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IDENTIFICATION OF STRUCTURAL PARAMETERS FOR ACTIVE VIBRATION CONTROL

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Abstract

The study of algorithms for active vibration control in flexible structures became an area of enormous interest for some researchers due to the innumerable requirements for better performance in mechanical systems, as for instance, aircrafts and aerospace structures. Intelligent systems, constituted for a base structure with sensors and actuators connected, are capable to guarantee the demanded conditions, through the application of diverse types of controllers. For the project of active controllers it is necessary, in general, to know a mathematical model that enable the representation in the space of states, preferential in modal coordinates to permit the truncation of the system and reduction in the order of the controllers. For practical applications of engineering, some mathematical models based in discrete-time systems cannot represent the physical problem, therefore, techniques of identification of system parameters must be used. The techniques of identification of parameters determine the unknown values through the manipulation of the input (disturbance) and output (response) signals of the system. Recently, some methods have been proposed to solve identification problems although, none of them can be considered as being universally appropriate to all the situations. This paper is addressed to an application of linear quadratic regulator controller in a structure where the damping, stiffness and mass matrices were identified through Chebyshev's polynomial functions.

INTRODUCTION

Vibration attenuation is an important goal in many engineering applications, particularly in aerospace industry. Active vibration control (AVC) in distributed structures is of practical interest because of the demanding requirement for

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guaranteed stability. This is particularly important in light structures, since, they have low degree of internal damping [1].

For control design is necessary, in general, to know the motion equation, therefore, this paper uses the technique of parameters and exciting forces identification. This technique provokes a known variation in some parameters of the structure. Thus, just measuring the signals before and after of the known parameters variation, the forces as well as the structural parameters can be identified.

The approach is based in two signal acquisitions: in the first one the signal is acquired from the original system and the second one the signal is obtained from the system with known variation in the parameters of the structure.

STRUCTURAL MODELLING

The dynamical behaviour of a structure can be described in terms of mass, stiffness and damping matrices, and displacement and velocity vectors as

$$\begin{aligned} &\ddot{\mathbf{q}}(t) + \mathbf{M}^{-1}\mathbf{D}\dot{\mathbf{q}}(t) + \mathbf{M}^{-1}\mathbf{K}\mathbf{q}(t) = \mathbf{M}^{-1}\mathbf{B}_0\mathbf{u}(t) \\ &\mathbf{y}(t) = \mathbf{C_{oq}}\mathbf{q}(t) + \mathbf{C_{ov}}\dot{\mathbf{q}}(t) \end{aligned} \qquad (1a,b)$$

where $\mathbf{q}(t)$ is the n-length displacement vector, $\mathbf{u}(t)$ is the s-length input vector, $\mathbf{y}(t)$ is r-length output vector, $\mathbf{M}$ is the n x n mass matrix, $\mathbf{D}$ is the n x n damping matrix, and $\mathbf{K}$ is the n x n stiffness matrix. $\mathbf{B}_0$ is the n x s input matrix, $\mathbf{C_{oq}}$ and the r x n output displacement matrix, and $\mathbf{C_{ov}}$ is the r x n output velocity matrix. The mass matrix is positive definite, and the stiffness and damping matrices are positive semi-definite, n is the number of degrees of freedom of the system (linearly independent coordinates describing the finite-dimensional structure), r is the number of outputs and s is the number of inputs.

Considering a linear time-invariant system, the space-state equations can be written in a vector-matrix format through the triple (**A**, **B**, **C**); it allows the equations to be manipulated more easily.

$$\begin{aligned} &\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ &\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \end{aligned} \qquad (2)$$

The related matrices are given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{B}_0 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_{\mathrm{oq}} & \mathbf{C}_{\mathrm{ov}} \end{bmatrix} \qquad (3a,b,c)$$

PARAMETERS IDENTIFICATION USING LEGENDRE POLYNOMIALS

In classical techniques of parameters identification, the identification of systems in permanent regimen was only possible from the previous knowledge of the forces that excited the same. However, in this work presents the development of a technique of parameters and exciting forces identification in systems with multi-degree-of-freedom, only considering the response signals of the system.

The technique developed in this work is based on the transformation of the system of differentials equations, that conducts the dynamic behavior of the mechanical systems, in a system of linear equations, whose resolution is much more simple. This transformation is made with the use of functions of orthogonal bases such as: the Chebyshev's Series.

Table 1 –Chebyshev's Series

Recursive formula in the interval $t \in [0, t_f]$	Operational matrix of integration
$(n+1)p_{n+1}(t) = (2n+1)\left(\frac{2t}{t_f}-1\right)p_n(t) - np_{n-1}(t)$,$n = 1, 2, 3, \ldots, r-1$ $p_0(t) = 1$ $p_1(t) = 2t/t_f - 1$	$\mathbf{P} = \frac{t_f}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ -\frac{1}{3} & 0 & \frac{1}{3} & 0 & \cdots & 0 \\ 0 & -\frac{1}{5} & 0 & \frac{1}{5} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{-1}{2r-3} & 0 & \frac{1}{2r-3} \\ 0 & 0 & \cdots & 0 & \frac{-1}{2r-1} & 0 \end{bmatrix}$

r = number of truncated terms

The following property, related to the successive integration of the vectorial bases, holds for a set of r orthonormal functions in the interval [0,*t*]:

$$\underbrace{\int_0^t \cdots \int_0^t}_{n\,vezes} \boldsymbol{\varphi}_\mathbf{m}(\tau)d\tau \cong \mathbf{P}^n \boldsymbol{\varphi}_\mathbf{m}(t) \tag{4}$$

where $P \in \Re^{r,r}$ is a square matrix with constant elements, called operational matrix and $\boldsymbol{\varphi}_\mathbf{m}(t) = \{\phi_0(t)\ \phi_1(t)\ \ldots\ \phi_r(t)\}^T$ is the vectorial bases of the orthonormal series. In fact, if a complete vectorial base is regarded, that is, if the series are not truncated, the relation obtained in equation (4) is equality. However, in the practice, it becomes not suitable, due to the high order of the matrix **P**.

The proposed identification method can exploit either free or forced time domain responses, in terms of displacements, velocities or accelerations. Since the formulations for these three types of responses are quite similar, only the formulation for forced systems, in terms of displacements, will be presented in this work.

The development of the method starts from the equation (1). This equation is then integrated twice in the interval [0,*t*]. Expanding the signals of displacement and excitation forces and applying the integral property given by equation (4), the following system of algebraic equations is obtained [2]:

$$[\mathbf{M} \quad -\mathbf{M}\mathbf{q}(0) \quad \{-\mathbf{M}\dot{\mathbf{q}}(0) - \mathbf{C}\mathbf{q}(0)\} \quad \mathbf{C} \quad \mathbf{K}] \begin{bmatrix} \mathbf{X} \\ \mathbf{e}^T \\ \mathbf{e}^T\mathbf{P} \\ \mathbf{XP} \\ \mathbf{XP}^2 \end{bmatrix} = \mathbf{FP}^2 \tag{5}$$

where $\mathbf{e} \in \Re^{r,1}$ is a constant vector whose value depends of the orthogonal function, $\mathbf{X} \in \Re^{N,r}$ is the matrix of the coefficients of expansion $\mathbf{x}(t)$, $\mathbf{F} \in \Re^{N,r}$ is the matrix of the coefficients of expansion $\mathbf{f}(t)$

Admitting an induced known variation in some structural parameters of the system, the equation (5) can be recast as

$$[\mathbf{M_M} \quad -\mathbf{M_M}\mathbf{q_M}(0) \quad \{-\mathbf{M_M}\,\dot{\mathbf{q}}_\mathbf{M}(0) - \mathbf{C_M}\mathbf{q_M}(0)\} \quad \mathbf{C_M} \quad \mathbf{K_M}] \begin{bmatrix} \mathbf{X_M} \\ \mathbf{e}^T \\ \mathbf{e}^T\mathbf{P} \\ \mathbf{X_M P} \\ \mathbf{X_M P}^2 \end{bmatrix} = \mathbf{FP}^2 \tag{6}$$

where $\mathbf{M_M}$, $\mathbf{K_M}$ and $\mathbf{C_M}$ are the inertia, stiffness and damping matrices respectively after some known modification in structural parameters, and $\mathbf{X_M}$ are the expansion coefficients of the output of the modified system. If it is considered that the input forces of the system do not change when the parameter variations are induced, then one can equal equations (5) and (6). After some mathematical manipulations the equations are represented by [3]:

$$[\mathbf{M_M} \quad 0 \;\; 0 \quad \mathbf{C_M} \quad \mathbf{K_M}] = [\Delta\mathbf{M} \quad 0 \quad 0 \quad \Delta\mathbf{C} \quad \Delta\mathbf{K}] \begin{bmatrix} \mathbf{X} \\ \mathbf{e}^T \\ \mathbf{e}^T\mathbf{P} \\ \mathbf{XP} \\ \mathbf{XP}^2 \end{bmatrix} \left\{ \begin{bmatrix} \mathbf{X} \\ \mathbf{e}^T \\ \mathbf{e}^T\mathbf{P} \\ \mathbf{XP} \\ \mathbf{XP}^2 \end{bmatrix} - \begin{bmatrix} \mathbf{X_M} \\ \mathbf{e}^T \\ \mathbf{e}^T\mathbf{P} \\ \mathbf{X_M P} \\ \mathbf{X_M P}^2 \end{bmatrix} \right\}^{-1} \tag{7}$$

where $\Delta\mathbf{M}$, $\Delta\mathbf{C}$ and $\Delta\mathbf{K}$ are, respectively, the known variations in the mass, in the damping and in the stiffness of the system.

LINEAR QUADRATIC REGULATOR CONTROL

The first step in the LQR control design process is the specification of a performance index, which can be defined by a quadratic cost function in the state and control variables

$$J = \int_0^{\infty} \left(\mathbf{u}^T \mathbf{R} \mathbf{u} + \rho \mathbf{x}^T \mathbf{Q} \mathbf{x} \right) dt \tag{8}$$

where **Q** is a symmetric and positive semi-definite weighting matrix on the states, **R** is a symmetric and positive definite weighting matrix on the controller outputs, and ρ is a weighting design parameter relative to state variables.

Considering the linear time-invariant system, described by equation (2)-(3), the control law is given by

$$\mathbf{u}(t) = -\mathbf{G}\mathbf{x}(t) \tag{9}$$

which minimizes J. If the regulator design is restricted to time-invariant control laws, **G** will be a constant coefficient matrix and **u** will be a linear combination of the states. It is assumed in the regulator design that all of the states are measured. It can be shown that the gain matrix **G** which minimizes the performance index is given by [4]

$$\mathbf{G} = \frac{1}{\rho} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{S} \tag{10}$$

where **S** is the solution of the steady-state matrix Riccati equation

$$-\dot{\mathbf{S}} = 0 = \mathbf{S}\mathbf{A} + \mathbf{A}^T \mathbf{S} - (1/\rho)\mathbf{S}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T \mathbf{S} + \mathbf{Q} \tag{11}$$

NUMERICAL APPLICATION

To verify the proposed methodology, a robotic arm, as shown on fig. 1, was considered. The system was used in Valer (1999) for design of the state observer [5]. The properties of the system are given in table 2. A disturbance how sine wave with frequency of the 30 rad/s and amplitude of the 12 N was considered as disturbance.

Table 2 – Properties of the system.

I	0.4 kg.m^2
K	1 N.m/rad
J_M	0.0424 kg.m^2
B_M	0.0138 N.m.s/rad
K_e	0.0403 N.m/v

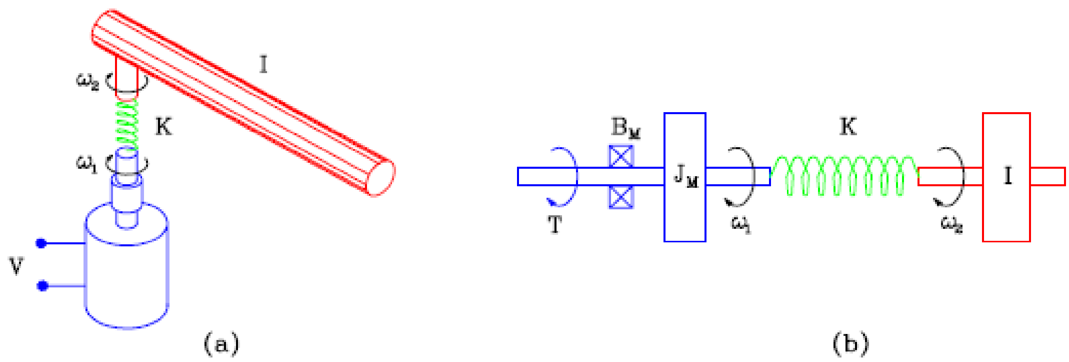


Figure 1 – Flexible arm of a robot.

Initially, the signal was obtained from the original system, and in a second phase the signal was obtained from the system with known variation in the parameters (0.01 kg.m^2 in I and 0.01 kg.m^2 in J_M). Figure 2a and 2b show the angular displacement of sensors 1 and 2 for the original and modified system.

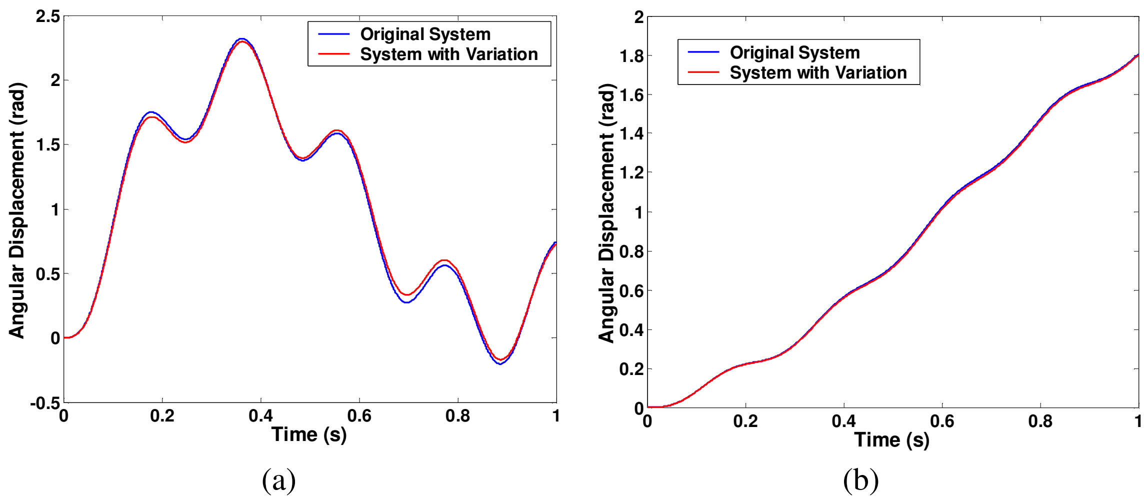


Figure 2 – Angular displacement of the sensors 1 and 2 for original system and system with variation.

The attenuation of flexible motion of the structure depends on the pole mobility on the left-hand side of the complex plane. Therefore, if a particular pair of poles is moved easily, the respective states are easy to control and estimate. On the other hand, if a particular pair of poles is difficult to move, the respective states are difficult to control and estimate. Figure 3 shows the pole location of the first two modes of both systems.

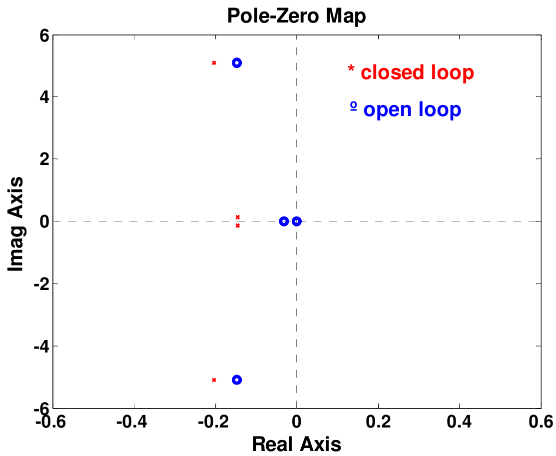
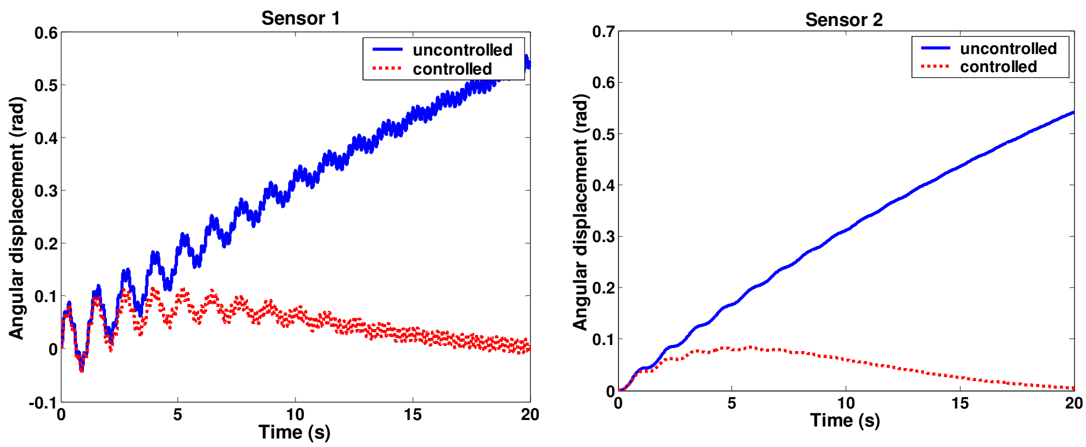


Figure 3 – Structural poles location of the flexible arm of a robot.

The response of the system with unknown input force was determined using the proposed identification procedure. The angular displacement is measured by sensor 1 and can be seeing in the time domain in Fig. 4a, Figure 4b shows the angular displacement measured by sensor 2.



(a)

(b).

Figure 4 – Angular displacement of controlled and uncontrolled system, (a): sensor 1 and (b): sensor 2.

FINAL REMARKS

An LQR feedback control strategy was used to actively control the robotic arm with two degrees of freedom. The system was described using the space state realization considering the dynamic equation of second order.

To obtain the motion equation was used a methodology of structural parameters identification by Chebyshev's polynomial function. In this technique, it is not necessary to know the exciting forces, but only a variation in some strucutral parameter. So, measuring the time response of the system before and after the known parameters variation, the forces as well as the structural parameters can be identified.

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