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TRANSVERSE MODES OF VIBRATION IN EUROPEAN CHURCH BELLS

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Abstract

The contribution of axial stiffness to the partial frequencies of radial modes of cylindrical and bell-like forms is discussed. Transverse axial modes in unconstrained church bells are reported and their behavior is correlated to radial mode frequencies as a measure of the contribution of axial stiffness to the behavior of these modes for changes in geometry.

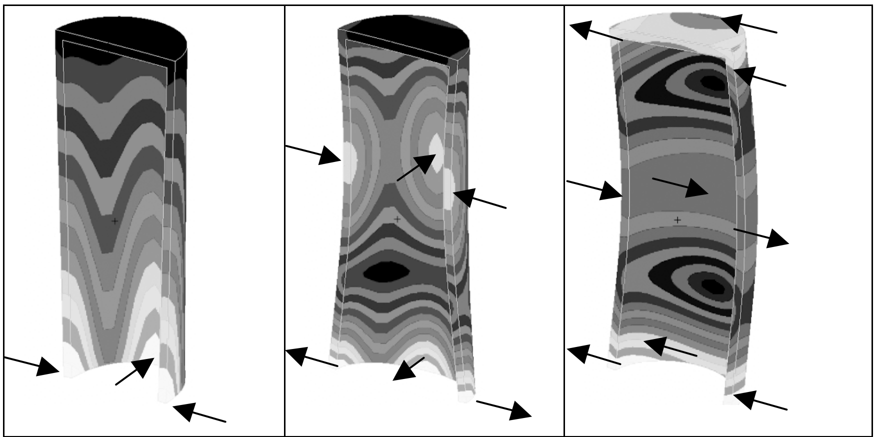
INTRODUCTION

It is common to use numerical methods such as Finite Element Analysis (FEA) to predict the behaviour of bells [1,5] and a number of 'major third' carillons have been designed using Finite Element Analysis (FEA) in the Netherlands [8]. Numerical optimization was applied to the bell geometry starting from a typical European carillon bell geometry. The frequency ratios of the first five, radiating, partial tones in carillon bells are usually 1; 2; 2.4; 3 and 4. Thus, the European bell could be tuned to a series of partials that included the first four harmonics. However the presence of the partial at 2.4 times the fundamental (a minor third musical interval) is likely to create complex acoustical percepts of pitch and dissonance that change in time as partial frequencies decay at varying rates [4]. In the 'major third' bells, the partial normally found at 2.4 times the fundamental frequency is tuned to 2.5. This was believed to be an advantage in limiting the dissonance caused by this partial in most musical contexts where major third intervals predominate.

Bells with up to the first 7 overtones tuned to the harmonic series have been designed and shown to produce sounds with clearer pitch percepts than European church or carillon bells [4]. Bells have also been designed to produce multiple perceived pitches due to tuning of their inharmonic overtones to include subsets of more than one harmonic series [3]. To design these bells it was essential to first study the behaviour of radiating modes for changes in a series of geometric parameters that defined bell-like geometries. Specifically it was important to understand how variations in mass and stiffness of a bell would affect the frequencies of the radiating modes. A study of these effects using general shape parameters so as to include a wide range of possible design variants was first undertaken and is described here. The results of this study then provide a new context in which to understand the modal behavior of European church bells.

RESULTS

The principle radiating vibratory modes of cylinders consist of modes in which the direction of motion of the cylinder wall is in the direction of the radius, and modes in which the direction of motion is transverse to the length of the cylinder [2]. In classifying radial modes it is common to use the wavenumbers m and n, where m gives the number of circumferential waves and n give the number of axial waves. The finite element model of ½ a capped cylinder in Figure 2 shows that 4 maxima and 4 minima of amplitude will occur around the open rim of the complete cylinder for m=2 modes, and one nodal line will appear as a ring of amplitude minima when n=1. The transverse mode is described by the number of nodal lines and the descriptor “t”.



2,0 radial mode	2,1 radial mode	t2 transverse mode

Figure 1. Vibratory modes of cylinders from FEA prediction of mode shape for an unconstrained capped cylinder (1/2 model). Arrows show the direction of recent motion.

FEA predicted results for radial and transverse modes are reported below for changing lengths, circumferences and cone angles of capped cylinders and cones. Nodal lines for transverse modes lie between two points diametrically opposite each other on the circumference of the cylinder. The location of the nodal lines for the t2 mode can be observed in Figure 1 at the centers of the concentric contours of amplitude. Figure 2 shows the variation in frequency of a series of modes due to changes in length of a 240 mm circumference capped cylinder, changes in circumference of a 200mm long capped cylinder, and changes in cone angle of a 200mm long by 240mm circumference capped cylinder. The cone angles were created by rotating the cylinder wall about its center point so that the average circumference remained approximately constant. The length of the wall was extended to intersect with the cap, which was then trimmed as shown in Figure 3d. The wall thickness was 10mm.

For variation of length, circumference and cone angle up to 20^0, changes in frequency of the 2,2 mode are very closely related to the sum of the frequency changes of the 2,0 and t2 modes. Assuming mass and circumferential stiffness changes for different geometries are the same for the 2,0 and 2,2 modes, this indicates that despite the difference in mode shape, changes in mass and bending stiffness in the axial direction has similar effects on the frequency of the 2,2 and t2 modes. For example, increasing the circumference increases the axial bending stiffness and hence the frequency of the t2 mode, but decreases the circumferential stiffness and hence decreases the frequency of the m,0 modes. These two effects negate each other to varying degrees in various higher order radial modes where n>0. For the 2,2 mode the effect of the changing stiffness in both directions is close to the sum of the effects on the t2 and 2,0 modes.

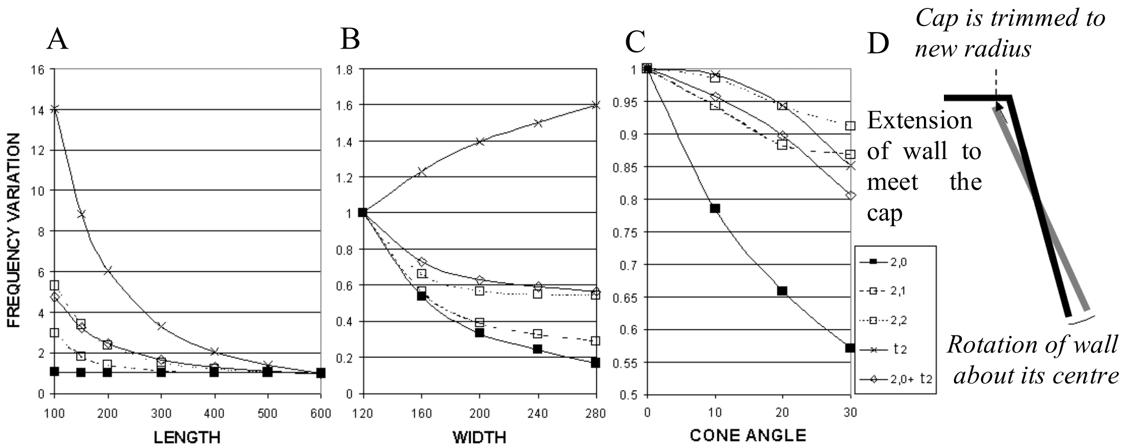


Figure 2 a) Calculated frequency variation of modes due to decreasing length of a capped cylinder, b) increasing circumference and c) increasing cone angle. d) The method of changing cone angles without increasing the average circumference.

Transverse modes in church bells

Transverse modes of vibration were not predicted or detected in the most extensive study of the vibration of church bells with unconstrained boundary conditions available in the literature by Perrin et al [5]. Researchers experienced in thin shell theory would not expect the presence of transverse modes in bells, as they are not usually described by shell theories except where the length to radius ratio is high [2]. Furthermore in most modal analyses bells are constrained at their crown, making transverse modes unlikely to occur [7]. Various extensions of bar theory have been used to describe these modes in pipes [2] and have been applied to tubular bells [1]. However the present data shown in Figure 5 indicates that these modes do appear amongst the low order partials in the relatively thick walled forms of bells.

In European bells, the modes described above as m,0 have been shown to exhibit small regions of anti-phase motion in the upper portion of the bell wall for m>2, leading to a new classification for just these modes. Whether m,0 modes with m>2 occur in a bell depends on its design. These modes do exist in Asian bells and modern bells of new designs [1,4]. They may also exist in some church bells, as their profiles vary widely and it can be difficult to be sure that small antiphase components of motion for these modes are not present. Since the m,0 modes defined in this paper behave consistently as a group across a wide variety of geometrical forms including church bells, it seems expedient to include them in one group, whilst noting where relevant that m,0 modes for m>2 may exhibit this behavior.

In their study employing physical measurement of vibration with FEA, Perrin et al proposed a theory to explain the behavior of modes by dividing them into "shell driven" and "ring driven" mechanisms. The m,0 modes are described by Perrin et al as "ring driven" as they are supposed to be driven by the relatively large stiffness of the thick rim of the bell. They are shown in Figure 3 as the 2,0, 3,0 and 4,0 modes. The remainder of the modes exhibiting radial motion is proposed by Perrin et al to be "shell driven". These modes have relatively low amplitudes in the rim of the bell, which in Perrin's theory was due to its greater thickness and mass. Chladni's law for thin plates modified for non-flat circular plates was fitted to the data obtained by Perrin et al for sets of modes described as either shell or ring driven, or including or excluding significant amplitudes in the shoulder of the bell [6].

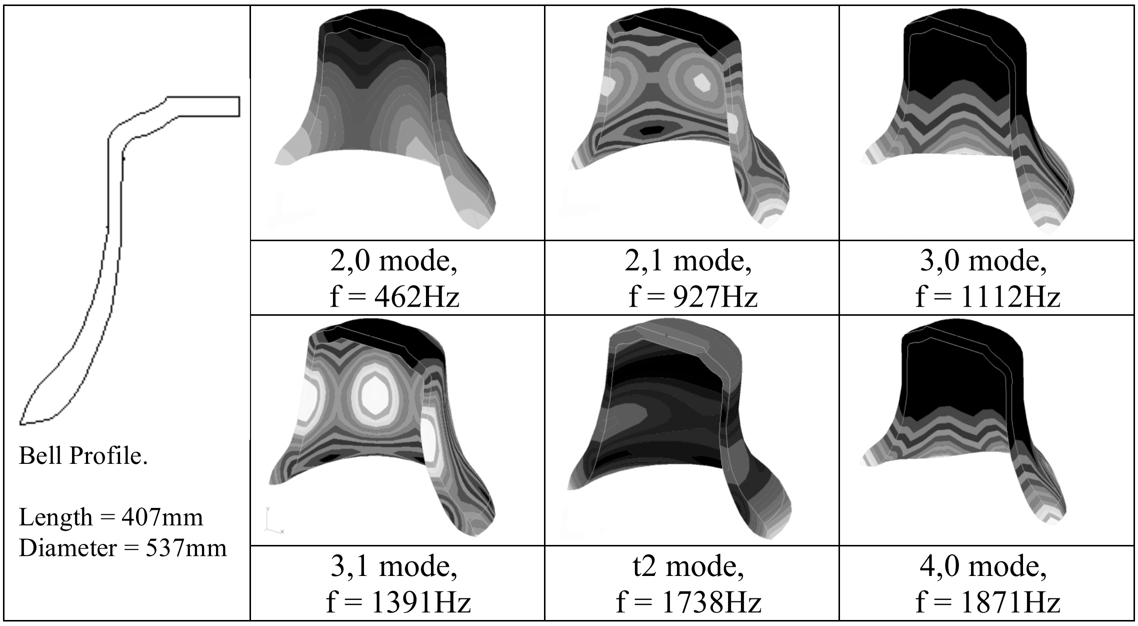


Figure 3. FEA plots of modal amplitude for an unconstrained model of a church bell.

The transverse t2 mode shape for the bell shown in Figure 3 is somewhat distorted compared to that shown in Figure 1 for a capped cylinder due to the thickness of the rim and complexity of the bell wall shape. The mode type can be more clearly identified in animated representations of the vibratory motion that cannot be presented here. Their presence at relatively low frequency ratios to the fundamental mode compared to cylinders and cones with similar length to width ratios is be due to bells having large masses at either end (the rim and crown) and relatively thin walled center portions where the elastic deformation is most acute.

Figure 4 shows FEA calculated amplitudes for a constant wall thickness cone with equal ratios of length and diameters at the open and closed ends to the church bell shown above. Very similar mode shapes as found in the church bell can be observed. This indicates that the proximity of the nodal ring to the rim for the 2,1 mode is not due to its increased thickness in church bells as suggested by Perrin et al, but to decreased stiffness in the open end of the form. A number of other mode types such as 1,m modes, axisymmetric bending modes in which the motion is along the axis of symmetry (i.e. gong like modes), extensional and torsional modes are also observed in the FEA models of bells and cones. These have been well described in Perrin et al and will not be discussed here.

Figure 5 shows the changes in FEA predicted frequencies for the first 6 modes of vibration of this bell when scaled in length and diameter. The overall diameter of the bell was changed by increasing the diameter at the head and leaving the profile otherwise unaltered to prevent large changes in rim thickness. As would be expected variation in length has much less effect on the m,0 radial modes than on the higher order radial modes where n>0 and the transverse mode, while variation in circumference has much less effect on the transverse mode and higher order radial modes where n>0 than the m,0 radial modes.

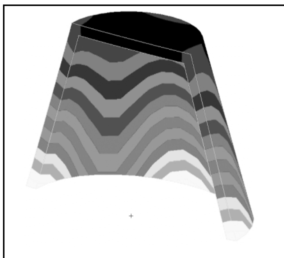
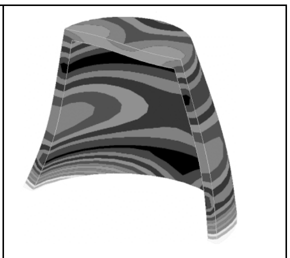
			
2,0 mode	2,1 mode	2,2 mode	t2 mode

Figure 4. FEA plots of modal amplitude for selected modes of an unconstrained model of a constant wall thickness cone approximating the church bell profile shown above.

As the length of the bell increased the t2 mode dropped in frequency as would be expected from analytical theories for cylinders. The higher order radial modes where n>0 exhibited similar behavior to the t2 mode with increased sensitivity to changes in length for the 2,1 and 2,2 modes respectively, and decreased sensitivity for the 3,1 mode. The m,0 radial modes showed little sensitivity to changes in length.

As the circumference of the bell increased the m,0 mode frequencies decreased, as would be expected from analytical theories and the FEA results for the capped cylinder shown above. Unlike the data for a cylinder, the t2 mode dropped slightly in frequency, suggesting that the increasing mass of the rim was having a larger effect on its frequency than increasing axial stiffness. The 2,1 and 2,2 mode behaviors correlated closely to the behavior of the t2 mode, whereas the 3,1 mode behavior correlated closely to the 2,0 and 3,0 modes.

Overall these results show that the behaviors of higher order radial modes where n>0 correlated more closely with purely transverse modal behavior as the number of nodal rings increased, and more closely with purely radial modal behavior as the number of nodal lines increased. The behavior of higher order radial modes where n>0 and m=2 was far more closely correlated to transverse modal behavior than radial modal behavior, suggesting that changes in bending stiffness in the axial direction was more influential than changes in radial bending stiffness.

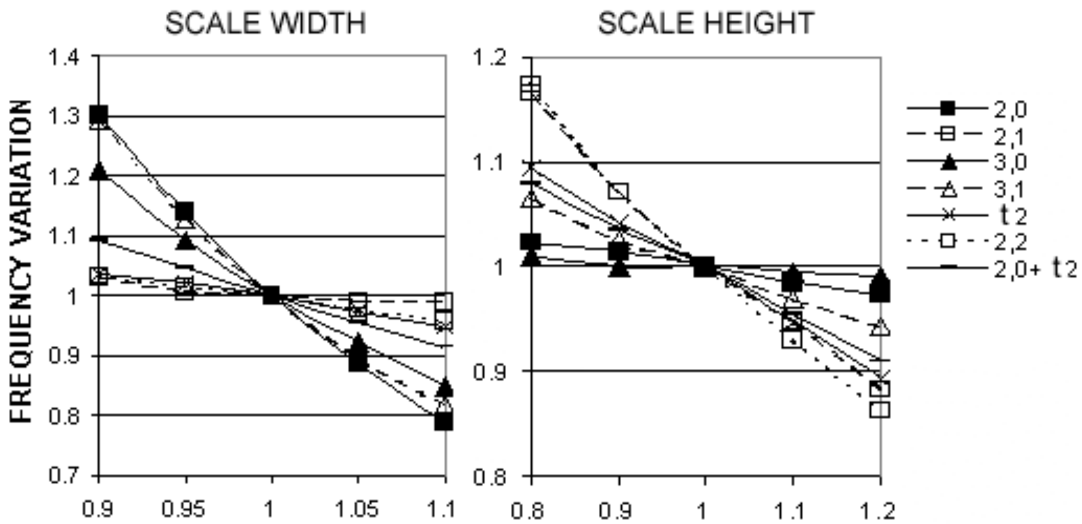


Figure 5. a) Variation of calculated frequency for various modes of a typical European church bell due to changing length and b) due to changing circumference.

Spectra of unconstrained church bell

Figure 6 shows spectra of a typical church bell with the profile shown above and a mass of about 60kg. The bell was hung by a short length of rope so as to allow the t2 mode to vibrate when the bell was struck by a 1kg steel hammer. Figure 6 also shows a spectrum for the same bell constrained at the head. The recordings were taken at a distance of 1 m from the surface perpendicular to the bell axis and in plane with the rim in a small room (volume approximately 40 m^3 and RT(60) in the 500Hz octave band = 0.68 s) with reflective floor and walls to ensure a broad array of acoustic transmission paths without excessive reverberation or resonance. The mallet velocity was such to produce an 'A weighted' sound pressure level of around 95 dB (fast response) in the early part of the sound. The FFT was produced at 50 milliseconds after the sound onset by using a Hamming window of 4,096 samples for a sample rate of 44.1kHz.

A large peak centered around 1,727Hz and a frequency ratio of 3.75 to the lowest frequency partial can be seen in both spectra where the head is free to move, but is absent when the head is constrained. This is very close to the predicted frequency ratio for the t2 mode calculated for a FE model of the bell. As would be expected from the FE modal amplitudes this peak does not diminish in amplitude when the bell is struck at the shoulder, whereas the predominantly radial modes at 461, 1107 and 1859Hz do substantially diminish.

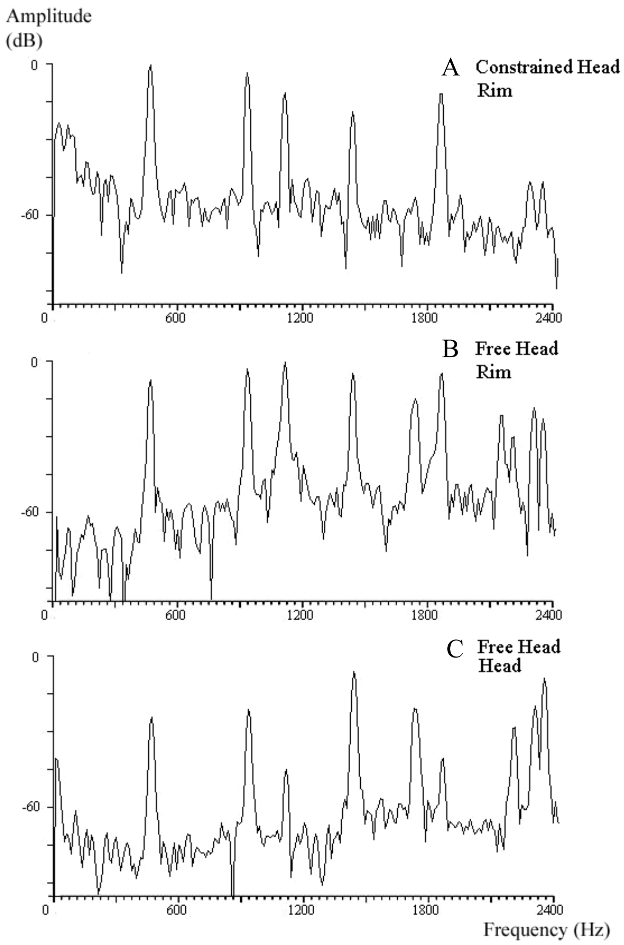


Figure 6. A) Church bell spectra for strike at rim and constrained head, B) strike at rim and free head, and C) strike at head and free head.

CONCLUSION

Despite the t2 mode asymmetry about the cylindrical axis, changes in its frequency could be closely correlated to changes in the frequency of the 2,2 radial mode for changing cylindrical and conical geometry. This indicates that changes in mass and axial stiffness due to the changing geometry had similar effects on both modes. The correlation was stronger for the 2,2 mode than the 2,1 mode as the axial wavelengths of the 2,2 and t2 modes are similar.

The t2 mode was discovered amongst the low order modes for unconstrained church bells. The behavior of the vibratory modes of bells for varying geometry may be understood to be due to changing contributions of transverse and radial stiffness in combination with mass effects. This understanding can assist bell designers to make informed decisions on changes to bell profiles when attempting to design a bell with a specific tuning using FEA and shape optimization algorithms. In particular this understanding could be applied to alter the frequencies of higher order radial modes where $n>0$ without substantially altering the frequencies of m,0 modes.

It is possible for the t2 mode to align with vibratory frequencies in the bell's supporting structure to cause the radiation of an un-tuned resonance of the combined mechanical system. This highlights the importance of well-designed isolation systems and well arranged modal separation strategies for bells and their supporting structures.

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