

# Elliptical side resonators for broadband noise reduction: theory and experiments

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# Abstract

Previous research of the authors pointed out that side-resonators can be applied to reduce fan noise. However, the noise reduction capabilities of most resonator geometries, e.g. tube resonators, cylindrical resonators (cylindrical air layers) and circular resonators (disc shaped air layers), are relatively narrow banded. This is disadvantageous in case resonators are used in combination with a noise source that emits broadband noise or tonal noise at varying frequencies (for instance a speed controlled fan). It was found that the choice of the resonator geometry influences the broadband reduction capabilities (circular resonators offering the best broadband reduction capabilities). In the present study, it is investigated to what extent elliptical resonators, consisting of an elliptically shaped air layer, can be used to achieve broadband noise reduction. A semi-analytical model is proposed that describes the wave propagation in the elliptically shaped air layer. This model is connected to the analytical solution for wave propagation in a tube. The dimensions of the elliptical resonator can be optimized for broadband noise reduction using this model. In addition, an experimental setup was built to verify the semi-analytical model of the elliptical resonator.

# **INTRODUCTION**

One of the main noise sources in computers are cooling fans. Tonal noise at a fixed multiple of the rotational frequency of the fan, the so-called blade passing frequency (BPF), and its higher harmonics are important in fan noise. A well-known solution to reduce the sound level at specific frequencies in a duct system housing an in-duct axial fan, is the application of so-called side-resonators. The side-resonator, a cavity of air connected to the circumferences of the duct, causes an impedance change in the duct. Having the proper dimensions and correct position, the side-resonator acts as an acoustic mirror reflecting the noise back to the fan.



Figure 1: Wave propagation in a system of coupled acoustical elements

When a side-resonator is applied at both sides of the fan, the noise is contained between the resonators and the level of the emitted noise can be reduced (see figure 1).

In case of speed controlled fans, the BPF is not fixed at a single frequency and a solution that offers broadband noise reduction is needed. Previous research pointed out that the geometry of a resonator influences both the level of noise reduction that is achieved and the width of the frequency range in which noise is reduced [3]. It was found that a circular resonator (consisting of a disc shaped air layer) offers noise reduction over a relatively wide frequency range compared to resonators with a sound field dominated by plain waves such as tube resonators.



Figure 2: Elliptical side-resonator in a duct system

In the present study, it is investigated to what extent elliptical resonators, consisting of an elliptically shaped air layer (see figure 2(a)), can be used to achieve broadband noise reduction. To this end, a modular model consisting of multiple acoustic elements, that describe viscothermal wave propagation in the duct and side-resonators of various geometry [3][4][5], is extended with an elliptical element. This new acoustic element and the experimental setup build for verification of the theory are described in this paper.

#### VISCO-THERMAL WAVE PROPAGATION IN ELLIPTICAL GEOMETRIES

The elliptical acoustic element used in the modular model describes the relation between the average pressures  $p_1$  and  $p_2$  and particle velocities  $v_1$  and  $v_2$  at the inlet and the hard back wall of the resonator as depicted in figure 2. To derive this relation, a semi-analytical model that describes the 2D wave propagation in the resonator is developed. The solution is derived from the set of equations that form the basis for the so-called *Low Reduced Frequency* model (LRF model). The equation governing pressure (in dimensionless coordinates) according to the LRF model as described by Beltman [2] is:

$$k^2 \nabla^2 p(\mathbf{x}^{pd}) - k^2 \Gamma^2 p(\mathbf{x}^{pd}) = -ikn(s\sigma)\Gamma^2 \Re$$
(1)

with p the pressure,  $\Gamma$  the propagation constant,  $n(s\sigma)$  the polytropic constant, s the shear wave number,  $\sigma$  the square root of the Prandtl number,  $k = \omega/c_0$  the wave number with  $c_0$ the speed of sound and  $\omega$  the angular frequency, and  $\Re$  the average normal velocity of the wall as a function of the coordinates in propagation direction  $\mathbf{x}^{pd}$ .

In case the walls that enclose the vibrating air are rigid, the right hand side of equation (1) equals zero and the Helmholtz differential equation is obtained. For a 2D air layer, there are two propagation directions, which are represented in a cylindrical coordinate system by the r- and  $\theta$ -coordinates. The thickness direction is described by the z-coordinate (see figure 2(a)). The Helmholtz equation becomes two-dimensional and equation (1) takes the form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} - \Gamma^2 p = 0$$
(2)

Separation of variables is achieved by substituting  $p(r, \theta) = R(r)\Theta(\theta)$  in the above equation. After separation, the solution for the  $\theta$ -dependent terms is found to be:

$$\Theta(\theta) = C_m e^{im\theta} + D_m e^{-im\theta} \qquad \text{with} \quad m \in \mathbb{Z}$$
(3)

Periodicity requires that  $\Theta(\theta) = \Theta(\theta + 2\pi)$ , so we find  $m \in \mathbb{Z}$ . Plugging this result back into equation (2) and multiplying through  $R/r^2$  yields:

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} - (\Gamma^2 + \frac{m}{r^2})R = 0 \qquad \text{with} \quad m \in \mathbb{Z}$$
(4)

This equation is a modified form of the Bessel differential equation, which has a solution:

$$R(r) = A_m J_m \left(-ir\Gamma\right) + B_m Y_m \left(-ir\Gamma\right)$$
<sup>(5)</sup>

where  $J_m$  and  $Y_m$  are Bessel functions of the first and second kind, respectively. The general solution for equation (2) is found by multiplying the *r*- and  $\theta$ -dependent parts of the solution, described by equation (3) and (5). Since  $m \in \mathbb{Z}$ , the general solution for  $p(r, \theta)$  is defined as the linear combination of the solutions for all possible *m*. Using the relations  $J_m(z) = -J_{-m}(z)$  and  $Y_m(z) = -Y_{-m}(z)$  (see [1]), it is possible to reduce the number of independent constants and the solution in semi-analytical form is written as:

$$p(r,\theta) = \sum_{m=-\infty}^{\infty} \left[ p_m^A J_m \left( -ir\Gamma \right) + p_m^B Y_m \left( -ir\Gamma \right) \right] e^{im\theta}$$
(6)

In order to calculate the corresponding particle velocities, the expression for pressure in the equation above is substituted into the LRF equation for the particle velocity given by [2]:

$$\mathbf{v}^{pd} = -\frac{i}{k\gamma} \nabla^{pd} p(\mathbf{x}^{pd}) A(s, \mathbf{x}^{pd})$$
(7)

with  $A(s, \mathbf{x}^{pd})$  a function describing the velocity profile in thickness direction and  $\gamma$  the ratio of specific heats. The  $\nabla^{pd}$  operator in cylindrical (dimensionless) coordinates is given by:

$$\nabla^{pd}p = k\frac{\partial p}{\partial r}\hat{\mathbf{r}} + k\frac{1}{r}\frac{\partial p}{\partial \theta}\hat{\theta}$$
(8)

with  $\hat{\mathbf{r}}$  and  $\hat{\theta}$  unity vectors in *r*- and  $\theta$ -direction, respectively. After substitution of  $p(r, \theta)$  we find the following expressions for the particle velocity in *r*- and  $\theta$ -direction:

$$v_{r}(r,\theta) = -\frac{\Gamma}{\gamma}A(s,z)\sum_{m=-\infty}^{\infty} \left(\frac{p_{m}^{A}}{2}\left[J_{m-1}\left(-ir\Gamma\right) - J_{m+1}\left(-ir\Gamma\right)\right] + \frac{p_{m}^{B}}{2}\left[Y_{m-1}\left(-ir\Gamma\right) - Y_{m+1}\left(-ir\Gamma\right)\right]\right)e^{im\theta}$$
(9)

$$v_{\theta}(r,\theta) = \frac{m}{\gamma} A(s,z) \sum_{m=-\infty}^{\infty} \frac{1}{r} \left[ p_m^A J_m(-ir\Gamma) + p_m^B Y_m(-ir\Gamma) \right] e^{im\theta}$$
(10)

From these velocity components, the velocity normal to the resonator boundary  $v_n$ , can be calculated.

## **APPLICATION OF BOUNDARY CONDITIONS**

To find the constants  $p_m^A$  and  $p_m^B$ , boundary conditions have to be applied. Assume that a pressure boundary condition  $p = p_1$  is applied at  $r = r_1(\theta)$ . The boundary condition is satisfied if equation (6), multiplied by a weighing function  $w(\theta)$  and integrated over the  $\theta$ -coordinate, holds for any arbitrary function  $w(\theta)$ . The arbitrary weighing function  $w(\theta)$  can be expressed as a linear combination of weighing functions  $w_k(\theta)$  that form a complete set of orthogonal functions:

$$w(\theta) = \sum_{k=-\infty}^{\infty} C_k w_k(\theta), \qquad w_k(\theta) = e^{ik\theta}$$
(11)

So by demanding that:

$$\int_{S} p_1(r_1(\theta), \theta) w_k(\theta) dS = \sum_{m=-\infty}^{\infty} \int_{S} \left[ p_m^A J_m \left( -ir_1(\theta) \Gamma \right) + p_m^B Y_m \left( -ir_1(\theta) \Gamma \right) \right] e^{im\theta} w_k(\theta) dS$$
(12)

with S the boundary defined by  $r_1(\theta)$ , is valid for  $k \in \mathbb{Z}$  the pressure boundary condition is met. To apply a velocity boundary condition, the same strategy is adopted as was used for the pressure. Applying a particle velocity  $v_n = v_2$  at  $r = r_2(\theta)$  yields the following set of equations:

$$\int_{S} v_2(\theta) w_k(\theta) dS = \int_{S} \frac{1}{\sqrt{\left(\frac{\partial r_2(\theta)}{\partial \theta}\right)^2 + 1}} \left[ \frac{\partial r_2(\theta)}{\partial \theta} v_\theta(r_2(\theta), \theta) - v_r(r_2(\theta), \theta) \right] w_k(\theta) dS$$
(13)

The system given by equations (12) and (13) is of infinite size. In order to solve this system, an approximation is made. The number of weighing functions that are taken into account is limited as is the number of terms in the summations on the right hand sides. It can be proven that for a complete set of orthogonal functions (as defined by equation (11)) the approximation error converges to zero for  $k \to \infty$ . In other words, the error of this approximation is controlled by the limit chosen for k. It can also be proven that the approximation error made by limiting the number of terms of the summations in equations (6), (9) and (10) is controlled by the limit chosen for m.

The boundary conditions can also be applied by means of collocation. In that case, the solution will converge to the exact solution if an infinite number of equidistant evaluation points are taken on the boundary.

### ELLIPTICAL ACOUSTIC ELEMENT

In the model described above, the pressure and particle velocity are not necessarily constant over the resonator boundaries. However, in order to apply boundary conditions a certain pressure or velocity profile over the resonator boundary has to be assumed. In this case, the pressure boundary condition over the inner (circular) boundary is set to a constant value  $p_1$  and the particle velocity  $v_2$  to zero at the outer (elliptical) boundary. The average pressure  $p_2$  and particle velocity and  $v_1$  are calculated on the outer (elliptical) boundary and inner (circular) boundary, respectively. Subsequently,  $p_1$ ,  $p_2$ ,  $v_1$  and  $v_2$  are all written as linear functions of  $p_m^A$  and  $p_m^B$ . The linear relations that are obtained can be represented schematically by:

$$\left\{ \begin{array}{c} v_1 \\ v_2 \end{array} \right\} = [K] \left\{ \begin{array}{c} p_m^A \\ p_m^B \end{array} \right\} \qquad \qquad \left\{ \begin{array}{c} p_1 \\ p_2 \end{array} \right\} = [L] \left\{ \begin{array}{c} p_m^A \\ p_m^B \end{array} \right\}$$
(14)

Using the latter equation,  $p_m^A$  and  $p_m^B$  are rewritten as a function of  $p_1$  and  $p_2$  and the result is substituted in the expressions for  $v_1$  and  $v_2$ . A system that describes the relation between  $p_1$ ,  $p_2$ ,  $v_1$  and  $v_2$ , needed for the acoustic element, can thus be obtained:

$$\left\{ \begin{array}{c} v_1 \\ v_2 \end{array} \right\} = [K] [L]^{-1} \left\{ \begin{array}{c} p_1 \\ p_2 \end{array} \right\}$$
(15)

#### **EXPERIMENTAL VALIDATION**

The acoustic element described by equation (15) can be used in a modular acoustical model combining duct elements, fan elements and resonators of different geometry. The experimental setup that was build to validate the new element is depicted schematically in figure 3. It



Figure 3: Experimental setup

consists of an impedance tube split up into two sections. Elliptical resonators of various size and shape (eccentricity) can be mounted between the two section which are equipped with pressure sensors ( $s_1$  to  $s_4$ ). The two pressure sensors in each duct section allow the back and forward traveling waves ( $p_A$  to  $p_D$ ) on each side of the resonator to be calculated with the 2p-method.

The setup was modeled with the acoustic elements described above. The (complex) amplitudes of the waves traveling toward and from the resonator, denoted by  $p_A$  to  $p_D$ , are also calculated and compared with the results from the measurements in figure 4. The amplitudes of all waves are scaled to  $p_B$ , the incident wave. The dimensions that where used for the setup are listed in figure 3. Note that the dimensions of the setup are in not optimized for noise reduction. The system was designed only for validation purposes. During calculation the boundary conditions are applied by collocation. On both boundaries, 50 equidistant points are evaluated for  $p_1 = 1$  and  $v_2 = 0$  with  $m \in [-10, \ldots, 10]$ .

The amplitudes of the calculated and measured pressure waves match reasonably well for the resonator dimensions that were chosen. Notice that the different peaks and dips in the plots are found at the same frequencies for calculations and measurements. However, the amplitude of the pressure waves differ. Special attention is paid to the amplitude of the emitted sound wave  $p_{em}$  when compared with the amplitude of the emitted sound wave for a system without a resonator  $p_{em,ref}$ . The amplitude of the emitted wave drops at the resonance frequencies of the resonator. The calculated noise reduction at these frequencies is considerably lower compared to the measured results. These differences are most likely caused by loss of accuracy during the calculation of the inverse of the system described by equations (12) and (13). The geometry of the boundaries and the nature of the solution obtained with separation of variables leads to an ill-conditioned matrix. When this matrix is inverted, the *Singular Value Decomposition* algorithm that is applied cannot calculate the contribution of all modes with enough accuracy. The number of participation factors of the different fields contributing to the total solution that can be calculated is therefore limited. In this case the contribution of



Figure 4: Amplitude of backward and forward traveling waves

only the first 11 fields can be calculated. As a consequence the solution that is found does not fit the prescribed boundary conditions best. This is demonstrated in figure 5. The fact that  $v_n$ 



Figure 5: Calculated pressure and particle velocity at the resonator boundaries

does not equal zero over the entire outer boundary, means that the resonator back wall can not be considered acoustically hard, but must be considered as a partially damping wall. Failure to calculate enough participation factors to meet the boundary conditions largely accounts for the aforementioned deviations of the model compared with the measurements. These deviations will increase for ellipses with higher eccentricity and for resonators that have a small ratio between the radius of the inner boundary ( $R_{in}$ ) and the smallest radius of the ellipse. It was found, that the condition of the matrix is not dependent of the method used to apply the boundary conditions. Increasing the number of participation factors (or collocation points) does not improve the condition. The solution can be improved by using an inversion algorithm that uses higher machine precision (switching from a 32 bit to a 64 bit algorithm will allow accurate calculation of the participation factors of approximately 30 additional fields).

A second possible cause for differences between measurements and results are the profile of the boundary conditions over the boundary. The pressure over the inner resonator boundary was assumed to be constant while this need not be the case. It is more likely that the resonator will locally cause a 3D sound field in the duct. This means both the duct model and the elliptical resonator model are inaccurate close to the duct/resonator interface. However, this effect is expected to be small compared to the inaccuracies caused by numerical issues described above.

#### SUMMARY

A semi-analytical model for elliptical resonators was derived using separation of variables of the LRF equations for visco-thermal wave propagation. An experimental setup was built to validate the elliptical resonator model. The sound field in the setup is predicted reasonably well, however, the model involves inverting an ill-conditioned matrix which prevents finding the solution that fits the boundary conditions best (given the number of available contributing sound fields). Calculating the participation factors with an algorithm of higher machine precision will offer better results.

#### REFERENCES

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