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NEW APPROXIMATIONS OF EXTERNAL ACOUSTIC-STRUCTURAL INTERACTIONS, PART I : MODEL DERIVATION

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Abstract
New approximate models for external acoustics interacting with flexible structures are developed. The basic form of the present models is obtained by a combination of the Laplace-transformed retarded and advanced potentials. It is shown that the maximum attainable time-derivative of convergent approximate models is two, hence any attempt to include higher orders will lead to non-convergent models. The basic model is subsequently modified from the consistency considerations of the resulting frequency response functions. The resulting model is expressed in terms of a free parameter that represents the weight of the retarded vs. advanced potential characteristics, and consistent in terms of capturing the correct impulse response. Hence, the present approximate model is well suited for acoustic-field determinations and inverse acoustic identification applications.

INTRODUCTION

Exact solutions of structures interacting with external acoustic fields are available for ideal geometries such as infinite cylinders, spheres, and plates. As engineering designs consist of irregular geometries, approximate acoustic models have been widely used. In most existing approximate acoustic models, first, the two limiting cases have been modeled: early-time response and late-time response by employing the initial-value and final-value theorems to the Kirchhoff's spherical acoustic integral wave equation [5]. Existing approximate models, known as Doubly Asymptotic Approximations (DAA), have been proven to be adequate for characterizing the acoustic radiation damping affecting the structural responses that are dominated by low-frequency components [7]. For medium and high-frequency transients, however, most existing approximate structure-external acoustic interaction models suffer from both frequency distortions and inaccurate radiation damping.

When one focuses on the pressure field modeling, not only the pressure magnitudes but also its phase information have to be obtained accurately. This is especially true for identifying sound sources as well as sound intensities. For example, a careful examination of most existing approximate models for ideal structural geometries, when compared with the exact models obtained via Kirchhoff's formula, reveals that the dominant acoustic scattering pressure modes are often not represented. Hence, most existing approximate models, while adequate for structural response calculations, may not be applicable to inverse acoustics problems wherein the primary objective is to identify the sound sources.

This has motivated the present authors to develop acoustic models that can capture predominant acoustic modes, as distinct from structural modes, and yet that are computationally attractive. From the theoretical point of view, the well-known Kirchhoff's retarded potential equation may be considered as the foundation of all the existing approximate models. Under this premise, different approximate acoustic models originate from the corresponding different approximations of the retarded (or delayed) operator.

In Part I of the present work, we will employ a combination of the retarded and advanced operators. In employing the advanced operator, we are keenly mindful of the disagreement between Ritz and Einstein[11] on the validity of the advanced potential and the subsequent discussions that appear to suggest that the use of the advanced potential may be untenable in relativistic electromagnetic theory [9]. Even to this date, the 1908 Ritz-Einstein disagreement continues to rouse intense arguments and counter-arguments[1, 4]. However, our use of the advanced potential in acoustics for the purpose of deriving approximate acoustic models is justified primarily by the observation that the classical laws of physics (to which the acoustics field belongs) discovered by Galileo, Newton and Einstein are time-symmetric and secondly by recent applications of the time-reverse concept in acoustics [3,6]. In other words, as acoustic signals are invariant under time-reversal, each packet of sound that comes from a source can be reflected, refracted or scattered. Consequently, a set of reflected waves can retrace all of the scattering paths, converging at the original source just as if time was going backwards. The rest of the paper is organized as follows.

To start off, the Kirchhoff's retarded potential and its corresponding Laplace-transformed equation are introduced and the inherent unstable nature of approximate models derived by approximating the retarded potential is discussed in Section 1. The use of the advanced operator is introduced in Section 2 and is subsequently combined with the retarded operator to form a combine. It is shown that the resulting basic approximate models are stable provided the weight parameter, β, is selected judiciously.

The present basic approximate models are then modified to address a consistency condition imposed on them in order for them to possess the important correct impulse response characteristics. The resulting approximate models thus derived show that: (1) the so-called added mass is associated only with the structural inertia acting on the acoustic boundary and not with the acoustic transient pressure as most existing models have assumed; (2) The maximum convergent order of the coupled acoustic pressure equation is at most two in terms of its time derivatives; (3) most existing approximate models fail to satisfy the initial impulse response condition, thus they may yield erroneous impulse responses that are important for inverse identification applications.

A critical evaluation of the present approximate models are reported in the companion paper, Part II[10].

KIRCHHOFF'S RETARDED POTENTIAL FORMULA

Kirchhoff's retarded potential formula for describing the expanding or radiating wave can be expressed as[2]:

$$4\pi\epsilon p(P,t) = \int_S \{ \frac{\rho}{R}\dot{u}(Q,t_r) - \frac{1}{R^2}\frac{\partial R}{\partial n}p(Q,t_r) - \frac{1}{cR}\frac{\partial R}{\partial n}\dot{p}(Q,t_r)\} dS_Q \tag{1}$$

where R is the distance from P to a typical point Q on the surface S; $\partial/\partial n$ denotes differentiation along the outward normal to S; ϵ is the solid angle that takes on $(1, 0.5, 0)$ depending on whether the point Q is within the acoustic domain, on the surface S, or inside the enclosed surface S, respectively. The t_r denotes the $retarded\ time\ t - \frac{R}{c}$ and the retarded potential terms are Laplace-Transformed as

$$\int_0^\infty e^{-st} g(Q,t_r)dt = e^{-sR/c}\overline{g}(Q,s), \quad \overline{g}(Q,s) = \int_0^\infty e^{-st} g(Q,t)dt \tag{2}$$

Hence, this formula(1) can be expressed in the Laplace Transform domain as

$$\int_S \overline{p}(Q,s)\frac{\partial R}{\partial n}\frac{1}{R^2}(1+Rs/c)\, e^{-Rs/c}dS_Q + 4\pi\epsilon\overline{p}(P,s) = \rho\int_S s\overline{u}(Q,s)\frac{1}{R}\, e^{-Rs/c}dS_Q. \tag{3}$$

The Laplace-transformed counterpart(3) states that the contributions of Kirchhoff's retarded potential formula(1) from the previous states are expressed in terms of the delay operator $e^{-sR/c}$. It should be noted that various approximations, both in the time and the Laplace domain methods, amount to how this delay operator is approximated.

An approximation of the Laplace-transformed Kirchhoff equation is to use the two-term Taylor series:

$$e^{-\mu} = 1 - \mu + O(\mu^2), \quad \mu = Rs/c \tag{4}$$

which is justifiable because as long as $Rs/c < 1$, it can be considered a good approximation for capturing the late-time (or low-frequency) responses.

Substituting (4) into (3), we obtain

$$4\pi\epsilon\overline{p}(P,s) + \int_S \overline{p}(Q,s)\frac{\partial R}{\partial n}\frac{1}{R^2}(1+Rs/c)(1-\frac{Rs}{c})dS_Q \approx \rho\int_S s\overline{u}(Q,s)\frac{1}{R}(1-\frac{Rs}{c})dS_Q \tag{5}$$

Observe that the above approximation does not contain any parasitic term, $\{O(R^p), p > 0\}$, so that the resulting approximation does not diverge as $R \to \infty$. Rearranging the above equation leads to

$$-s^2\left[\frac{1}{c^2}\int_S \overline{p}(Q,s)\frac{\partial R}{\partial n}dS_Q\right] + 4\pi\epsilon\overline{p}(P,s) + \int_S \overline{p}(Q,s)\frac{\partial R}{\partial n}\frac{1}{R^2}dS_Q$$
$$= s\rho\int_S \overline{u}(Q,s)\frac{1}{R}dS_Q - s^2\frac{\rho}{c}\int_S \overline{u}(Q,s)dS_Q \tag{6}$$

When the preceding equation is discretized and transformed back to the time domain , its discrete form may be symbolically expressed as

$$-\,[M_d]\,\ddot{p}_d + [K_d]\,p_d = B_d\dot{u}_d - A_d\ddot{u}_d \tag{7}$$

where the subscript d refers to the discrete quantities; and, $[M_d]$ and $[K_d]$ physically represent the projected area matrix and the static pressure stiffness matrix, respectively, hence non-negative definite matrices. This means that the pressure approximation equation using the two-term Taylor approximation of the delay operator leads to an unstable model.

NEW APPROXIMATIONS OF KIRCHHOFF POTENTIAL FORMULA

It is recalled that the Taylor series expansion of the retarded operator, $e^{-Rs/c}$, has led to an unstable approximation in time. This suggests that one may profit by employing the advanced potential defined as

$$\phi_a = \phi(Q,t_a) = \phi(Q,t+\frac{R}{c}),\;\; \int_0^\infty e^{-st}\phi(Q,t_a)dt = e^{sR/c}\overline{\phi}(Q,s). \tag{8}$$

Although the use of the advanced potential continues to rouse intense arguments and counter-arguments [1, 4], in the view of acoustics, their signals are invariant under time-reversal. Furthermore, each packet of sound that comes from a source can be reflected, refracted or scattered. Consequently, a set of reflected waves can retrace all of the scattering paths, converging at the original source just as if time was going backwards.

Combining the two-term Taylor series approximate Laplace-transformed expressions of the retarded and advanced equation in accordance of the weighting rule stipulated in [11],

$$\phi = \beta\phi_r + (1-\beta)\phi_a \tag{9}$$

we obtain

$$\begin{aligned}&\int_S \overline{p}(Q,s)\frac{\partial R}{\partial n}\frac{1}{R^2}(1+Rs/c)\;\{1+(1-2\beta)Rs/c\}dS_Q\\ +4\pi\epsilon\overline{p}(P,s) \approx \rho &\int_S s\overline{u}(Q,s)\frac{1}{R}\;\{1+(1-2\beta)Rs/c\}dS_Q\end{aligned} \tag{10}$$

It should be noted that as briefly stated in the previous section, inclusions of the terms $O(\mu^p, p>1)$ would yield parasitic models and those terms would result in a large value of the coefficient matrices as $R\rightarrow\infty$.

Rearranging the above approximate equation in the order of s-variable, we obtain

$$\begin{aligned}(1-2\beta)s^2B\overline{p}(P,s) + 2(1-\beta)scB_1\overline{p}(P,s) + c^2B_2\overline{p}(P,s)\\ = (1-2\beta)s^2\rho cA\overline{u}(P,s) + s\rho c^2A_1\overline{u}(P,s)\end{aligned} \tag{11}$$

where

$$B\overline{p}(P,s) \;=\; \int_S \frac{\partial R}{\partial n}\overline{p}(Q,s)dS_Q,\quad B_1\overline{p}(P,s) \;=\; \int_S \frac{1}{R}\frac{\partial R}{\partial n}\overline{p}(Q,s)dS_Q$$

$$B_2\overline{p}(P,s) = \int_S \frac{1}{R^2}\frac{\partial R}{\partial n}\overline{p}(Q,s)dS_Q + 4\pi\varepsilon\delta(P-Q) \tag{12}$$
$$A\overline{u}(P,s) = \int_S \overline{u}(Q,s)dS_Q, \quad A_1\overline{u}(P,s) = \int_S \frac{1}{R}\overline{u}(Q,s)dS_Q$$

It is observed that the present approximate model equation(11) would be stable provided

$$(1-2\beta) \geq 0 \quad \Rightarrow \quad \beta \leq 1/2. \tag{13}$$

MODIFICATIONS FOR EARLY TIME RESPONSES

The approximate model for external acoustic field interacting with flexible structures derived in (11) has been obtained by assuming that Rs/c is less than 1. This implies that the approximate model thus derived would offer higher model fidelity for the late-time response than in early-time response by virtue of the initial($s \to \infty$) and final($s \to 0$) value theorems of the Laplace transform. This means that among the five coefficient operators (B, B_1, B_2, A, A_1) in the basic approximate acoustic model(11), the two zeroth-order terms (B_2, A_1) should need no further modifications. In addition, as $s^2\overline{u}$ may be regarded as an applied pressure to the pressure equation, we will accept A with no further modifications well. This leaves the two remaining operators, viz., (B, B_1), as potential modification operators in order to improve the model fidelity for the early-time response.

Plane wave approximation

The plane wave approximation was investigated for the early-time responses by Mindlin and Bleich. Fellipa[10] also derived the first order early time approximate, which is the plane wave approximation of Mindlin and Bleich. So, we apply it to the present approximation model for the early time responses.

Using the plane wave approximation,($\frac{\partial R}{\partial n} \to 1$), B and B_1 are modified as :

$$B_1\overline{p}(P,s)|_{\frac{\partial R}{\partial n}\to 1} = A_1\overline{p}(P,s), \quad B\overline{p}(P,s)|_{\frac{\partial R}{\partial n}\to 1} = A\overline{p}(P,s) \tag{14}$$

This is because, in physical terms, the direction of the wave path and the normal to the interaction surface remain parallel for plane waves.

With the above modifications, we arrive at the present approximate second-order external acoustic model given by

$$\begin{aligned} &\alpha s^2 A\overline{p}(P,s) + (1+\alpha)scA_1\overline{p}(P,s) + c^2B_2\overline{p}(P,s) \\ = \;& \alpha s^2\rho cA\overline{u}(P,s) + s\rho c^2A_1\overline{u}(P,s), \quad \alpha = 1-2\beta > 0 \end{aligned} \tag{15}$$

which considers the early-time response and yields the quasi-steady state scattering pressure response as the corresponding terms in the expansion of the Kirchhoff's retarded potential has not been altered due to the introduction of the advanced potential.

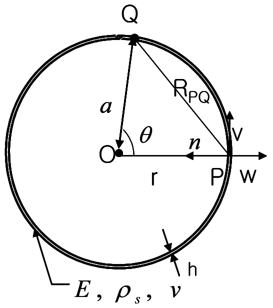

Figure 1: A spherical shell surrounded by acoustic medium

Modal Form of the Present Approximate Model for a Spherical Shell

For the simplicity of subsequent algebra, dimensionless variables are used: $t = Tc/a$, $w = W/a$, $v = V/a$, $r = R/a$ and p is normalized by ρc^2, where T is the time(sec).

To this end, first, we utilize the spherical surface as shown Fig 1 for which the analytical modal equation for the scattering pressure is known[8]. Second, we use the three operators (B_2, A, A_1) , calculated in the companion paper Part II, in terms of the Legendre functions.

In a spherical shell, the scattered pressures are expressed as :

$$p^s\ (r,\theta,t) = \sum_{n=1}^{\infty} p_n^s(r,t)\ P_n(cos\theta) \tag{16}$$

where P_n is the nth order Legendre polynomial and p_n^s is the component of p^s for nth order.

Using the above scattered pressure and three operators for a spherical shell, the present approximate model in terms of the Lgendre functions is described in Laplace domain as:

$$\alpha s^2 \overline{p}_n + (1+\alpha)s\overline{p}_n + (1+n)\overline{p}_n = \alpha\overline{u}_n + \overline{u}_n \tag{17}$$

The DAA$_1$ and DAA$_2$ can be assessed in the same way. Geers et al [7] gives the following Laplace-transformed equations:

$$\text{DAA}_1 \quad : \quad s\overline{p}_n + (1+n)\overline{p}_n = s\overline{u}_n \tag{18}$$

$$\text{DAA}_2 \quad : \quad s^2\overline{p}_n + (1+n)s\overline{p}_n + (1+n)^2\overline{p}_n = s^2\overline{u}_n + (1+n)s\overline{u}_n. \tag{19}$$

In this case, the analytical Legendre mode-by-mode relation the external pressure vs. the radial particle velocity[7, 8] is derived as

$$\frac{\overline{p}_n(s)}{\overline{u}_n(s)} = [G_n(s)]_{ex} = \begin{cases} s/(s+1), & \text{for n=0} \\ (s^2+s)/(s^2+2s+2), & \text{for n=1} \\ \frac{s^3+3s^2+3s}{s^3+4s^2+9s+9}, & \text{for n=2} \end{cases} \tag{20}$$

Early-Time Consistency Analysis

The early time consistency is important for inverse acoustic problems. Applying the initial value theorem to the impedances of the exact, present approximate model, one obtains

$$\lim_{t\to 0}\left[\frac{p_n(t)}{u_n(t)}\right] = \delta(0) - 1, \qquad \text{for the exact and present model cases} \tag{21}$$

Hence, the present model satisfies the early-time consistency requirement. On the other hand, the DAA_1 and DAA_2 have the initial values determined as

$$\lim_{t\to 0}\left[\frac{p_n(t)}{u_n(t)}\right]_{DAA} = \begin{cases} \delta(0) - (n+1) & \text{for } DAA_1 \\ \delta(0) & \text{for } DAA_2 \end{cases} \tag{22}$$

Neither the DAA_1 nor the DAA_2 satisfies the early-time consistency requirement. The only exception is for the case of the DAA_1 with the breathing spherical mode($n = 0$).

OBSERVATIONS OF THE PRESENT APPROXIMATION

We now offer the following observations:

Remark 1: The case of $\beta = 1/2(\alpha = 0)$ in the present modified model(15) is further simplified to a first-order model which is identical to the DAA_1:

$$scA_1\overline{p}(P,s) + c^2B_2\overline{p}(P,s) = s\rho c^2 A_1\overline{u}(P,s) \tag{23}$$

From the present model derivation viewpoint, thus, the DAA_1 is obtained when the equal strength of the retarded and advanced potentials is chosen in the model. However, the above first-order model (the DAA_1) fails to satisfy the early-time response consistency requirement, except for the case of the breathing mode ($n = 0$) for the spherical geometry. Nevertheless, the DAA_1 has been extensively utilized as a baseline approximation for the transient response analysis of submerged structures[7].

Remark 2: The case of $\beta = 0(\alpha = 1)$ corresponds to the use of only the advanced potential.

Remark 3: As pointed out in Section 2, the case of $\beta = 1(\alpha = -1)$ corresponds to the use of only the retarded potential in the approximate model(6) which is unstable.

Finally, a two-parameter model can be developed by a linear combination of the first-order model(23) and the second-order model(15):

$$\begin{aligned} &\alpha(1-\gamma)s^2A\overline{p}(P,s) + [(1+\alpha)(1-\gamma)+\gamma]\, scA_1\overline{p}(P,s) + c^2B_2\overline{p}(P,s) \\ &= \alpha(1-\gamma)s^2\rho cA\overline{u}(P,s) + s\rho c^2A_1\overline{u}(P,s), \quad \{\alpha > 0, \quad \gamma < 1\} \end{aligned} \tag{24}$$

SUMMARY

New approximate model of external acoustic pressure field interacting with flexible structures has been derived. The approximate model derived therein has shown that it is consistent with respect to initial value of impedance. This consistency condition is considered an important property for inverse identification applications, in addition to transient acoustic-structure interaction simulations, because accurate determination of impulse response functions in time(or frequency) domain identification methods is critical.

It has been shown that the maximum order of the pressure model equations is two in time ($\frac{d^2}{dt^2}$). The model includes a free parameter (β) that represents the participation weight of the retarded potential (β) vs. that of the advanced potential ($1 - \beta$) in the approximate model, where $\beta = 1/2$ is a special case of their equal participation. However, this seemingly reasonable choice of the equal weight case ($\beta = 1/2$) reduces the present approximate model to a first-order model in time that is not consistent with respect to initial conditions. This has motivated us to conduct a thorough parametric study vs. the corresponding model characteristics so that the model can be tailored for various particular pressure loading/incident waves. This is presented in Part II[10].

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