ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

NEW APPROXIMATIONS OF STRUCTURE-EXTERNAL ACOUSTIC INTERACTIONS, PART II : MODEL EVALUATION

Moonseok Lee*[1], Youn-Sik Park[1], and K. C. Park[2]

[1]Center for Noise and Vibration Control, Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Daejeon, Korea 305-701.
[2]Department of Aerospace Engineering Sciences, Center for Aerospace Structures, University of Colorado, Boulder, CO 80309-0429, USA,
Email:kcpark@colorado.edu.

Abstract
A second-order coupled acoustic pressure model interacting with flexible structures derived in the companion paper, Part I, is evaluated for its fidelity. For evaluation purposes, an isotropic elastic spherical shell that is subject to a plane step wave and impulse velocities distributed over the shell surface is employed as this problem has its exact mode-by-mode solutions in terms of the Legendre functions. The mode-by-mode characteristic roots of the coupled systems are computed for the exact solutions, Geers' Doubly Asymptotic Approximation (DAA_2) model and the present second-order model. The results show that: (1) the present second-order model captures the dominant acoustic scattering pressure modes; (2) the DAA_2 model fails to satisfy the important initial impulse responses; (3) the model parameter can be adjusted to capture certain desirable pressure modes more accurately depending on the problem at hand; (4) hence, the present second-order model is recommended for use in inverse problems for identifying the sound sources, in addition to the computations of the structural responses subject to external acoustic shocks and explosions.

INTRODUCTION

In the companion paper, Part I[7], new approximate models of external acoustic pressure field interacting with flexible structures have been derived. Therein, the models have been derived by a linear combination of the retarded and advanced potentials, which has heretofore not been tried. The approximate models derived therein have shown that it is consistent with respect to general admissible initial conditions such as impulse incident wave and other types of incident pressure conditions. This early-time consistency condition is considered an important property for inverse identification applications, in addition to transient acoustic-structure interaction simulations, because accurate determination of impulse response functions in time or frequency domain identification methods is critical.

In Part I, the model includes free parameter($\alpha = 1 - 2\beta$) of which β represents the participation weight of the retarded potential (β) vs. that of the advanced potential ($1 - \beta$) in the approximate model. The case of $\alpha = 0$ corresponds the equal weight case of the two potentials and leads to a first-order model in time that is not consistent with respect to various initial conditions. This has motivated us to conduct a thorough parametric study vs. the corresponding model characteristics so that the model can be tailored for various particular pressure loading/incident waves.

To this end, we have employed two approaches: the root-locus method to examine the effect of the free parameter (α) vs. desirable characteristic root locations, and transient responses by the present model subjected to the various loading (both incident step pressure and incident impulse velocities). Following the previous investigators[1,5], we have adopted a uniform-thickness spherical shell that is subjected to various initial conditions as there has been mode-by-mode analytical solution for this case to guide us in assessing the effect of the free parameter weight. In doing so, whenever possible, we have compared the root loci and the transient responses of two methods: the analytical, the Doubly Asymptotic Approximation method, DAA_2[6].

ELASTIC SPHERICAL SHELL SURROUNDED BY ACOUSTIC MEDIUM

Figure 1 shows a thin elastic spherical shell of radius a, thickness h, an isotropic material with Young's Modulus E, density ρ_s, and Poisson's ratio ν. The shell thickness-to-ratio h/a is small enough to apply thin shell theory and the longitudinal wave velocity of the shell is denoted by $c_0 = \sqrt{E/\rho_s(1-\nu^2)}$. The shell geometry is described using a spherical coor-

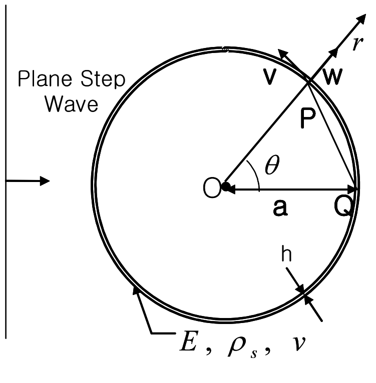


Figure 1: Spherical shell excited by a incident step plane wave

dinate (R, θ) with its origin at O and an in-vacuo condition of its interior. The radial and meridional displacements of the shell are denoted by $W(\theta,t)$ and $V(\theta,t)$, respectively. For subsequent analysis, dimensionless variables are introduced: $t = Tc/a$, $w = W/a$, $v = V/a$, $r = R/a$ and p is normalized by ρc^2, where T is the time(sec). The external wave pressure, $p(r,\theta,t)$, is the sum of the known incident wave pressure $p^0(r,\theta,t)$ and an unknown scattered wave pressure $p^s(r,\theta,t)$. The coupling of the pressure to the interface particle velocity is governed by potential formula. Using the potential formula and geometric compatibility on the shell surface, one obtains the following non-dimensional relation between the pressure and the particle velocity normal to the shell surface as:

$$dp/dr(1,t) \quad = \quad dp^0/dr(1,t) + dp^s/dr(1,t) = -\dot{u}^0 - \dot{u}^s = -\ddot{w}. \tag{1}$$

Modal Expansion of Exact Scattering Pressure

Scattered wave pressures for a spherical shell are expressed in a spherical coordinate as :

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial p^s}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial p^s}{\partial\theta}) = \frac{\partial^2 p^s}{\partial t^2} \tag{2}$$

whose modal solution can be expressed as:

$$p^s\ (r,\theta,t) = \sum_{n=1}^{\infty} p_n^s(r,t)\ P_n(cos\theta) \tag{3}$$

where P_n is the nth order Legendre polynomial and p_n^s is the component of p^s for nth order. The Laplace transformed $\overline{p}_n^s(r,s)$ in (3) is given by[3] as

$$\overline{p}_n^s(r,s) = B_n(s)\kappa_n(rs) \tag{4}$$

where s is the Laplace Transform variable, $B_n(s)$ is the constant determined from the geometrical compatibility conditions and $\kappa_n(rs)$ is the nth order modified spherical Bessel function of the third kind, $\kappa_n(z) = \frac{\pi}{2}e^{-z}\sum_{m=1}^{n}\Gamma_{nm}z^{-(m+1)}, \quad \Gamma_{nm} = (n+m)!/[2^m m!(n-m)!]$.

Present Approximate Scattering Pressure Equation

The approximate external acoustic equation derived in Part I(Park, Lee and Park, 2006) is given as

$$\alpha A\ddot{p}(t) + (1+\alpha)A_1\dot{p}(t) + B_2 p(t) = \alpha A\ddot{u}(t) + A_1\dot{u}(t) \tag{5}$$

where A, A_1 and B_2 are the surface integrals defined in Part I. For evaluation of the present approximate model for the scattering pressure and to compare its performance with respect to the analytical scattering pressure response, we must obtain the Laplace-transformed modal form of (5). This requires the evaluation of the three surface integral expressions. Utilizing the orthogonality and normalization conditions of the Legedre polynomials, the modal forms of B_2, A_1 and A can be expressed as

$$B_2\overline{p} \quad = \quad \int_S \frac{1}{R^2}\frac{\partial R}{\partial n}\overline{p}(Q,s)dS = \sum_{n=0}^{\infty} B_2^n\overline{p}_n(r,s), \quad B_2^n = \frac{4\pi a^2(n+1)}{(2n+1)}\frac{r^n}{a^{n+2}} \tag{6}$$

$$A_1\overline{p} = \int_S \frac{1}{R} p(Q,t)dS = \sum_{n=0}^{\infty} A_1^n \overline{p}_n(r,s), \quad A_1^n = \frac{4\pi a^2}{(2n+1)} \frac{r^n}{a^{n+1}} \tag{7}$$

$$A\overline{p} = \int_S p(Q,t)dS = \sum_{n=0}^{\infty} A^n \overline{p}_n(r,s), \quad A^n = \frac{4\pi a^2}{(2n+1)}. \tag{8}$$

Using (6), (7) and (8), the Laplace-transformed form of the present approximate acoustic model(5) can be expressed in the following modal form for the spherical surface:

$$\alpha s^2 \overline{p}_n + (1+\alpha)s\overline{p}_n + (1+n)\overline{p}_n = \alpha s^2 \overline{u}_n + s\overline{u}_n \tag{9}$$

Coupled Modal Acoustic-Structure Interaction Equations

For an elastic spherical shell, the radial and meridional displacements (w, v) can be expanded in term of the Legendre polynomials as [1]:

$$w\ (\theta,t) = \sum_{n=0}^{\infty} w_n(t)\ P_n(\cos\theta),\ \ v\ (\theta,t) = -\sum_{n=1}^{\infty} v_n(t)\ \frac{d}{d\theta} P_n(\cos\theta) \tag{10}$$

where $P_n(\cos\theta)$ is the Legendre polynomial of nth order. Using the preceding series expansions, the corresponding modal equations of motion for an elastic spherical shell with uniform thickness and isotropic material for each mode are given as [4, 5]:

$$\lambda_n \ddot{v}_n + A_n^{vv} v_n + A_n^{vw} w_n = 0,\ \ A_n^{vw} v_n + \ddot{w}_n + A_n^{ww} w_n = -\mu p_n \tag{11}$$

where $\mu = (\rho/\rho_s)(a/h)$, $\gamma_0 = c_0^2/c^2$, $\beta = (h/a)^2/12$, $\lambda_n = n(n+1)$, $\xi_n = \lambda_n - 1 + \nu$ and $A_n^{vv} = \lambda_n(1+\beta)\xi_n\gamma_0,\ \ A_n^{vw} = \lambda_n(1+\nu+\beta\xi_n)\gamma_0,\ \ A_n^{ww} = [2(1+\nu)+\lambda_n\beta\xi_n]\gamma_0$.

Combining (11), the exact solutions, the present and the $2nd$ order Doubly Asymptotic Approximation(DAA$_2$)[5], the coupled modal acoustic-structure interaction equations for a spherical shell can be expressed in the following matrix form:

$$\begin{bmatrix} \lambda_n s^2 + A_n^{vv} & A_n^{vw} & 0 \\ A_n^{vw} & s^2 + A_n^{ww} & \mu \\ 0 & -sQ_n(s) & R_n(s) \end{bmatrix} \begin{Bmatrix} v_n \\ w_n \\ p_n^s \end{Bmatrix} = \begin{Bmatrix} 0 \\ -\mu p_n^0 \\ -Q_n(s)u_n^0 \end{Bmatrix}. \tag{12}$$

where $Q_n(s)$ and $R_n(s)$ are given by

$$\begin{aligned} Exact &: \ Q_n(s) = -\kappa(s),\ \ R_n(s) = \kappa'(s) \\ Present &: \ Q_n(s) = s(\alpha s + 1),\ \ R_n(s) = \alpha s^2 + (1+\alpha)s + (n+1) \\ DAA_2 &: \ Q_n(s) = s(s+(n+1)),\ \ R_n(s) = s^2 + (1+n)s + (1+n)^2 \end{aligned}$$

NUMERICAL RESULTS

Free vibration of a spherical shell surrounded by acoustic medium

This section presents the responses of a steel spherical shell surrounded by air medium, with the model properties of $h/a = 0.01$, $\rho_s/\rho = 5137$, $c_0/c = 4.12$ and $\nu = 0.3$.

Figures 2 shows the coupled roots of the exact solution, DAA$_2$ and the present approximation with $a = 0.5$ for $n = 2$. Both of approximations can accurately approach the roots from structure roots. On the other side, in case of the high damped roots from pressure roots, through appropriate α, the present model can more accurately approximated than DAA$_2$. Table 1 summarizes the best α tailored to the order of the modes for a spherical shell in air medium. Whether this trend would hold for general complex structural surfaces remains to be determined. It can be seen that in general the higher the modes, the lower the value of α should be for accuracy considerations.

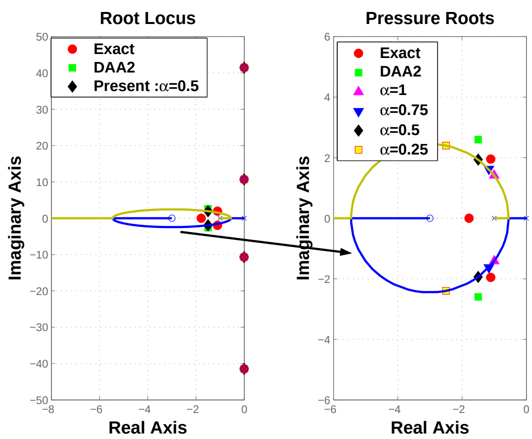


Figure 2: Free-vibration roots of a spherical shell for n=2 in air medium

Legendre polynomial order	The best α
0^{th}	–
1^{st}	1
2^{nd}	0.65
3^{rd}	0.48
4^{th}	0.37
5^{th}	0.26
6^{th}	0.23

Table 1: Mode-by-mode best α for a spherical shell surrounded by air

A spherical shell excited by a plane step wave

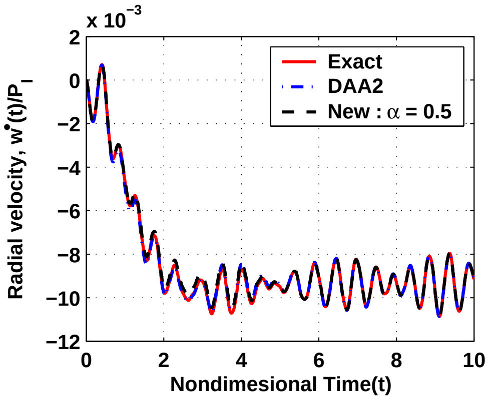
We will study the response of an elastic spherical shell surrounded by acoustic medium wherein a plane step pressure wave is applied to excite a spherical shell as shown in Fig. 1. This problem has been studied by a number of investigators in the past and the incident pressure and velocity of the plane step wave[2] are expressed as

$$p^0(r,\theta,t) = P_I H(t - r\cos\theta - 1), \quad u^0(r,\theta,t) = P_I \cos\theta H(t - r\cos\theta - 1) \tag{13}$$

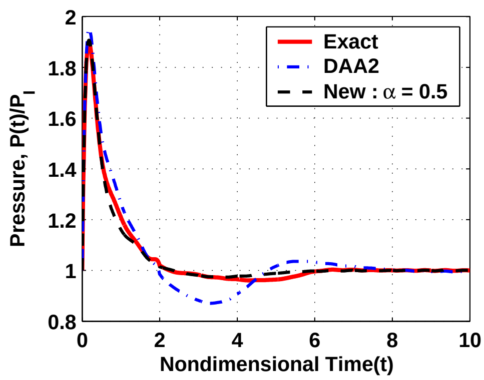
where P_I is the magnitude of the incident wave and $H(t)$ is the Heaviside step function[2]. The modal components of the incident pressure and velocity can be calculated as

$$\{p_n^0(t), u_n^0(t)\} = \frac{2n+1}{2} \int_0^\pi \{p^0(r,\theta,t), u^0(r,\theta,t\} P_n(\cos\theta) \sin\theta d\theta. \tag{14}$$

The scattered pressure and shell displacement are obtained to apply (14) to (12). Finally, the external pressure and shell velocities on the spherical shell are obtained using modal summation from[2]. For this problem, $\alpha = 0.5$ is used and the modal summations up to $N = 6$ was utilized. Figures 3.(a) shows the velocity responses at the front with respect to the plane wave direction. A close examination of the velocity history during the early time ($0 \leq T \leq 2$) indicates that the coupled pressure roots dominate during the rise time period where $T = 2$ is the time for the plane wave traverse from the front to the back. Afterwards, the high-mode coupled structural modes continue to oscillate. As both the present and the DAA_2 possess accurate coupled structural roots, their structural velocities trace closely the exact response. In Figures 3.(b), the pressure response at the front is displayed. Note that the



(a) Radial velocity



(b) Pressure

Figure 3: Shell radial-velocity and pressure at $\theta = \pi$ of a spherical shell excited by a plane step wave

front-end pressure response ($\theta = \pi$) typifies a parabolic response corresponding to the high-damping and low-frequency oscillations, viz., a rapid rise to the peak pressure then settling to a steady-state pressure level. It is interesting to observe that the present approximation traces closely that of the exact solution whereas the DAA$_2$ experiences a hump after the plane wave traverses to the back end, primarily affected by the low damping and high oscillatory coupled pressure roots.

A spherical shell excited by impulsive incident waves

A second example problem chosen for evaluating the present approximate external acoustic model is an elastic spherical shell subject to impulse incident velocities. Physically, the incident impulse wave on the shell surface corresponds to an explosion at time ($T = 0$) of charges distributed over the half of the sphere. The mode-by-mode decomposition of the half-cosine impulse can be obtained in terms of the Legendre polynomials as

$$u_I^n = \frac{2n+1}{2} \int_0^{\pi} (\cos\theta - \cos\theta \; U(\cos\theta))\delta(t) P_n(\cos\theta) \sin\theta d\theta \tag{15}$$

where $U(\cos\theta)$ is the unit spatial step input. Figure 4 presents the pressure responses pre-

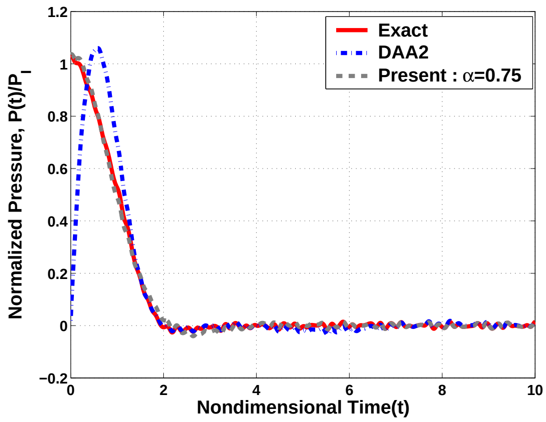


Figure 4: Pressure response at $\theta = \pi$ for a spherical shell excited by impulsive incident waves

dicted by the exact solution, DAA$_2$ and the present approximate model with $\alpha = 0.75$ at the front side ($\theta = \pi$) where the impulsive incident wave is applied. The case of $\alpha = 0.75$ gives the well accurate response. Clearly, the DAA$_2$ is not consistent with the exact solution at early time for this case. One finds this reason from mode-by-mode unit impulse response for the n-th mode using the initial value theorem of the Laplace Transform as applied to the coupled structure-external acoustic equation(12) as :

$$\lim_{t \to 0} \left[\frac{p_n(t)}{u_I^n(t)} \right] = \begin{cases} \delta(0) + 1 + \mu & \text{for exact and the present models} \\ \delta(0) + \mu & \text{for } DAA_2 \end{cases} \tag{16}$$

CONCLUSIONS

In the present Part II, the new approximate acoustic model derived in Part I has been evaluated for its fidelity by comparing its solutions with the exact solutions for the problem of an elastic spherical shell surrounded by acoustic medium.

The present approximate model(5) can be constructed by employing the three standard boundary integral kernels, A, A_1 and B_2. Hence, the present model can be constructed without having to develop new surface integral evaluations.

The present approximate model employs the model parameter, α, which can be adjusted with computational experience to improve its model fidelity. For example, when the low modes dominate the interactions, it has been found that $\alpha \to 1$ is desirable for the case of an elastic spherical shell surrounded by acoustic medium. This is summarized in Table 1. It has been found that $\alpha = 1/2$ would yield good overall accuracy.

The importance of the initial response consistency examined in Part I has been demonstrated for problems excited by impulsive incident wave. It has been shown analytically in Part I and in the present Part II that the present approximate model satisfies the initial consistency condition. This will have important consequence for inverse identification problems in which a primary task is to construct impulse (or frequency) response functions.

Future studies are needed to evaluate it for wider interaction problems.

ACKNOWLEDGMENTS

This work was sponsored by NRL No. M10500000112-05J0000-11210 and BK21.

References

[1] H. Huang, ”Transient Interaction of Plane Acoustic Wave with a Spherical Elastic Shell”,J.Acoust.Soc.Am. 45,661-670,1969

[2] M. A. Sprague, T. L. Geers, ”Response of empty and fluid-filled, submerged spherical shells to plane and spherical, step-exponential acoustic waves”,Shock and Vibration, Vol 6, 147-157, 1999

[3] M.Abramowitz, I.A.Stegun, *Handbook of Mathematical Functions*. (NBS Appl. Math. Ser.55 U.S.Dept of Commerce, Washington, DC, 1964)

[4] M.C.Junger, D.Feit,*Sound, Structures, and Their Interaction*. (MIT, Cambridge, 1972)

[5] P. Zhang, T. L. Geers, ”Excitation of a fluid-filled, submerged spherical shell by a transient acoustic wave”,J.Acoust.Soc.Am. 93,696-705,1993

[6] T. L. Geers, ”Doubly Asymptotic Approximation for Transient Motion of Submerged Structures”,J.Acoust.Soc.Am. 64,1500-1508,1978

[7] K. C. Park, M. S. Park, Y. S. Park, New Approximations of Structure-External Acoustic Interactions, Part I : Model Derivation, *this Proceedings*.