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PREDICTION OF VIBRATION ENERGY TRANSMISSION IN A STRUCTURE CONSISTING OF THICK ELEMENTS

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Abstract

Regarding the prediction of structure-borne sound in large structures consisting of thick plates, there are some difficulties in applying the conventional SEA (statistical energy analysis) based on thin plate bending and in-plane waves. This paper examines the applicability of WIA (wave intensity analysis) to the prediction of the vibration energy level in an L-shaped structure assembled by thick plates. The bending wave in plates is thus expressed by Mindlin's theory, taking the shear deformation and rotary inertia into consideration. The new model is introduced to improve the accuracy of wave transmission coefficients for line junctions between thick plates. As a result of comparison of the present method with FEM (finite element method), reasonable results are obtained for the vibration energy level in the L-shaped structure, within approximately 2dB up to 2kHz.

INTRODUCTION

SEA (statistical energy analysis) has been widely used to predict structure-borne sound in buildings made of concrete and brick. As the floors and walls for large structures such as power plants and multi-story car parks are often thicker than those of general buildings, acoustic designers have been faced with severe difficulty in estimating structure-borne sound by means of conventional SEA in which the plates are assumed to be thin. The thin plate is generally defined as the condition that wavelength λ is large compared to the plate thickness and the validity of the thin plate theory is $kh<1.2$ [1], where h is the plate thickness and k is the wave number. Conventional SEA is therefore applicable only to "thin" plate structures, and two specific obstacles must be overcome to estimate the dynamic response of "thick" plate structures. First, the modal density of bending waves for thick plates is much larger than that obtained from the thin plate theory, because the phase speed of bending waves propagating through thick elements is shifted from thin plate bending waves to shear waves within the frequency range of interest. This means that the modal density used in SEA is different for thin and thick plates. Second, the mechanism of vibration energy transmission between thick plates is more complicated than the corresponding mechanism for thin plates.

Several papers dealing with power transmission for general buildings have already been published. Buildings having three floors have been studied using SEA, and good agreement was generally found between the measured and predicted vibration levels [2]. In the high frequency range, however, the coupling loss factors were slightly larger than were measured due to the omission of longitudinal and shear waves. A general method [3] [4] for calculating the coupling loss factors at a rectangular slab junction was described using pure bending and in-plane waves. Springs are also taken into consideration at the connecting line between slabs. In the majority of studies of wave transmission in plate/beam structures, these coefficients [5] have been formulated based on the thin plate theory by means of either pure bending waves only or pure bending and in-plane equations. Accordingly, the coupling loss factors may be inaccurate either in the high frequency range or for thick plate structures. Only a few researchers have dealt with structure-borne sound for thick plates [6][7]. In another approach, FEM (finite element method) has been studied to evaluate the energy transmission of complicated junctions by using local finite elements with wave absorbing boundary conditions [8]. However, this computer-based method requires substantial time and cost.

The objective of the study reported here is to develop a prediction method for structure-borne sound in structures assembled from thick plates. Thus, WIA (wave intensity analysis) [9] has been extended to consider the dynamic effects of thick plates, i.e., the bending waves propagating on the thick plate are expressed by Mindlin's theory [10], taking the shear deformation and rotary inertia into consideration. The wave transmission coefficients between sub-systems used in the wave intensity technique are newly formulated by means of Mindlin's bending and in-plane wave theory.

The modal density of the thick plate element obtained by Mindlin's theory is first compared with exact figures yielded by the three-dimensional wave theory, such that the applicability of Mindlin's theory is demonstrated. In the second example for validating the present method, the vibration levels of an L-shaped assembly consisting of thick concrete plates with and without a beam at the junction are calculated and the results are then compared with those calculated by the conventional SEA and FEM. As a result of comparison of the present method with FEM, reasonable results are obtained for the vibration energy level in the L-shaped structure, within approximately 2dB up to 2kHz. This demonstrates that the wave intensity technique considering Mindlin's bending and assuming a beam at the junction brings forth an improved prediction of structure-borne sound in structures consisting of thick plates.

OUTLINE OF THEORY

In summary, the procedure is explained for determining the vibration energy transmission in an L-shaped structure by means of the wave intensity technique considering Midlin's wave theory instead of the conventional thin plate theory as follows:

(1) The L-shaped assembly is divided into two semi-infinite plates and a beam at the junction shown in Fig.1.
(2) A dynamic stiffness approach [5] is derived for each thick plate and the beam in order to calculate the transmission coefficients. In the dynamic stiffness matrix, the tractions per unit length at the edge of each plate are related to the edge displacements.
(3) A global equation is assembled to apply the appropriate equilibrium and compatibility conditions, after which the transmission coefficients can be calculated.
(4) These coefficients are used to predict the vibration energy in the wave intensity analysis as well as the statistical energy analysis.

The methodology for evaluating the energy transmission is basically the same as that used by McCollum and Cuschieri [6], but in this study a beam is newly considered at the junction instead of a line coupling with two plates directly. This model successfully improves the accuracy of the vibration energy in the next numerical examples.

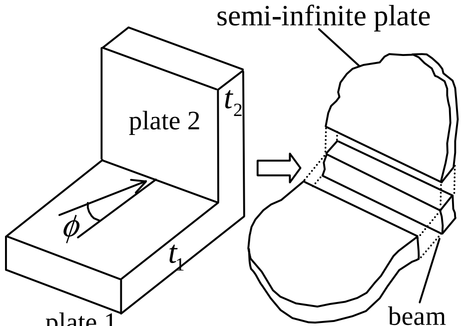


Figure 1 - Model of an L-shaped structure

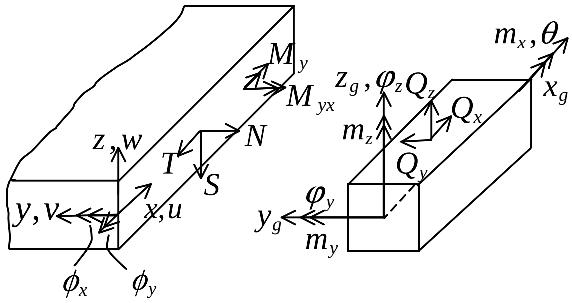


Figure 2 - Coordinate system

Governing equations

Mindlin's theory [10] is deduced from the three-dimensional wave equations in order to consider the effects of rotary inertia and shear deformation in the same manner as for Timoshenko's beam. In Mindlin's bending, three wave equations are represented by the averaged deflection *w* and two rotations, ϕ_x and ϕ_y of plate cross sections about the in-plane coordinates as shown in Fig.2. Mindlin's wave equations can be expressed in the form:

$$\frac{D}{2}\left[(1-\nu)\,\nabla^2\phi_x + (1+\nu)\frac{\partial\Phi}{\partial x}\right] - G'h\left(\phi_x + \frac{\partial w}{\partial x}\right) = \frac{\rho h^3}{12}\frac{\partial^2\phi_x}{\partial t^2} \tag{1}$$

$$\frac{D}{2}\left[(1-\nu)\,\nabla^2\phi_y + (1+\nu)\frac{\partial\Phi}{\partial y}\right] - G'h\left(\phi_y + \frac{\partial w}{\partial y}\right) = \frac{\rho h^3}{12}\frac{\partial^2\phi_y}{\partial t^2} \tag{2}$$

$$G'h\left(\nabla^2 w + \Phi\right) = \rho h\frac{\partial^2 w}{\partial t^2}, \qquad \Phi = \frac{\partial\phi_x}{\partial x} + \frac{\partial\phi_y}{\partial y} \qquad (3), (4)$$

where G' is the corrected shear modulus defined by $G' = \kappa^2 G$, G is the shear modulus, κ is the shear correction factor[10], D is the bending rigidity, ρ is the material density, ν is Poisson's ratio and h is the plate thickness.

The in-plane displacement, u and v, of each plate are defined in the x and y coordinates, respectively in Fig.1. The in-plane equations [5] are described by the in-plane shear and the longitudinal deformations. The displacement and corresponding forces and moments which act at the junction edge of the plane are shown in Fig.2. The beam modelled at the junction between thick plates is characterized not only by the bending, longitudinal and torsion deformations, but also the shear deformation and rotary inertia.

Dynamic stiffness matrix approach

The dynamic stiffness matrix $\mathbf{K}_\ell$ can be derived for a single semi-infinite "thick" plate l on which the transmitted wave propagates from the connected edge as shown in Fig.2.

$$\mathbf{E}_\ell = \mathbf{K}_\ell \mathbf{b}_\ell \tag{5}$$

where $\mathbf{E}_\ell = \begin{bmatrix} T & N & S & M_y & M_{yx} \end{bmatrix}_\ell^T$, $\mathbf{b}_\ell = \begin{bmatrix} u_e & v_e & w_e & \phi_y & \phi_x \end{bmatrix}_\ell^T$.

The total tractions $\mathbf{Q}$ which are applied to the beam by N semi-infinite plates may be expressed in the form

$$\mathbf{Q} = \sum_{\ell=1}^{N} \mathbf{R}_\ell \mathbf{E}_\ell \tag{6}$$

where $\mathbf{R}_\ell$ is the transfer matrix from the local coordinate of each plate to the global one. For the connecting beam, the dynamic stiffness matrix $\mathbf{C}$ may be derived in the same manner as for the plate.

$$\mathbf{Q} = \mathbf{C}\mathbf{a} \tag{7}$$

where $\mathbf{Q} = \begin{bmatrix} Q_x & Q_y & Q_z & m_x & m_y & m_z \end{bmatrix}^T$, $\mathbf{a} = \begin{bmatrix} u & v & w & \theta & \varphi_y & \varphi_z \end{bmatrix}^T$

The global equation can be derived from equations (5)-(7) in the following form:

$$\left\{\mathbf{C} + \sum_{\ell=1}^{N} \mathbf{R}_\ell \mathbf{K}_\ell \mathbf{R}_\ell^{\,T}\right\}\mathbf{a} = \mathbf{R}_m \mathbf{f}_m \tag{8}$$

The right-hand side of equation (8) is an external force by the incident wave propagating on the plate m towards the junction; equation (8) is then solved to yield the displacement $\mathbf{a}_\ell$ of the beam. The displacement $\mathbf{b}_\ell$ of each plate edge is derived by the relation $\mathbf{b}_\ell = \mathbf{R}_\ell^T \mathbf{a}_\ell$. Finally, the transmission coefficients are calculated as the ratio of the power transmitted by the generated wave to the total power by the incident wave.

Wave intensity analysis

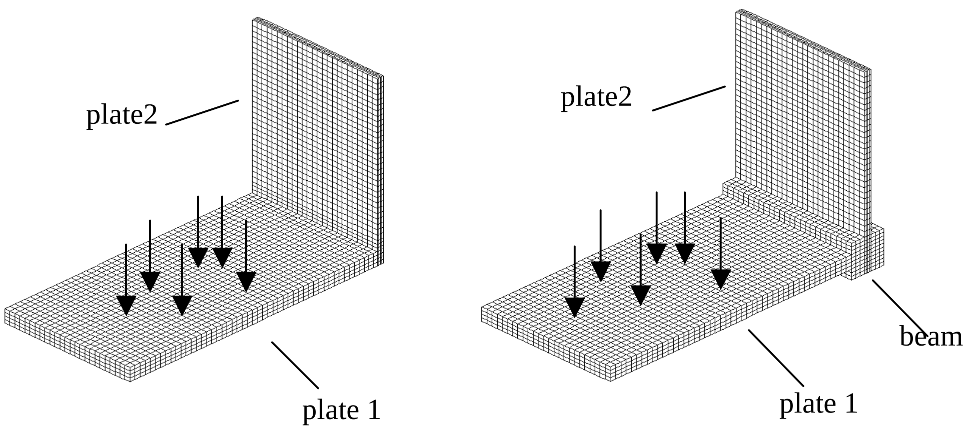
Wave intensity analysis [9] is based on the conventional SEA assumptions considering homogeneous reverberant wave fields. However, in contrast to SEA, the wave field in each sub-system is decomposed into wave intensity components with directional dependency. This allows the energy densities to be represented in the form of a Fourier series. For the purpose of applying the wave intensity technique to the prediction of dynamic response of "thick" plate structures, the modal density and the transmission coefficients should be introduced by assumption based on Mindlin's wave theory.

NUMERICAL SIMULATIONS

The calculations were carried out for two L-shaped structures made of concrete with Young's modulus E=2.3$\times 10^{10}$Pa, density ρ=2,300kg/m^2, Poisson ratio ν=0.22, loss factor η=0.01 and width B=4m in Fig.1. The dimensions of the plates are shown in Table 1. The model of case-1 is shown in Fig.3 (a) and case-2 shown in Fig.3 (b) by adding the beam at the junction of case-1. The cross section of the beam is square, size 1.0m$\times$1.0m.

Table 1 Plate dimensions

Plate	L(m)	B(m)	t(m)
1	8	4	0.4
2	5	4	0.2



(a) case-1 (8,600 solid elements) (b) case-2 (10,100 solid elements)

Figure 3 - Finite element model of an L-shaped structure

Finite element analysis

The performance of the present technique is assessed by comparison with the finite element method in which the thick plates are divided into extremely fine meshes with solid elements to express an appropriate wave field in the high frequency of interest. The model is shown in Fig.3. The first plate is excited by point loads, which are chosen at random to achieve delta-correlated random loading. The time and space averaged vibration energy of each plate is thus obtained from the out-of-plane deformation.

Modal density

The modal density of out-of-plane waves for the thick plate (L=7.9m, B=4.0m, t=0.4m) using Mindlin's theory is compared with that obtained by the three-dimensional wave theory to investigate the applicability of Mindlin's wave. As shown in Fig.4, the number of modes from Mindlin's theory corresponds to the exact results in almost all frequency bands. The results of the thin plate theory, however, are seen to underestimate the number of modes, and the discrepancy from the exact results increases with frequency. The total number of modes consisting of out-of and in-plane waves are also compared with that obtained by finite element analysis shown in Fig.5. It can be concluded from a modal density perspective that Mindlin's theory is capable of evaluating the number of modes for thick plates up to 4kHz.

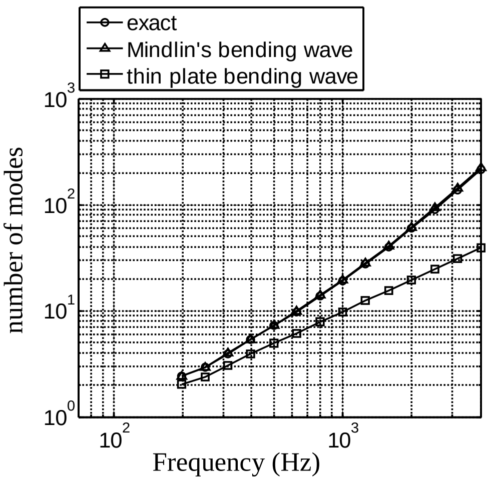


Figure 4 Number of modes of thick plate (L=7.9m, B=4.0m, t=0.4m)

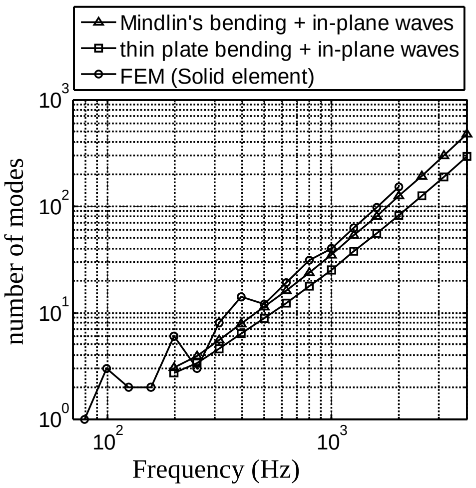


Figure 5 Comparison of the total number of modes of thick plate (L=7.9m, B=4.0m, t=0.4m)

Comparison of energy level for L-shaped plates

The energy levels are compared using the "thick plate" wave intensity technique with finite element analysis and statistical energy analysis for thick L-shaped assemblies.

The assumptions of simulation are shown in Table 2. The ratios of vibration energies are calculated between the second plate and the excited one, up to 2kHz. In approximate analyses such as WIA and SEA, the input power is injected into the out-of plane waves, namely bending waves on the first plate.

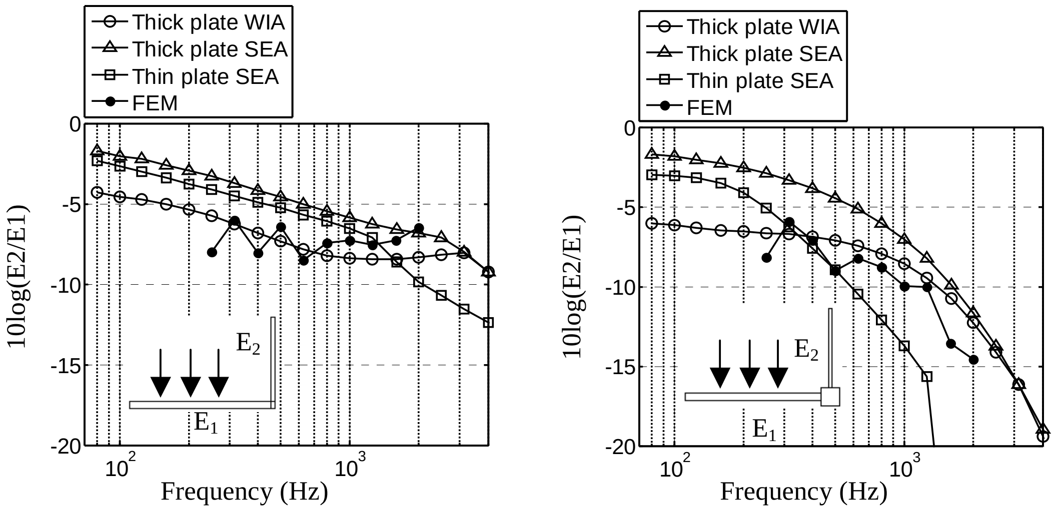
Table 2 Assumption of simulation

	Modal Density	Transmission coefficient	Wave field
Conventional SEA	Thin plate theory		Diffuse
Thick plate SEA	Mindlin's theory		
Thick plate WIA			Directional dependency

The calculations have been conducted for a "thick" L-shaped structure by means of wave intensity analysis, both with and without a beam as the junction model. The discrepancies in the energy ratios between FEM and both thin and thick SEA increase with frequency shown in Fig.6 (a). The trend for FEM energy ratios is constant in almost all frequency bands, whereas for SEA the attenuation increases approximately 2dB/Oct as the frequency increases. In the case of "thick" WIA, the energy ratios are in agreement with finite element analysis within approximately 2dB up to 2kHz.

As shown in Fig.6 (b), the attenuation of case 2 increases approximately 3dB in comparison with case 1 at 1kHz, because of the existence of the beam at the junction. The energy ratios are also in agreement with WIA and finite element analysis within approximately 2dB up to 2kHz.

It is thus established that the WIA technique with the modelling at the junction as presented here is applicable to energy transmission for the "thick" L-shaped structures.



(a) case-1 (b) case-2

Figure 6 - Energy ratio E_2/E_1 for an L-shaped structure

CONCLUSIONS

Wave intensity analysis has been applied in predicting vibration response of L-shaped structures consisting of thick plates in the broad band of audible frequencies. Calculations based on transmission coefficients obtained from Mindlin's theory have verified that accurate response predictions can be made to compare the preset analysis with the finite element method. The attenuation of vibration energy for the L-shaped plates with an inserted beam was found to be 8dB at 1kHz by the present method. Predictions made using conventional statistical energy analysis were found to overestimate the attenuation by 14dB. This deviation is caused by the limits of applicability of the thin plate theory for out-of-plane waves. Further development of this technique considering multi-element systems is the subject of an ongoing study.

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