4|y_\nu|^2 + 2\rangle^2 \tag{29}$$

$$y_\nu = \sum_{i=1}^{3} u_{\nu i}(g) z_i \quad (\nu = 1, 2, 3 \;\; g \in \mathrm{SU}(3)) \tag{30}$$

Maximization of the cost function with respect to $g = (\alpha_i, \beta_i, \gamma_i)(i = 1, 2, 3)$ can be made by the gradient method in search for zero gradient point. There are some properties of gradient for the absolute value in the cost function such as

$$\frac{\partial}{\partial \phi}|y_\nu|^{2m} = m|y_\nu|^{2(m-1)} \frac{\partial}{\partial \phi}|y_\nu|^2, \;\; (m = 0, 1, 2, 3, \cdots) \tag{31}$$

$$\frac{\partial}{\partial \phi}|y_\nu|^2 = \frac{\partial \overline{y}_\nu}{\partial \phi} y_\nu + \overline{y}_\nu \frac{\partial y_\nu}{\partial \phi} \tag{32}$$

$$\frac{\partial y_\nu}{\partial \phi} = \sum_{i=1}^{3} \frac{\partial u_{\nu i}(g)}{\partial \phi} z_i \quad (g \in \mathrm{SU}(3)) \tag{33}$$

where ϕ corresponds to each angle α_i, β_i and γ_i.

Finally, the iterative algorithm of gradient method searches for the optimal angles:

$$\phi(t+1) = \phi(t) + \mu \frac{\partial}{\partial \phi} \sum_{\nu=1}^{3} E[|y_\nu(g)|^4 - 4|y_\nu(g)|^2 + 2]^2 \tag{34}$$

with μ being a positive constantD

3 Computer simulation

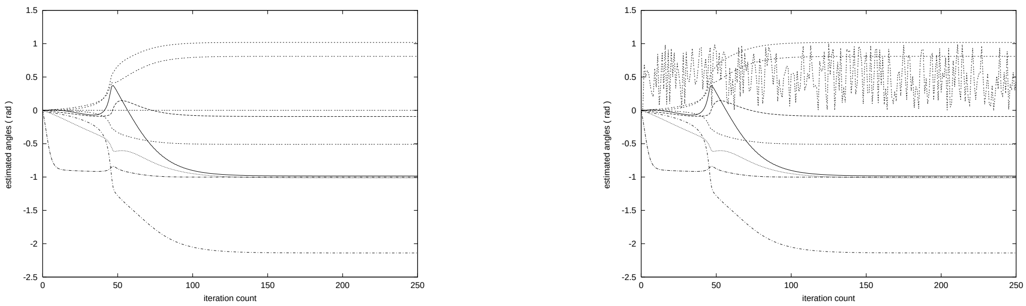
The validity of the proposed method is confirmed through computer simulation.

3.1 Simulation with Random Numbers

As 3 original signals z_i $(i = 1, 2, 3)$, complex random numbers with 0 mean and unit variance are generated using the relationship $z = r\cos\psi + ir\sin\psi$ (data length: 80,000), where ψ denotes the uniform random number $[0, 2\pi)$ and r [0, $\sqrt{6}/3$) (i.e., the variance of r is $1/2$). The observed signals are mixed these original signals by the mixing matrix in Eq.(14). Here, we set all $\alpha_i, \beta_i, \gamma_i$ $(i = 1, 2, 3)$ to be 1.0 [rad].

Eq.(14) is also adopted as a unitary matrix for signal separation and each parameter is estimated by the gradient method in Eq.(34). All of initial values are set as 0.0 and the learning rate $\mu = 1$. Figure 2(a) shows an estimation process for 9 angle parameters. All of estimated values converge to constant values and there is only one parameter remaining in the initial value 0, i.e., it's $\hat{\gamma}_3$. As mentioned above, $\hat{\gamma}_3$ of 9 parameters cannot be estimated.

To confirm this further, the random numbers are substituted in the estimated value $\hat{\gamma}_3$ every iteration (see Fig.2(b)). The randomness of $\hat{\gamma}_3$ gives no effect on the estimated values for the other parameters. This means that $\hat{\gamma}_3$ may take arbitrary values.



(a) Convergence for 9 parameters. (b) Randomly given $\hat{\gamma}_3$

Figure 2: Estimation process by gradiend method.

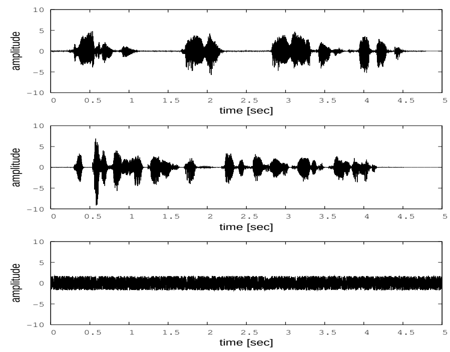


Figure 3: Original signals (top: male, middle: female, bottom: random number)

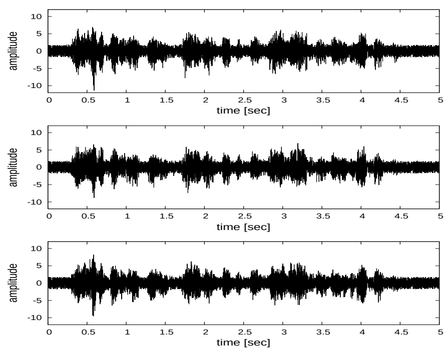


Figure 4: Observed signals.

The separation performance is evaluated using the absolute value of each entry with respect to the product $\widehat{\mathbf{U}}\mathbf{A}$ of the separation matrix $\widehat{\mathbf{U}}$ and the mixing matrix $\mathbf{A}$, as follows:

$$\begin{pmatrix} 0.999689 & 0.019094 & 0.016040 \\ 0.019113 & 0.999816 & 0.001445 \\ 0.016017 & 0.001679 & 0.999870 \end{pmatrix} \tag{35}$$

Since this matrix contains only approximately 1 value at each column and each row, we find the successful separation.

Next, for the mixed signals similar to the previous ones, the separation matrix is given in Eq.(12) and the separation performance is as follows:

$$\begin{pmatrix} 0.007924 & 0.005954 & 0.999951 \\ 0.999919 & 0.009931 & 0.007930 \\ 0.009936 & 0.999933 & 0.005946 \end{pmatrix} \tag{36}$$

This means that the mixed signals can be separated even if the type of separating matrix is different from that of mixing matrix.

3.2 Simulation with Speech Signals

To confirm the effectiveness of our method further, it is applied to the actual speech data. As shown in Fig.3, male and female speeches (s_1, s_2) and random number (s_3) are utilized as sound sources (length of data: 5 sec., sampling frequency: 16kHz).

By taking the time-delay in observations into consideration, the mixing matrix is given as follows:

$$\mathbf{A}(z) = \begin{pmatrix} z^{-10} & z^{-10} & z^{-30} \\ z^{-20} & z^{-10} & z^{-20} \\ z^{-30} & z^{-10} & z^{-10} \end{pmatrix} \tag{37}$$

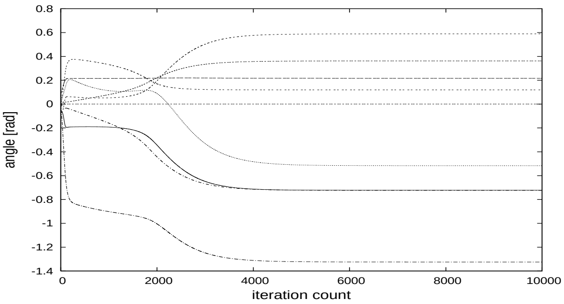


Figure 5: The estimation process of angle parameters at 170[Hz].

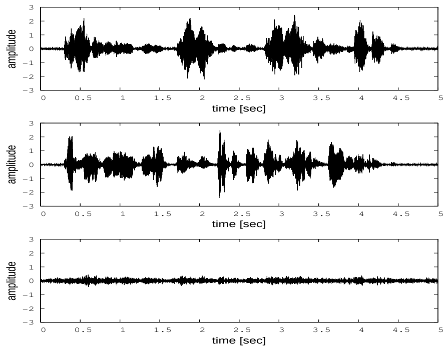


Figure 6: Synthesized separated signals.

where z^{-i} denotes the i-step time delay.

The observed signals are shown in Fig.4. To make them narrow-banded, the complex filter of 1st order with 20Hz bandwidth is introduced. The estimation process of the angle parameters are shown in Fig.5 for the frequency band of center frequency 170Hz as an example.

In this case, the permutation problem is solved using the correlation between the envelopes of separated signals in adjacent frequency bands. Figure6 shows the separated signals synthesized for all frequency bands. To evaluate the separation performance numerically, the Log-Spectral Distance (LSD) is introduced as:

$$\mathrm{LSD} = \sqrt{\frac{1}{W}\sum_{\omega=\omega_{min}}^{\omega_{max}} (20\log_{10}|S(\omega)| - 20\log_{10}|\hat{S}(\omega)|)^2} \quad [\mathrm{dB}] \;\text{ with }\; W = \omega_{max} - \omega_{min}, \quad (38)$$

where $S(\omega)$ denotes the spectrum of source signal and $\hat{S}(\omega)$ that of separated signal (see Table 1).

Table 1: Separation performance by LSD.

	Observed signals [dB]	Separated signals [dB]
male	24.03	13.32
female	21.95	14.81

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