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A NEW CRITERION TO OPTIMIZE THE PARAMETERS OF DINAMIC VIBRATION ABSORBERS

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Abstract

The dynamic vibration absorbers (DVA) are simple and traditional devices used very commonly for vibration suppression. In this work a theoretical analysis of a dynamic vibration absorber attached to a viscously damped beam is presented.

The DVA is composed by several one degree of freedom systems, its principal advantage is that it is possible to control several modes of the principal structure simultaneously.

A new criterion is introduced to optimize the parameters of the device with the objective of reducing the motion of certain points of the beam.

The mathematical model is based on the Assumed-Modes Method which makes use of Lagrange's multipliers to take into account the constraints introduced by the absorber and the optimization is performed by means of the "simulated annealing" technique.

The proposed method consists on minimizing the amplitude of vibration in predetermined points of the beam considering a broadband attenuation and using Den Hartog's proposals.

Cases studied show the influence of the position of the DVA relative to the point where the motion is desired to be reduced is considered.

INTRODUCTION

The dynamic vibration absorber (DVA) is a technical device, in principle, quite simple, which advantageously couples vibrations between the main system (discrete or continous) and a discrete, subsidiary system having as a goal the reduction of vibrational phenomenon in the main system. It is important to point out that some authors define the device as “vibration neutraliser”[1] since the vibrations of the main system are not absorbed but reduced in it. On the other, hand the vibrational levels are increased in the subsidiary system. It could also be called “dynamic vibration redistributer”(DVR) which philosophically corresponds to the actual phenomenon if damping is not considered.

The simplest version of a DVR is an undamped one degree of freedom system. The device allows for a considerable reduction of the vibrational levels of the main system but two resonant frequencies appear, close to the sintonization frequency at which the undesired vibrational levels occurs. Interestingly, if one considers a DVR but now includes viscous damping, it is possible to choose values of spring and damping constants, *k* and *c* respectively, in such a manner that the vibrational level of the main system is reduced over a range of frequencies and not only at the disturbing, sintonization frequency.

This was shown by Den Hartog in a now classical paper [2] taking advantage of the existence of “fixed” or “invariant” points in the frequency response function (FRF). The desired natural frequency of DVR must be given by

$$\omega_a = \frac{\omega_0}{1+\mu}$$

where ω_a is the DVR’s frequency that gives equal amplitude at the fixed points, ω_0 is the host’s resonance frequency, and μ is the ratio between the DVR’s and the host’s mass. The expression for the DVR’s optimal damping ratio is

$$\xi_{op} = \sqrt{\frac{3\mu}{8(1+\mu)}}$$

where ξ_{op} is the DVR’s damping ratio defined as $\xi = c/(2mf)$, c, m and f are the DVR’s damping, mass and frequency, respectively. As a consequence the frequency response function (FRF) of the whole system is relatively flat with no new resonance appearing.

Later Jacquot [3] obtained the optimized governing parameters of a one degree of freedom DVR when the main system is a Bernoulli-Euler beam without internal damping by modelling the behavior of the beam using a single mode of vibration. Jacquot’s approach makes use of the fixed point methodology in the frequency response function. One obtains now

$$\omega_a = \frac{\omega_0}{1+\mu\,\phi_i^2(x_i)}$$

being ϕ_i is the mode shape of the structure and x_i is the point where the DVR is attached. On the other side, the damping coefficient is giving by

$$\xi_{op} = \sqrt{\frac{3\mu\,\phi_i^2(x_i)}{8(1+\mu\,\phi_i^2(x_i))}}$$

Thompson [4] analysed a primary system with internal damping by means of the frequency locus method. The construction of the frequency loci for the general system leads to the determination of graphical criteria for the optimization problem. A different concept, the use of a cruciform dynamic vibration absorber made of two free-free beams rigidily interconnected at the middle cross section is due to Snowdon [5].

The problem of a tuned mass damper attached to a multiple-degree-of-freedom (MDOF) main system has also been studied extensively. In a very recent paper Ozer and Royston [6] presented an extension of Den Hartog approach to multi-degree-of-freedom undamped main system. Multiple dynamic absorbers have been proposed by several authors. In a recent work Rice [7] showed the design of multiple discrete vibration absorber systems for broadband applications. Rade and Steffen [8] proposed a general methodology for the optimum selection of DVR parameters so as to guarantee the efficiency of those devices over a previously selected frequency band

The goal of the present study is to present a new optimization criterium for intervening parameters of a DVR by extending the concept of fixed points in the case where the main system is an internally damped beam and the DVR is composed of a multiple degree of freedom discrete system with viscous damping.

PROBLEM STATEMENT

In this paper the theory is applied to the case where the main structure is a beam with simply supported boundary conditions and the DVR consists on 1 or 2dof system, see figure 1. It is well known that the dynamic of a continuous system coupled with a discrete one can be developed by the assumed modes method [9].

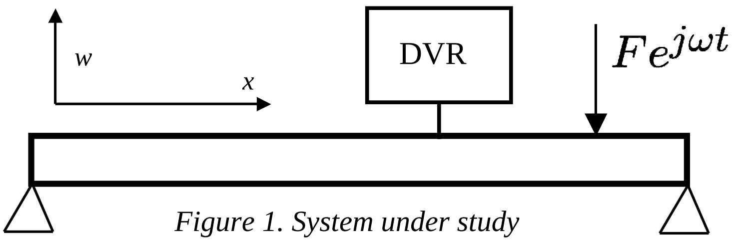


Figure 1. System under study

The basic steps are simple: *w(x,t)* is expressed as superposition of modal amplitudes of host structure

$$w(x,t)=\sum_{i=1}^{n}\phi_i(x)\,q_i(t) \tag{1}$$

where $\phi_i(x)$ is the mode shape of the simple supported beam and $q_i(t)$ are generalized coordinates, which depend on t. The total kinetic energy, strain and dissipation functions are

$$T=\frac{1}{2}\sum_{i=1}^{n}M_i\,\dot{q}_i^2+\frac{1}{2}\sum_{t=1}^{r}m_t\dot{z}_t^2 \qquad V=\frac{1}{2}\sum_{i=1}^{n}M_i\,\omega_i^2\,q_i^2+\frac{1}{2}\sum_{t=1}^{r}k_t(z_t-z_{0t})^2 \tag{2-3}$$

$$D=\frac{1}{2}\sum_{i=1}^{n}c_i\,\dot{q}_i^2+\frac{1}{2}\sum_{t=1}^{r}c_t\,(\dot{z}_t-\dot{z}_{0t})^2 \tag{4}$$

z_t and z_{0t} are the coordinates of the mass and the other end of the spring. The Lagrange equations are

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{s}_k}\right)-\frac{\partial D}{\partial \dot{s}_k}+\frac{\partial V}{\partial s_k}=Q_k+\sum_{t=1}^{r}\lambda_t\frac{\partial f_t}{\partial s_k} \qquad k=1,\dots n+2r \tag{5}$$

where λ_t are Lagrange's multipliers and subscript r is the number of restrictions of the problem, the generalized force Q_k is $F\,e^{i\omega t}\,\phi_k(x_f)$ being x_f the point where force is applied. Replacing the expression of T, V, D and Q_k in eq.(5), Lagrange's equations are obtained.

The restrictions are given by

$$f_t=\sum_{t=1}^{r}\phi_i(x_t)\,q_i(t)-z_{0t}=0 \tag{6}$$

which allow the eliminations of redundant coordinates of the problem. Substituting the equations (1) to (4) in Lagrange's equations the equation of motion are obtained.

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{f_p\}+\{f_s\} \tag{7}$$

$[M]_{n\,x\,n}=diag\,(M_i)$ is the modal mass matrix, $[C]_{n\,x\,n}=diag\,(2\,\xi_i\,\omega_i\,M_i)$ the modal damping and $[K]_{n\,x\,n}=diag\,(\omega_i^2 M_i)$ the stiffness matrix. The primary force is an external harmonic excitation $\{f_p\}_{n\,x\,1}=F\,e^{i\,\omega t}\,\{\phi(x_f)\}$, $\{\phi(x_f)\}_{n\,x\,1}=\{\phi_1(x_f)\dots\ \phi_n(x_f)\}^T$ $\{f_s\}_{n\,x\,1}=[\phi]_{n\,x\,r}\{F_e\}_{n\,x\,1}$ $\{F_e\}_{r\,x\,1}=-[K_{abs}]_{r\,x\,r}[\phi]^T_{r\,x\,n}\{q\}_{n\,x\,1}$
Assuming harmonic motion $q_k=\bar{q}_k\,e^{i\omega t}$ the final expression is

$$\{\bar{q}\}=\left([A]+[\phi][K_{abs}][\phi]^T\right)^{-1}\,F\{\phi(x_f)\} \tag{8}$$

where $[A]_{nxn}=\left([K]-\omega^2[M]+i\,\omega[C]\right)$ and $[\phi]_{nxr}$ is the matrix of modal amplitudes Finally, the displacement of the system at point x_a can be expressed as

$$W(x_a)=\sum_{i=1}^{n}\phi_i(x_a)\,\bar{q}_a$$

The performance of both, uncoupled and coupled by inertia 2dof DVR will be compared in the present study, see figure 2.

(a)

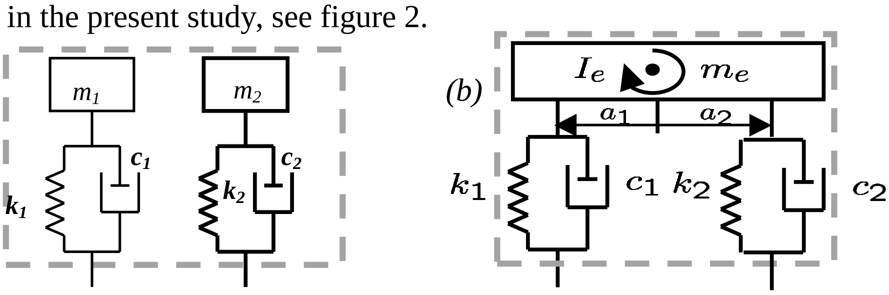


Figure 2. (a) uncoupled 2 dof DVR (b) coupled 2 dof DVR

One must use the dynamic stiffness of each type of DVR in expression (8)

a) uncoupled 2dof $[K_{abs}]=diag\,(K_i)$ where

$$K_i=\frac{-\omega^2 m_i\left(1+i\,2\,\xi_{ai}(\omega/\omega_{ai})\right)}{1-\left(\omega^2/\omega_{ai}^2\right)+i\,2\xi_{ai}(\omega/\omega_{ai})} \quad \xi_{ai}=\frac{c_i}{2m_i\omega_{ai}} \quad \omega_{ai}=\sqrt{k_i/m_i}$$

b) coupled 2dof

$$[K_{abs}]=\begin{pmatrix} k_1^*(1-k_1^*\,\beta_{11}) & -k_1^*k_2^*\beta_{12} \\ -k_2^*k_1^*\beta_{21} & k_2^*(1-k_2^*\beta_{22}) \end{pmatrix} \quad [\beta]=[\alpha]^{-1}$$

$$\alpha=\begin{pmatrix} k_1^*-\omega^2\dfrac{\left(m_e a_2^2+I_e\right)}{\left(a_1+a_2\right)^2} & \omega^2\dfrac{\left(I_e-m_e a_1 a_2\right)}{\left(a_1+a_2\right)^2} \\ \omega^2\dfrac{\left(I_e-m_e a_1 a_2\right)}{\left(a_1+a_2\right)^2} & k_2^*-\omega^2\dfrac{\left(m_e a_1^2+I_e\right)}{\left(a_1+a_2\right)^2} \end{pmatrix} \quad k_i^*=k_i+i\,\omega c_i$$

CRITERION OF OPTIMIZATION

Using the model previously described it is possible to study the influence of the parameters of DVR in its performance, from the control of vibration viewpoint.

The determination of the optimization parameters of 2 dof DVR is a task which from an optimization viewpoint requires minimization of an objective function which depends upon k_1,k_2,c_1,c_2, m_1, m_2 *(or* m_e, I_e*)*, x_1, x_2. This constitutes a rather complex problem which can be simplified taking into account some practical considerations. Mass or inertia properties of the DVR constitutes a parameter whose influence is uniform since to a larger mass corresponds a larger reduction, in the entire range of frequencies. It is convenient to fix its value as 0.1 of the mass of the principal structure. Regarding the position of the absorber one must clarify that it is a parameter influenced by the dynamic of the system since according to the mode of vibration one will have a resulting amplitude of the DVR's force. On the other hand there may be locations which are not readily accessible. Due to these reasons it was decided that the objective function depends only upon the elastic and damping constants.

The existence of fixed points in the FRF allows for the uncoupling of the problem in two phases:

- sintonization stage
- flattering of the FRF curve

Sintonization

If the main structure does not possess damping one determines k_1 and k_2 in such a manner that the fixed points P and Q acquire the same ordinate, see figure 3 (a). On the other hand, if the main structure possesses internal damping the fixed points does not exist. However, if damping is small, say ξ=0.01, the curves corresponding to different values of damping pass through near points. Once two values of damping are selected and provided that the ordinates be equal, at points where they cross, k_1 and k_2 can be calculated. Figure 3 illustrates the situation where $c_1=c_2=0$ and $c_1=c_2=\infty$.

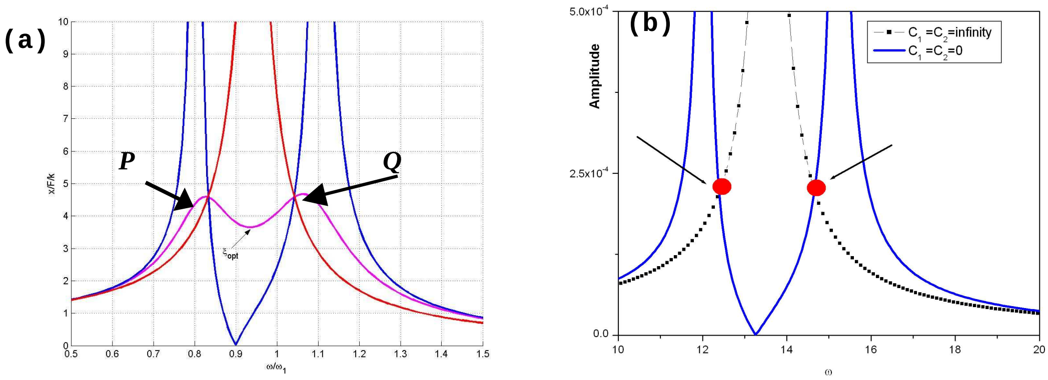


Figure 3.-Fixed points in FRF. Beam without internal damping (a)and with internal damping(b)

Flattering of the FRF curve

Once the sintonization stage is completed the next step consists on obtaining a robust solution of the problem. In order to accomplish this one must select the appropriate damping coefficients. The idea is fairly simple since once P and Q acquire the same ordinate value one must obtain c_1 and c_2 such that the curve is flattened between the crossing points.

To carry this out one performs the optimization in two stages. The first one uses as objective function

$$f_1(c_1,c_2)=\sum_{i=1}^{N}\left|W\left(\omega=\omega_i,c_1,c_2\right)\right|+\sum_{i=1}^{N}\left|W\left(\omega=\omega_i,c_1,c_2\right)\right|$$

where in each summation one selects N frequencies close to each one of the resonances that are pretended to control.

In this stage one obtains values of c_1 and c_2 which minimize the objective function f_1 but do not satisfy the robustness requirements, see figure 5 (a). Then, it is decided to perform a second stage which minimizes the “distance” between the displacement amplitude and the average between the maximum and minimum of the curve in the neighborhood of each resonance frequency. The second objective function f_2 is given by

$$f_2(c_1,c_2)=\sum_{i=1}^{N}\left|W\left(\omega=\omega_i,c_1,c_2\right)-A_1\right|+\sum_{i=1}^{N}\left|W\left(\omega=\omega_i,c_1,c_2\right)-A_2\right|$$

the results are shown in figure 5 (b)

RESULTS

In the previous section, the new criterion to optimise the parameters of dynamic vibration absorber was presented. Here this criterion is applied to show the versatility and usefulness of it.

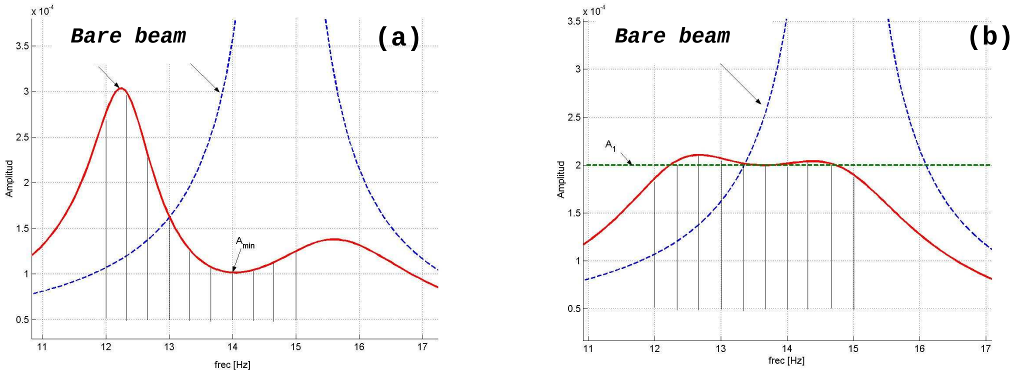


Figure 5.- (a) FRF after the first optimization. (b)FRF after the sencond optimization

Influence of position of 1 dof DVR

First we applied the methodology to study the influence of position of a 1 dof DVR to control the first resonance frequency of a simply supported beam.

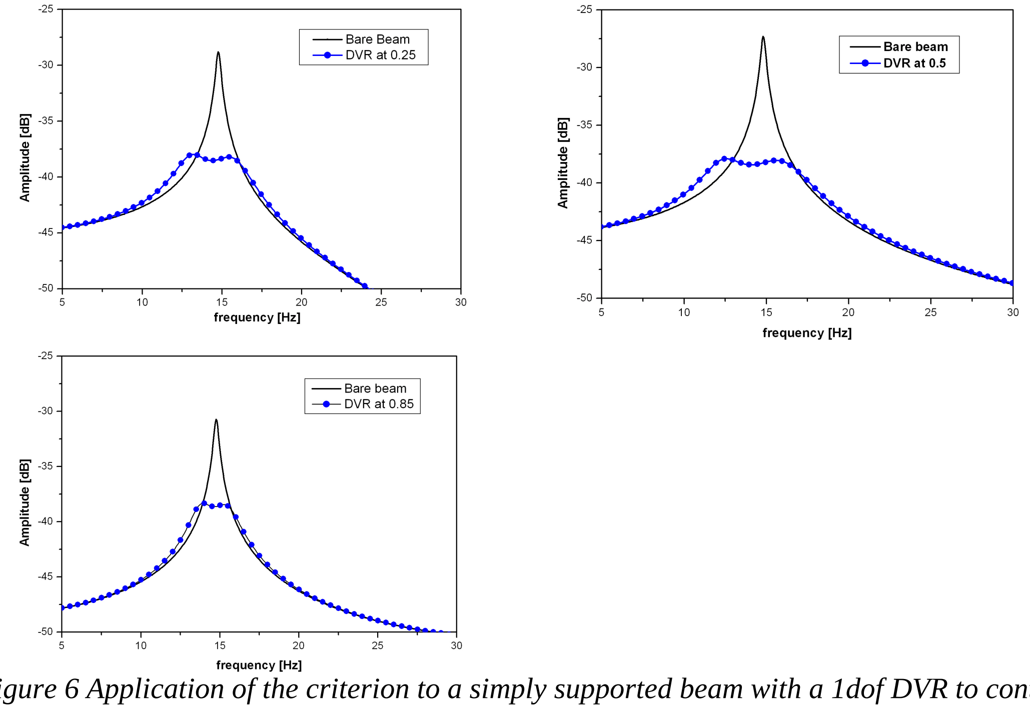
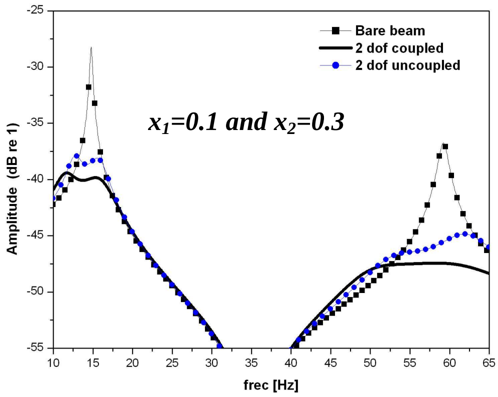


Figure 6 Application of the criterion to a simply supported beam with a 1dof DVR to control the first frequency at different positions

Comparison between coupled and uncoupled DVR

As a second implementation of the proposed criterion a comparison between coupled and uncoupled 2 dof DVR, at different position, are shown.



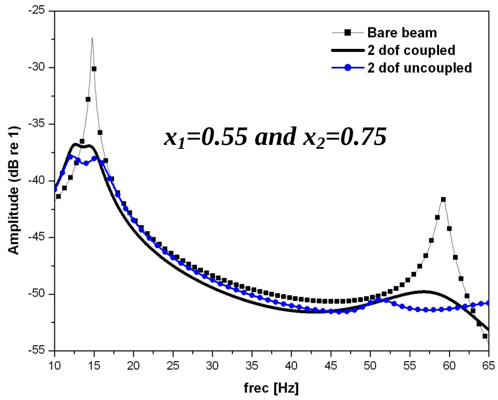


Figure 7.- Comparison of reduction between 2dof coupled and uncoupled DVR .

CONCLUSIONS

In this work, a new criterion to optimize the parameters of dynamic vibration redistributers is presented. This criterion extends the fixed points theory to the case where internal damping is present in the main structure. It was used successfully to investigate the performance of the 1 dof and 2 dof DVR at different localizations.

The theory does not require and analytical formulation to obtain a solution, for this reason FEM data would be equally used as a model of the structure with similar results.

For the sake of simplicity the main structure and the DVR were considered of a particular type but others systems could be studied in a straightforward manner.

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