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A FINITE ELEMENT APPROACH TO THE PREDICTION OF SOUND TRANSMISSION THROUGH PANELS WITH ACOUSTIC RESONATORS

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Abstract

Previous research by the authors has shown that sound radiated by a vibrating panel can be reduced considerably by using tuned acoustic resonators. The length of the tube resonators determines the frequency range in which sound is reduced. The shape of the spectrum is determined by the ratio of the cross-sectional areas of the resonators to the area of the panel. Maximum sound reduction is achieved if the volume velocities at the surface of the vibrating panel and those at the entrance of the resonators are equal in magnitude but opposite in phase. Up to now, the effect of the resonators on the radiated sound has been studied with a one-dimensional analytical model. In this paper, a three-dimensional acousto-elastic model is developed using the finite element method. The purpose of this model is to study the influence of the flexibility and the boundaries of the panel, as well as the presence of rooms behind and in front of the panel on the sound transmission. Modelling the complete structure, including the resonators and the interaction with the air inside the resonators, is computationally expensive. Therefore, an alternative approach is developed. Because of the repetitive pattern of resonators in the panel, the structural part of the panel is modelled with superelements. To enable coupling between the structural part of the model and the air behind and in front of the panel, a new interface element is derived. The formulation of this interface element also includes the acoustic behaviour of the resonators. Sound transmission loss calculations are made for one configuration and the results are compared with the results obtained with a one-dimensional analytical model.

INTRODUCTION

Previously, the effect of tube resonators on the sound radiated by a vibrating panel was studied with a one-dimensional analytical model [2]. It was shown that the radiated sound can be reduced considerably by tuning the length and the radius of these resonators. In this paper, an acousto-elastic model, based on the finite element method (FEM), is developed to study the effect in a three-dimensional setup. The model can be used to calculate the sound transmission loss of panels with resonators for different geometries and boundary conditions. Figure 1 shows a resonator panel for the situation studied in this paper. At the incident side, the panel is acoustically excited by applying a harmonic pressure perturbation on the panel. The sound is transmitted into a room with sound absorbing walls. Using a fully coupled FEM model of the complete structure, the air inside the resonators, and the room, is computationally expensive. Therefore, an alternative approach is developed.

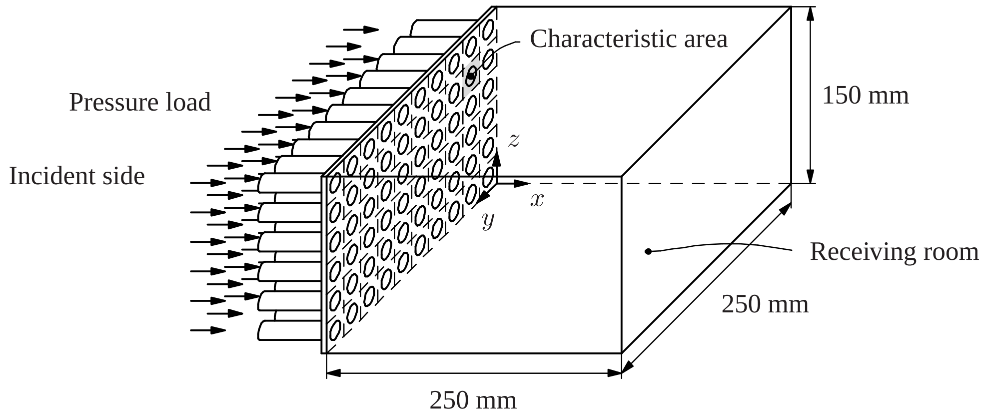

Figure 1: Quarter of resonator panel with a room at one side.

The structure is characterised by a repetitive pattern of resonators (see Figure 1). A part of the plate containing one resonator is called a characteristic area. By means of Guyan reduction, a superelement is generated which represents the structural part of these characteristic areas. Figure 2(a) shows the mesh used to generate the superelement for the calculations in this paper. Because of the FEM code that is adopted, the meshes of the structural and the acoustic domain have to coincide. The air in the room is modelled with 20-node hexahedral fluid elements (see Figure 2(b)). Therefore, the superelement can only contain eight master nodes at the plate boundaries (see Figure 2(a)). A ninth master node is located at the end of the resonator to take into account the inertia effects of the tube. All master nodes have six degrees of freedom; three displacements and three rotations. To enable coupling between the structure and the air behind and in front of the panel, a new interface element is derived, which also includes the acoustic behaviour of the resonators. In this paper, the formulation of this interface element is derived and the effect of the interface element is demonstrated by means of an example.

FINITE ELEMENT FORMULATION OF INTERFACE ELEMENTS

Figure 3 shows a schematic representation of a characteristic area with a resonator of length L. p_l and p_r are the pressure perturbations at the left-hand side and the right-hand side of the panel, respectively. The purpose of the interface element is to relate the pressure perturbations

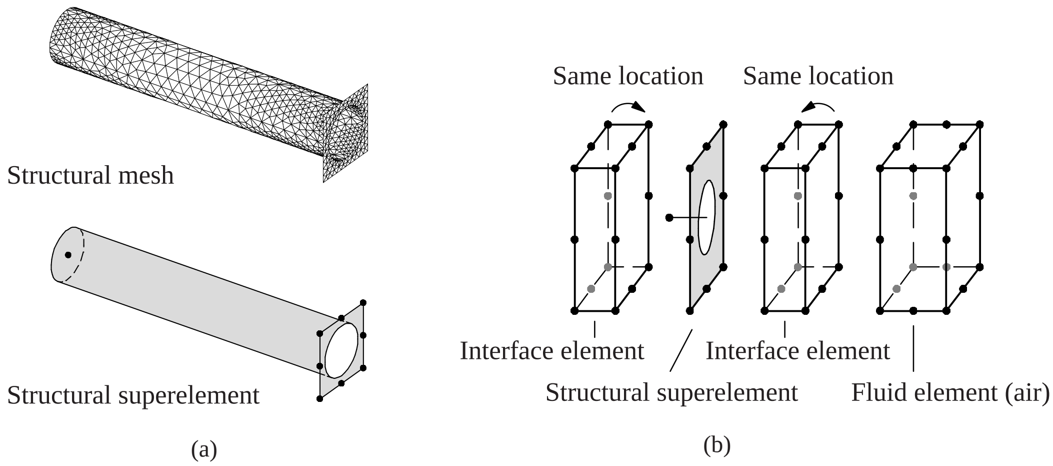

Figure 2: Structural mesh and superelement (a) and the element types to model a resonator panel with a room at one side (b).

to the structural displacements of the panel (see also Figure 2(b)).

For the derivation of the interface element, both the plate and the tube resonator are assumed to have the same normal structural velocity amplitude $\dot{u}^s$. Within the characteristic area, no local effects of acousto-elastic interaction are taken into account. The amplitude p_j of the pressure perturbation and the amplitude $\dot{u}^a_j$ of the particle velocity perturbation in axial direction, in and around the resonators, are described by:

$$p_j(x) = A_j e^{\mathrm{i}kx} + B_j e^{-\mathrm{i}kx} \qquad j = 1, 2 \tag{1}$$

$$\dot{u}^a_j(x) = -\frac{1}{\rho_0 c_0}\left(A_j e^{\mathrm{i}kx} - B_j e^{-\mathrm{i}kx}\right) \qquad j = 1, 2 \tag{2}$$

where i is the imaginary unit, ρ_0 is the density of air, c_0 is the speed of sound, and $k = \omega/c_0$ is the wave number, with ω the angular frequency. A_j and B_j are the amplitudes of the backward and forward travelling sound waves in and around the resonators, respectively, determined by the boundary conditions of these domains.

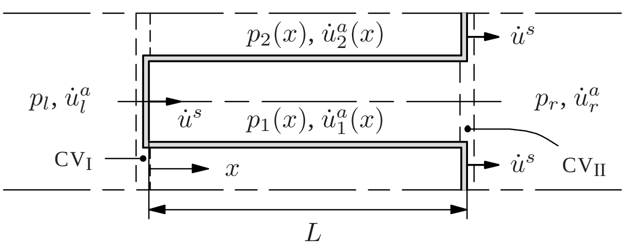

Figure 3: Characteristic area with parameters used to formulate the interface element.

Boundary conditions at the right-hand side of the panel require that the pressure perturbation is continuous at the entrance of the resonator; the particle velocity at the end of the resonator is equal to the structural velocity; and conservation of mass holds for the control volume CV$_{\mathrm{II}}$

at the resonator entrance (see Figure 3). At the left-hand side of the panel similar boundary conditions have to be satisfied. The conditions are formulated as follows:

$$p_2(0) = p_l \tag{3}$$

$$\dot{u}_2^a(L) = \dot{u}^s \tag{4}$$

$$S\dot{u}_l^a = S_l\dot{u}_2^a(0) + (S - S_l)\dot{u}^s \tag{5}$$

$$p_1(L) = p_r \tag{6}$$

$$\dot{u}_1^a(0) = \dot{u}^s \tag{7}$$

$$S\dot{u}_r^a = S_r\dot{u}_1^a(L) + (S - S_r)\dot{u}^s \tag{8}$$

where S is the characteristic area, and S_r and S_l are the cross-sectional areas of the resonator and the space around the resonators, respectively. By substitution of equations (3) and (4) into equations (1) and (2), the pressure amplitudes A_2 and B_2 can be solved. Substituting these solutions back into equation (1), gives the following expression for the pressure perturbation at the panel:

$$p_2(L) = p_l \sec(kL) - \rho_0 c_0 \dot{u}^s \mathrm{i} \tan(kL) \tag{9}$$

where $\sec(kL) = 1/\cos(kL)$. The total force at the left-hand side of the characteristic area is:

$$F_l = (S - S_l)\, p_l + S_l\, p_2(L) \tag{10}$$

Assuming the force to be uniformly distributed over the area, the distributed load is written as $q_l = F_l/S$. Introducing the porosity $\Omega_l = S_l/S$ and substitution of equation (9) into the expression for the distributed load, gives:

$$q_l = p_l[1 - \Omega_l + \Omega_l \sec(kL)] - \rho_0 c_0 \dot{u}^s \mathrm{i}\Omega_l \tan(kL) \tag{11}$$

Using equations (2) and (5), the particle velocity perturbation at the left-hand side of the panel can be written as:

$$\rho_0 c_0 \dot{u}_l^a = \rho_0 c_0 \dot{u}^s[1 - \Omega_l + \Omega_l \sec(kL)] + p_l \mathrm{i}\Omega_l \tan(kL) \tag{12}$$

By assuming harmonic structural velocity perturbations, relations (11) and (12) can be formulated as:

$$q_l = p_l[1 - \Omega_l + \Omega_l \sec(kL)] + \rho_0 c_0 u^s \omega \Omega_l \tan(kL) \tag{13}$$

$$\rho_0 c_0 \ddot{u}_l^a = \rho_0 c_0 \ddot{u}^s[1 - \Omega_l + \Omega_l \sec(kL)] - p_l \omega \Omega_l \tan(kL) \tag{14}$$

In the same way, by using equations (6) to (8), similar expressions are found for the distributed load and the particle acceleration at the right-hand side of the panel. The only difference is the change of sign of the accelerations, velocities and displacements. So at both sides of the panel the same interface elements can be used. From now on the subscripts l and r will be omitted. For the FEM discretisation, the pressure perturbations p and the normal structural displacements u^s are written in terms of vectors with nodal pressures $\mathbf{p}$ and nodal displacements $\mathbf{u}^s$, and vectors with interpolation functions $\mathbf{N}^a$ and $\mathbf{N}^s$:

$$p = [\mathbf{N}^a]^{\mathrm{T}}\, \mathbf{p}, \quad u^s = [\mathbf{N}^s]^{\mathrm{T}}\, \mathbf{u}^s \tag{15}$$

The element matrices for the acoustic part of the interface element are obtained by discretising the wave equation using the Galerkin method. Multiplying the equation by a virtual pressure perturbation δp and integrating over the volume of the domain yields the variation of a functional [4]. The mass coupling matrix follows from the contribution $\delta\Pi^a$ of the pressure on the boundary of the domain to this variation:

$$\delta\Pi^a = \int_{\Gamma^{as}} \rho_0 \, \delta p \, \ddot{u}^a \, \mathrm{d}\Gamma \tag{16}$$

where $\ddot{u}^a$ is the outward (out of the air domain) normal component of the particle acceleration perturbation, and Γ^{as} is the interface area. Substitution of equations (14) and (15) into equation (16) gives for the characteristic area:

$$\delta\Pi^a = [\delta\mathbf{p}]^{\mathrm{T}} \, \mathbf{M}^{as}_{pu}(\omega) \, \ddot{\mathbf{u}}^s + [\delta\mathbf{p}]^{\mathrm{T}} \, \mathbf{M}^{as}_{pp}(\omega) \, \ddot{\mathbf{p}} \tag{17}$$

with the mass coupling matrices formulated as:

$$\mathbf{M}^{as}_{pu}(\omega) = \int_{\Gamma^{as}} \rho_0 \, [1 - \Omega_l + \Omega_l \sec(kL)] \, \mathbf{N}^a \, [\mathbf{N}^s]^{\mathrm{T}} \, \mathrm{d}\Gamma \tag{18}$$

$$\mathbf{M}^{as}_{pp}(\omega) = \int_{\Gamma^{as}} \frac{\Omega \tan(kL)}{\omega c_0} \, \mathbf{N}^a \, [\mathbf{N}^a]^{\mathrm{T}} \, \mathrm{d}\Gamma \tag{19}$$

The stiffness coupling matrices follow similarly from the formulation of the structural part of the standard acousto-elastic interaction problem. By using equations (13) and (15), the contribution $\delta\Pi^s$ of the pressure on the interface to the variation of the functional can be written as:

$$\delta\Pi^s = -\int_{\Gamma^{as}} \delta u^s q \, \mathrm{d}\Gamma = [\delta\mathbf{u}^s]^{\mathrm{T}} \, \mathbf{K}^{as}_{up}(\omega) \, \ddot{\mathbf{p}} + [\delta\mathbf{u}^s]^{\mathrm{T}} \, \mathbf{K}^{as}_{uu}(\omega) \, \ddot{\mathbf{u}}^s \tag{20}$$

with the stiffness coupling matrices formulated as:

$$\mathbf{K}^{as}_{up}(\omega) = -\int_{\Gamma^{as}} [1 - \Omega_l + \Omega_l \sec(kL)] \, \mathbf{N}^s \, [\mathbf{N}^a]^{\mathrm{T}} \, \mathrm{d}\Gamma \tag{21}$$

$$\mathbf{K}^{as}_{uu}(\omega) = -\int_{\Gamma^{as}} \rho_0 c_0 \omega \Omega \tan(kL) \, \mathbf{N}^s \, [\mathbf{N}^s]^{\mathrm{T}} \, \mathrm{d}\Gamma \tag{22}$$

The resulting set of FEM equations for the interface element is given by:

$$\begin{bmatrix} \mathbf{M}^s & \mathbf{0} \\ \mathbf{M}^{as}_{pu}(\omega) & \mathbf{M}^a + \mathbf{M}^{as}_{pp}(\omega) \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}^s \\ \ddot{\mathbf{p}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}^s + \mathbf{K}^{as}_{uu}(\omega) & \mathbf{K}^{as}_{up}(\omega) \\ \mathbf{0} & \mathbf{K}^a \end{bmatrix} \begin{Bmatrix} \mathbf{u}^s \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}^s \\ \mathbf{0} \end{Bmatrix} \tag{23}$$

where $\mathbf{M}^s$, $\mathbf{M}^a$, $\mathbf{K}^s$ and $\mathbf{K}^a$ are the structural and acoustic mass and stiffness matrices, respectively. If $\Omega = 0$ or $L = 0$, $\mathbf{M}^{as}_{pp}$ and $\mathbf{K}^{as}_{uu}$ become zero and $\mathbf{M}^{as}_{pu}$ and $\mathbf{K}^{as}_{up}$ change into the standard ω-independent coupling matrices.

SOUND TRANSMISSION LOSS

The transmission coefficient τ is defined as the fraction of incident acoustic energy which is transmitted through the panel. The sound transmission loss TL is subsequently defined as:

$$TL = 10\log_{10}(1/\tau) = 10\log_{10}\frac{(\bar{I}_i)_{\text{avg}}}{(\bar{I}_t)_{\text{avg}}} \tag{24}$$

where $(\bar{I}_i)_{\text{avg}}$ and $(\bar{I}_t)_{\text{avg}}$ are the time- and space-averaged incident and transmitted sound intensities, respectively. The incident and transmitted sound intensities are calculated normal to the panel and spatially averaged over the surface area of the panel[1]. In case of harmonic time dependence, the time-averaged sound intensity $\bar{I}_n(\mathbf{r})$ at position $\mathbf{r}$ in direction n is given by:

$$\bar{I}_n(\mathbf{r}) = \frac{1}{2}\text{Re}[p(\mathbf{r})\dot{u}_n^{a*}(\mathbf{r})] \tag{25}$$

where * denotes the complex conjugate. At the incident side, a distinction has to be made between incident and reflected acoustic energy. However, from the finite element calculations only total pressure and velocity perturbations are known. Usually, in case of plane wave excitation and a fully reflective surface, the incident part of the pressure perturbation on the panel is approximately half the total pressure [1]. However, in case of a resonator panel the reflection coefficient is not equal to one. Therefore, the pressure amplitude of the incident sound wave is estimated by means of an alternative, one-dimensional approach here.

For normal incident plane waves, the pressure perturbation p_l can be written in terms of incident and reflected sound waves according to equation (1). The boundary conditions at the incident side of the panel are then given by:

$$A_l + B_l = p_l \tag{26}$$

$$A_2 + B_2 = p_l \tag{27}$$

$$S(A_l - B_l) = (S - S_l)(A_2 - B_2) - S_l\rho_0 c_0 \dot{u}^s \tag{28}$$

$$A_2 e^{\text{i}kL} - B_2 e^{\text{i}kL} = -\rho_0 c_0 \dot{u}^s \tag{29}$$

From these equations, the unknown pressure amplitude B_l of the incident sound wave is solved as function of the quantities p_l and $\dot{u}^s$:

$$B_l = \frac{p_l}{2}[1 + \text{i}\Omega_l\tan(kL)] + \frac{\rho_0 c_0 \dot{u}^s}{2}[1 - \Omega_l + \Omega_l\sec(kL)] \tag{30}$$

The values of the quantities p_l and $\dot{u}^s$ result from the FEM analysis. Using equation (25), the incident sound intensity on the panel is calculated by:

$$\bar{I}_i = \frac{\text{Re}(B_l)^2 + \text{Im}(B_l)^2}{2\rho_0 c_0} \tag{31}$$

If $\Omega_l = 0$ or $L = 0$, the solution converges to the solution for a panel without resonators.

[1] Spatially averaging the sound intensities may be done since a regular grid is used here. If this is not the case, the surface integral of the normal sound intensities has to be taken to calculate the sound powers.

EXAMPLE

The transmission loss is calculated for an aluminium resonator panel in the setup as shown in Figure 1. The panel is a simply supported plate with dimensions 500 $\times$ 300 $\times$ 2.0 mm, consisting of 20 $\times$ 12 characteristic areas. The resonators have a length of 110 mm, a radius of 9.5 mm, a wall thickness of 0.3 mm, and a thickness of the end of 0.6 mm. The properties of aluminium are: density $\rho = 2700$ kg/m^3, Young's modulus $E = 70 \cdot 10^9$ N/m^2, and Poisson's ratio $\nu = 0.3$. The dimensions of the room are 500 $\times$ 300 $\times$ 250 mm, and the properties of air are $\rho_0 = 1.2$ kg/m^3 and $c_0 = 343$ m/s.

Finite element model

The system is modelled using an in-house FEM code under MATLAB, in which the new interface element has been implemented. The element matrices of the 9-node superelement are generated in ANSYS, and subsequently imported in MATLAB. To mesh the acoustic domain, 20-node hexahedral fluid elements are used. The superelements are coupled to the acoustic domain via the eight nodes on the plate (see Figure 2(b)). The node at the end of the resonator remains uncoupled. At the incident side, a uniformly distributed pressure load of 1 Pa, representing a normal incident plane wave, is applied directly on the interface elements. The five sound absorbing walls of the room are modelled by prescribing a dimensionless impedance of one. Since both structure, acoustic domain and pressure load are symmetric, only a quarter of the panel is modelled. The frequency range that is studied is 500-3000 Hz.

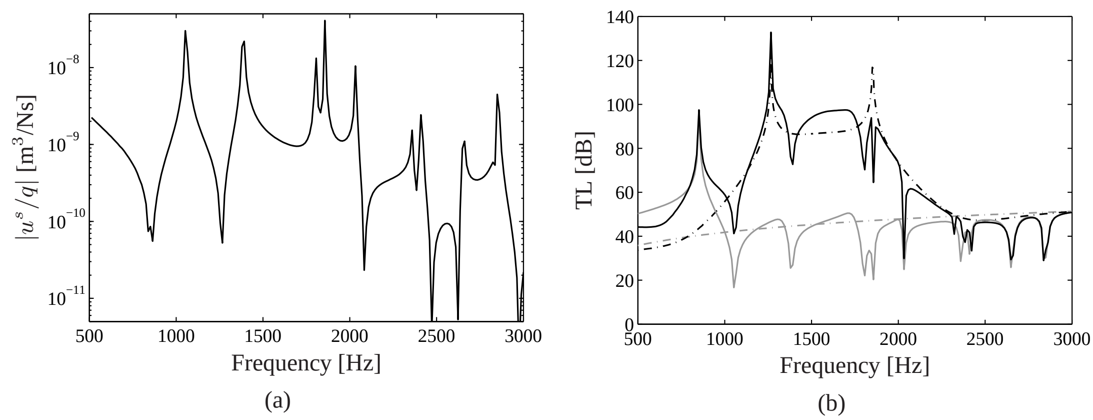

Figure 4: Frequency response function (a) and sound transmission loss (b): mass law (-··-··), one-dimensional analytical model (-··-··-), FEM without resonators (—), and FEM with resonators (—).

Sound transmission loss

Figure 4(a) shows the response of a point on the panel ($y = 75$ mm, $z = 125$ mm) to the pressure load. Figure 4(b) shows the sound transmission loss for the considered resonator panel. The transmitted sound intensity is calculated at a distance of 88 mm from the panel. The particle velocities in the acoustic domain are determined using the derivatives of the shape functions to calculate the pressure gradients at the centroids of the elements and subsequently applying Euler's equation. The pressures are also calculated at the centroids of the elements. Figure 4(b) also shows the normal incidence transmission loss for the same configuration calculated with a one-dimensional analytical model [3]. The results are compared with the normal incidence mass law, which is given by [1]:

$$TL = 10 \log_{10} \left[1 + \left(\frac{m\omega}{2\rho_0 c_0} \right)^2 \right] \tag{32}$$

where m is the mass per unit area of the panel. Furthermore, the sound transmission loss is shown for a panel without resonators but with the same mechanical properties. It is seen that the trends of the curves obtained with the FEM model and the one-dimensional analytical model are the same. The structural eigenfrequencies of the panel have a large, negative, influence on the sound transmission loss. For other frequencies, the structural behaviour of the panel has a positive effect on the calculated sound transmission loss. The structural behaviour of the panel seems to have the same influence on both the panel with resonators and the panel without resonators.

CONCLUSIONS

To study sound transmission through a panel with tube resonators, a three-dimensional coupled model was developed using the FEM. A new type of interface element was formulated, which not only enables coupling between the structural part of the model and the air behind and in front of the panel, but also includes the acoustic behaviour of the resonators. Normal incidence transmission loss calculations for a specific resonator panel configuration showed the same trend as the results obtained with a one-dimensional analytical model. A large increase in sound transmission loss is predicted in a wide frequency range. The structural eigenfrequencies of the panel have a negative effect on the predicted sound transmission loss, while for other frequencies the structural behaviour of the panel has a positive effect on the calculated sound transmission loss.

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