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ACTIVE CONTROL IN FLEXIBLE PLATES WITH PIEZOELECTRIC ACTUATORS USING LINEAR MATRIX INEQUALITIES

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Abstract

The study of algorithms for active vibrations control in flexible structures became an area of enormous interest, mainly due to the countless demands of an optimal performance of mechanical systems as aircraft, aerospace and automotive structures. Smart structures, formed by a structure base, coupled with piezoelectric actuators and sensor are capable to guarantee the conditions demanded through the application of several types of controllers. The actuator/sensor materials are composed by piezoelectric ceramic (PZT - *Lead Zirconate Titanate*), commonly used as distributed actuators, and piezoelectric plastic films (PVDF - *PolyVinyliDeno Floride*), highly indicated for distributed sensors. The design process of such system encompasses three main phases: structural design; optimal placement of sensor/actuator (PVDF and PZT); and controller design. Consequently, for optimal design purposes, the structure, the sensor/actuator placement and the controller have to be considered simultaneously. This article addresses the optimal placement of actuators and sensors for design of controller for vibration attenuation in a flexible plate. Techniques involving linear matrix inequalities (LMI) to solve the Riccati's equation are used. The controller's gain is calculated using the linear quadratic regulator (LQR). The major advantage of LMI design is to enable specifications such as stability degree requirements, decay rate, input force limitation in the actuators and output peak bounder. It is also possible to assume that the model parameters involve uncertainties. LMI is a very useful tool for problems with constraints, where the parameters vary in a range of values. Once formulated in terms of LMI a problem can be solved efficiently by convex optimization algorithms.

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INTRODUCTION

Lightweight space structures are the future of space vehicles and satellite technology. Possessing ideal space launching characteristics, such as minimal storage volume and minimal mass, these lightweight structures will propel the space industry into the next generation of space satellite technology. Space satellites must be expertly controlled from a vibration standpoint because signal transmission to and from the earth mandates tight tolerances. Active vibration control (AVC) is critical to mission success as well as satellite longevity [1].

Various methods have been developed to AVC. Application of AVC in flexible structures has been increasingly used as a solution for space structures to achieve the degree of vibration suppression required for precision pointing accuracy and to guarantee the stability. In this work, it is used LQR controller by linear matrix inequalities (LMI) to attenuate vibration signal in a flexible plate using a PZT actuator. LMI contributes to overcome many difficulties in control design. In the last decade, LMI has been used to solve many problems that until then was unfeasible through other methodologies, due mainly to the emerging of powerful algorithms to solve convex optimization problem, as for instance, the interior point method [2; 3].

STRUCTURAL MODELLING

The method is theoretically though a numerical application. The dynamical behaviour of a structure can be described in terms of mass, stiffness and damping matrices, and displacement and velocity vectors as

$$\begin{aligned}
&\ddot{\mathbf{q}}(t)+\mathbf{M}^{-1}\mathbf{D}_{\mathbf{a}}\dot{\mathbf{q}}(t)+\mathbf{M}^{-1}\mathbf{K}\mathbf{q}(t)=\mathbf{M}^{-1}\mathbf{B}_0\mathbf{u}(t)\\
&\mathbf{y}(t)=\mathbf{C}_{\mathbf{oq}}\mathbf{q}(t)+\mathbf{C}_{\mathbf{ov}}\dot{\mathbf{q}}(t)
\end{aligned} \tag{1a,b}$$

where $\mathbf{q}(t)$ is the n-length displacement vector, $\mathbf{u}(t)$ is the s-length input vector, $\mathbf{y}(t)$ is r-length output vector, $\mathbf{M}$ is the n x n mass matrix, $\mathbf{D_a}$ is the n x n damping matrix, and $\mathbf{K}$ is the n x n stiffness matrix. $\mathbf{B_0}$ is the n x s input matrix, $\mathbf{C_{oq}}$ and the r x n output displacement matrix, and $\mathbf{C_{ov}}$ is the r x n output velocity matrix. The mass matrix is positive definite, and the stiffness and damping matrices are positive semi-definite, n is the number of degrees of freedom of the system (linearly independent coordinates describing the finite-dimensional structure), r is the number of outputs and s is the number of inputs. Using the classic procedure of modal analysis [4], it is possible to write the equations of motion in modal coordinates, $\mathbf{q}_{\mathrm{m}}(t)$. Thus, the modal model of second order is given by

$$\begin{aligned}
&\mathbf{q}(t)=\mathbf{\Phi}\mathbf{q}_{\mathbf{m}}(t)\\
&\ddot{\mathbf{q}}_{\mathbf{m}}(t)+2\mathbf{Z}\mathbf{\Omega}\dot{\mathbf{q}}_{\mathbf{m}}(t)+\mathbf{\Omega}\mathbf{q}_{\mathbf{m}}(t)=\mathbf{B}_{\mathbf{m}}\mathbf{u}(t)\\
&\mathbf{y}(t)=\mathbf{C}_{\mathbf{mq}}\mathbf{q}_{\mathbf{m}}(t)+\mathbf{C}_{\mathbf{mv}}\dot{\mathbf{q}}_{\mathbf{m}}(t)
\end{aligned} \tag{2a,b,c}$$

where $\mathbf{\Phi}$ is the modal matrix and $\mathbf{Z}$ is the matrix of damping coefficients (ζ_i), given by

$$\mathbf{Z} = 0.5\mathbf{M}_\mathbf{m}^{-1}\mathbf{D}_\mathbf{m}\mathbf{\Omega}^{-1} = 0.5\mathbf{M}_\mathbf{m}^{-1/2}\mathbf{K}_\mathbf{m}^{-1/2}\mathbf{D}_\mathbf{m} \qquad (3)$$

where $\mathbf{\Omega}^2 = \mathbf{M}_\mathbf{m}^{-1}\mathbf{K}_\mathbf{m}$ is the matrix of natural frequencies. The matrices $\mathbf{M}_\mathrm{m}$, $\mathbf{K}_\mathrm{m}$ and $\mathbf{D}_\mathrm{m}$ are diagonal matrices of modal mass, stiffness and damping, respectively, which are given by

$$\mathbf{M}_\mathbf{m} = \mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}, \quad \mathbf{K}_\mathbf{m} = \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}, \quad \mathbf{D}_\mathbf{m} = \mathbf{\Phi}^T\mathbf{D}_\mathbf{a}\mathbf{\Phi} \qquad (4a,b,c)$$

The matrix $\mathbf{D}_\mathbf{a}$ is assumed to be proportional to mass and stiffness matrices, so that

$$\mathbf{D}_\mathbf{a} = \alpha\mathbf{M} + \beta\mathbf{K} \qquad (5)$$

Matrix $\mathbf{B}_\mathrm{m}$ in equation (2b) is the input modal matrix, or participation modal matrix and is given by

$$\mathbf{B}_\mathbf{m} = \mathbf{M}_\mathbf{m}^{-1}\mathbf{\Phi}^T\mathbf{B}_\mathbf{0} \qquad (6)$$

$\mathbf{C}_\mathbf{mq}$ and $\mathbf{C}_\mathbf{mv}$ are the output displacement and velocity modal matrices given by

$$\mathbf{C}_\mathbf{mq} = \mathbf{C}_\mathbf{oq}\mathbf{\Phi}, \quad \mathbf{C}_\mathbf{mv} = \mathbf{C}_\mathbf{ov}\mathbf{\Phi} \qquad (7a,b)$$

The state equations can be written in a vector-matrix format through the triple (**A**, **B**, **C**); it allows the equations to be manipulated more easily. The related matrices are given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega}^2 & -2\mathbf{Z}\mathbf{\Omega} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_\mathrm{m} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_\mathrm{mq} & \mathbf{C}_\mathrm{mv} \end{bmatrix} \qquad (8a,b,c)$$

The equations (8) are not a modal state representation (although it was obtained using modal displacements, $\mathbf{q}_\mathrm{m}$). The modal state-space representation has a triple ($\mathbf{A}_\mathrm{m}$, $\mathbf{B}_\mathrm{m}$, $\mathbf{C}_\mathrm{m}$) characterized by the block-diagonal dynamic matrix, $\mathbf{A}_\mathrm{m}$, and the related input and output matrices [4]

$$\mathbf{A}_{\mathrm{m}} = \mathrm{diag}\left(\mathbf{A}_{\mathrm{mi}}\right), \qquad \mathbf{B}_{\mathrm{m}} = \begin{bmatrix} \mathbf{B}_{\mathrm{m1}} \\ \mathbf{B}_{\mathrm{m2}} \\ \vdots \\ \mathbf{B}_{\mathrm{mn}} \end{bmatrix}, \qquad \mathbf{C}_{\mathrm{m}} = \begin{bmatrix} \mathbf{C}_{\mathrm{m1}} & \mathbf{C}_{\mathrm{m2}} & \cdots & \mathbf{C}_{\mathrm{mn}} \end{bmatrix} \tag{9a,b,c}$$

where i=1,2,…,n; $\mathbf{A}_{\mathrm{mi}}$, $\mathbf{B}_{\mathrm{mi}}$ and $\mathbf{C}_{\mathrm{mi}}$ are 2 x 2, 2 x s and r x 2 blocks, respectively. These blocks can take several different forms and also it is possible to convert from one form to another by a linear transformation. One possible form to block $\mathbf{A}_{\mathrm{mi}}$ is:

$$\mathbf{A}_{\mathrm{mi}} = \begin{bmatrix} -\zeta_{\mathrm{i}}\omega_{\mathrm{i}} & \omega_{\mathrm{i}} \\ -\omega_{\mathrm{i}}(\zeta_{\mathrm{i}}^2 - 1) & -\zeta_{\mathrm{i}}\omega_{\mathrm{i}} \end{bmatrix} \tag{10}$$

The state vector $\mathbf{x}(t)$ in modal coordinates consists of n independent components, $\mathbf{x}_{\mathrm{i}}(t)$, that represent a state of each mode. The $\mathbf{x}_{\mathrm{i}}(t)$ (ith state component), related to equation (10), is given by [4].

$$\mathbf{x}_{\mathrm{i}}(t) = \begin{Bmatrix} \mathbf{q}_{\mathrm{mi}}(t) \\ \mathbf{q}_{\mathrm{moi}}(t) \end{Bmatrix}, \qquad \mathbf{q}_{\mathrm{moi}}(t) = \zeta_{\mathrm{i}}\mathbf{q}_{\mathrm{mi}}(t) + \dot{\mathbf{q}}_{\mathrm{mi}}(t) \big/ \omega_{\mathrm{i}} \tag{11}$$

LINEAR QUADRATIC REGULATOR BY LMI APPROACH

The first step in LQR control design process is the definition of a performance index, which can be defined by a quadratic cost function in state and control variables. This index can be written as

$$J = \int_0^{\infty} \left(\mathbf{u}^T \mathbf{R} \mathbf{u} + \rho \mathbf{x}^T \mathbf{Q} \mathbf{x}\right) dt \tag{12}$$

where $\mathbf{Q}$ is a symmetric and positive semi-definite weighting matrix on the states, $\mathbf{R}$ is a symmetric and positive definite weighting matrix on the controller outputs, and ρ is the relative state to control weighting design parameter.

Considering the linear time-invariant system in state-space form, and the matrices described by equations (8)-(9), the object of the regulator design is to find a linear control law of the form

$$\mathbf{u}(t) = -\mathbf{G}\mathbf{x}(t) \tag{13}$$

which minimizes J. If the regulator design is restricted to time-invariant control laws, $\mathbf{G}$ will be a constant coefficient matrix and $\mathbf{u}$ will be a linear combination of the states. It is assumed in the regulator design that all of the states are measured. It can be shown that the gain matrix **G** which minimizes the performance index is given by [5]

$$\mathbf{G} = \frac{1}{\rho}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \tag{14}$$

where **P** is the solution of the steady-state matrix Riccati equation

$$-\dot{\mathbf{P}} = 0 = \mathbf{PA} + \mathbf{A}^T\mathbf{P} - (1/\rho)\mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} \tag{15}$$

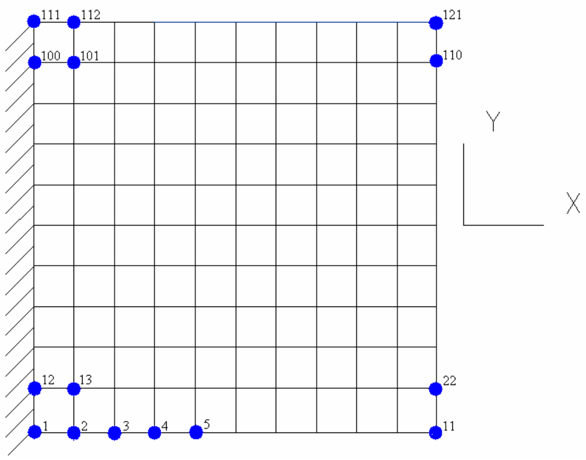
Using LMI technique, the linear quadratic regulator can be writing by [6]

$$\min_{\mathbf{Y},\mathbf{P},\mathbf{X}} Tr(\mathbf{QP}) + Tr(\mathbf{X}) + Tr(\mathbf{YN}) + Tr(\mathbf{N}^\mathbf{T}\mathbf{Y}^\mathbf{T})$$
$$\text{subject to} \quad \mathbf{AP} - \mathbf{BY} + \mathbf{PA}^\mathbf{T} - \mathbf{Y}^\mathbf{T}\mathbf{B}^\mathbf{T} + \mathbf{EE} < 0, \quad \begin{bmatrix} \mathbf{X} & \mathbf{R}^{1/2}\mathbf{Y} \\ \mathbf{Y}^\mathbf{T}\mathbf{R}^{1/2} & \mathbf{P} \end{bmatrix} > 0 \tag{16}$$

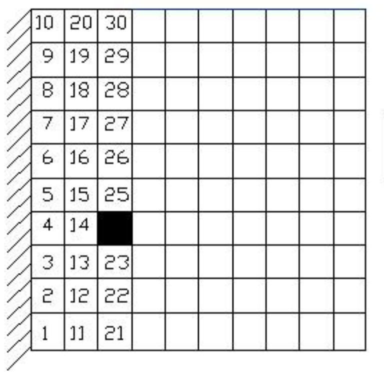
where $Tr()$ is the trace of the matrix, **E** and **N** is the disturbance and noise vectors, respectively, and **X** and **Y** are the solution of the LMI.

NUMERICAL APPLICATION

To verify the proposed methodology, a flexible aluminum plate, as shown in fig. 1a, was considered. The plate is discretized by FEM in 100 elements and three degree of freedom per node. The structure has 363 structural dof's (121 nodes). The plate is clamped in one end, so considering this boundary condition, the system has N=660 states. Table 1 shows the physic and geometric properties of the plate used in the FEM modeling. PZT actuator is considered in optimal position obtained using H_∞ norm of the system solved by LMI (Fig. 1b).



(a) cantilever plate



(b) plate with PZT actuator

Figure 1 – Finite element model for a cantilever plate.

Table 1. Geometric and physic properties of the plate.

Dimensions (m)	Length	Width	Thickness
	0.5	0.03	0.005
Density (kg.m^{-3})	2710		
Young's Modulus (GPa)	70		

Figure 2 shows the time response for the system controlled and uncontrolled. Fourth first modes are considered in control design. The frequency response functions (FRF) for the reduced model and residuals modes are shown in Figure 3.

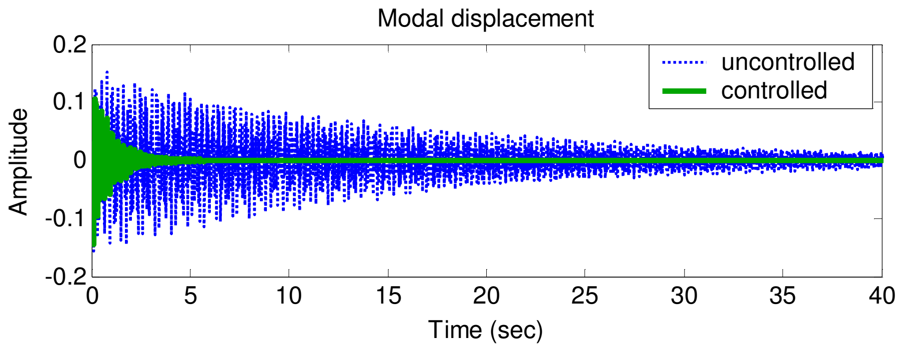


Figure 2 – Modal displacement controlled and uncontrolled system.

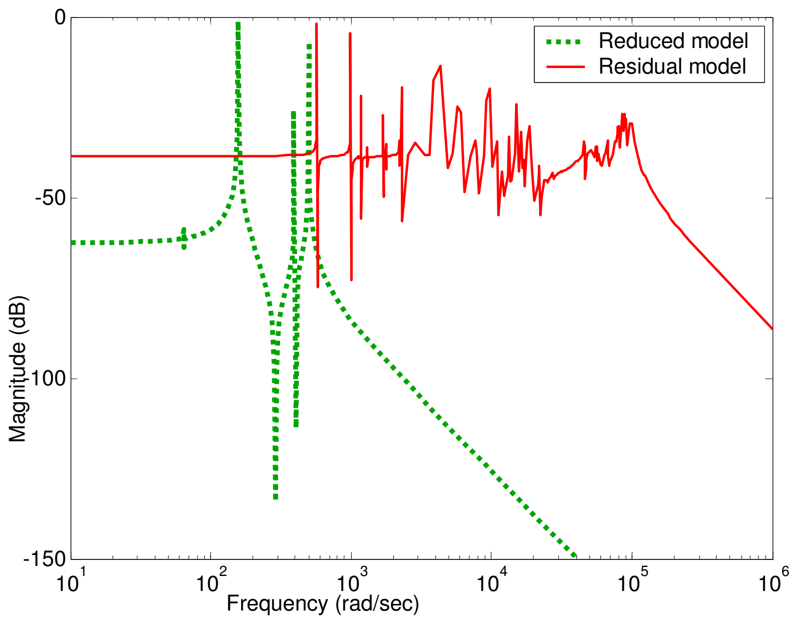


Figure 3 – FRF of the system: reduced and residual modes.

Figure 4 shows the FRF of the controlled model with the application of the LQR-LMI controller. The impulse disturbance was applied in the PZT position. The amplitude of the second mode was significantly attenuated, while the others ones are nearly the same as those when the control system is open loop. The modal force in PZT actuator is shown in figure 5.

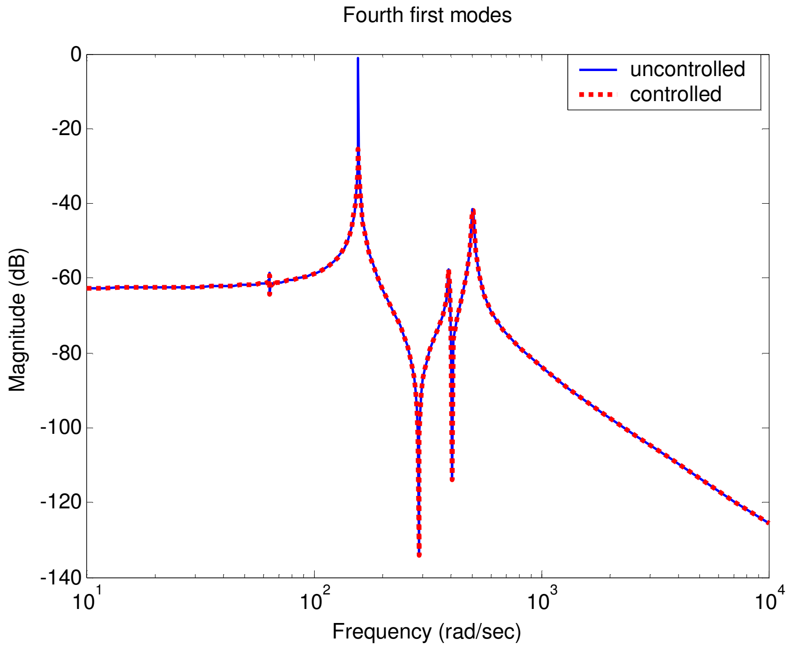


Figure 4 – FRF of the reduced model: controlled and uncontrolled.

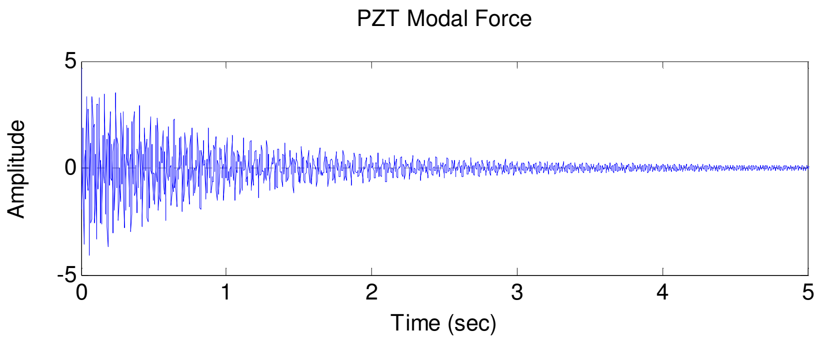


Figure 5 – Modal force in PZT actuator.

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FINAL REMARKS

An LQR feedback control strategy solved through LMI approach was used to actively control the fourth first modes of a aluminum plate. The model of the structure was discretized by FEM. Residual modes can cause spillover effects; however, it did not happen in the numerical application.

The use of piezoelectric material, coupled in flexible structure as actuator and sensor, has shown to be a good solution in order to reduce structural mechanical vibration. The LMIs techniques (classified by some authors as postmodern control) present many advantages, mainly due the facilities to solve numerical problems for complex structure, where the analytical solution should be difficult to implement. Uncertainties in dynamics parameters for the design of the robust active vibration control.

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