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MODELLING WAVE PROPAGATION IN THIN PLATES FOR THE LOCALIZATION OF TACTILE INTERACTIONS

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Abstract

Tactile interactions with thin plates generate modal propagation of elastic waves that, in the frequency range of interest (*0-10 kHz*), is made of only the two lowest-order modes (flexural and longitudinal). Using sensors that acquire just vertical displacements, the flexural mode is, in fact, the only one that is recorded. In order to localize the interaction from the acquired signals using such sensors, we consider two classes of solutions: TDOA (Time Delay Of Arrival) and CC (localisation by Cross-Correlation). In both cases a method that predicts the response of the board to a tactile excitation would be extremely useful, as it would enable TDOA to greatly improve its accuracy; in addition, it would greatly simplify the learning phase of CC techniques. With these goals in mind, we developed two methods for predicting the board response. The former computes the exact flexural modal solution by setting free boundary conditions only on the interaction surfaces while assuming the plate to be of infinite extension. The plate's borders are then accounted for through ray tracing. The latter is based on the numerical solution of the elastic wave equation, which inherently takes into account the reflections at the plate's border. We tested and compared the two methods on real data, which proved effective on a wide range of situations. The former turned out to excel with larger plates and in the presence of weak reverberations, while the second proved more accurate in complementary situations. As both methods need to know the excitation signal and the elastic parameters of the plate, we also developed a method for computing the excitation through wave dispersion compensation based on inverse propagation, and a method for estimating the elastic parameters from the acquired data. This latter solution was validated through measurements in the ultrasonic range.

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INTRODUCTION

Nicholson at al. [1] analyzed the elastic wave propagation in panels considering two different types of geometry: a semi-infinite solid half-space with the transducers on the free surface, and infinite solid plates, with different thicknesses and with the transducers on the upper free surface. In the semi-infinite solid half-space the situation is clear: we can recognize the longitudinal (*P-wave*) and shear (*S-wave*) wavefronts, the Rayleigh wave localized close to the surface and the lateral or head wave [2]. In the infinite solid plate with thicknesses of the same order of the wavelength the situation is much more complex, due to the presence of guided waves generated by the interaction of the different wavefronts with the surfaces of the plate. Finally for infinite solid plates with smaller thicknesses, only guided wave arrivals are visible. Tucker [3] showed that the previous behaviour can be found in panels made up of many materials, like Medium Density Fiberboard (MDF), plexiglass (PLX), aluminium, etc. The boards are “perceived” by the wave as homogeneous (through the thickness), orthotropic plates as long as the wavelength λ remains much larger than the panel thickness h. The “perception” of the material greatly reduces the complexity of the equations needed to describe the plate wave propagation (also commonly termed *Lamb* or *guided wave propagation*) [4].

There are two distinct types of plate waves: symmetric (s) and antisymmetric (a), each of which have an infinite number of modes (s_0, s_1, s_2, …, s_n and a_0, a_1, a_2, …, a_n) at higher frequencies. The lowest modes are also called extensional (s_0) and flexural (a_0) Lamb modes. Plate waves are dispersive by nature, meaning that different frequencies travel at different speeds (*phase velocities*). The phase velocity v_{ph} is the fundamental characteristic of the Lamb wave and once it is known we can determinate the wave number and calculate the stresses and displacements at any point of the plate.

Extensional and flexural modes should be distinguished in the same received signal but it can be proved that the extensional mode does not propagate below certain frequencies [3] and we therefore only consider the flexural mode in this paper.

SOLUTION FOR WEAKLY REVERBERATING FLEXURAL PLATES

In large plates (for example *1m* square boards) or in small (for example *30cm* square boards) weakly reverberating (made up of MDF, PLX, etc.) ones the exact flexural modal solution can be found by solving a characteristic equation after setting free boundary conditions on the interaction surfaces, while assuming the plate to be of infinite extension. The plate’s borders can then be accounted for through ray tracing.

The Characteristic Equation

If $h = 2d$ is the thickness of the plate, k_β is the *S-wave* number, v_α and v_β are the *P-wave* and *S-wave* velocities and v_a is the phase velocity of the antisymmetric Lamb waves, then the characteristic equation for the antisymmetric modes is:

$$\frac{tg\left(\bar{d}\sqrt{1-\zeta_a^2}\right)}{tg\left(\bar{d}\sqrt{\xi^2-\zeta_a^2}\right)}+\frac{\left(2\zeta_a^2-1\right)^2}{4\zeta_a^2\sqrt{1-\zeta_a^2}\sqrt{\xi^2-\zeta_a^2}}=0 \tag{1}$$

where $\bar{d}=k_\beta d$, $\zeta_{s,a}^2=\frac{v_\beta^2}{v_{s,a}^2}$ and $\xi^2=\frac{v_\beta^2}{v_\alpha^2}$.

Many authors have performed calculations of the phase velocities and their dependence on the plate thickness and frequency (*dispersion curves*). To achieve this purpose the elastic properties of the medium (in our tests the bulk velocities v_α and v_β) are necessary. In the next section we show how to estimate these properties and how to calculate the dispersion curves for the s_0 and a_0 modes.

Estimation Of The Elastic Properties Of The Plate

A transducer *T* converts electrical energy into mechanical energy, propagated through a thin panel in form of elastic waves. Two sensors Rx_1 and Rx_2, placed a known distance apart, can be used to receive the signals associated to these waves, to calculate their phase difference and thus to measure their phase velocity (Fig. 1). Then different phase velocity observations can be used to estimate the elastic properties (v_α and v_β) of the panel by solving a bidimensional optimization problem. A more detailed description of this method is reported in the following.

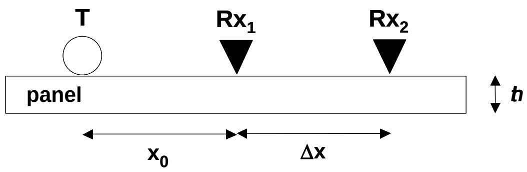

Figure 1 - Technique to calculate the dispersion curves of a thin panel.

Estimation Of The Signature Of The Excitation Signal

In this section we propose a simple scheme to estimate the finger touch signature. Let us consider a receiver *Rx*, located at the centre of the board, and the corresponding acquired signal *s*. This latter is thus not affected by problems of overlap between the direct arrival and the signals reflected from the borders of the panel (*edge reflections*). If the position of the touch (x_T, y_T) is known, the transmitted signature can be estimated by *inverse propagating s* of the exact distance between source and receiver. The inverse propagation is obtained by filtering and the filter is designed in the frequency domain by using the knowledge of the plate wave theory and of the estimated elastic properties of the panel.

In an experiment on a MDF panel with the thickness of *5mm*, the estimated signature of the finger touch, after inverse propagating the signal *s*, is shown in Fig. 2. It is impulsive with a time duration of about *6 ms*.

Validation Of The Method With The Observed Data

The knowledge of the transmitted signal, of the plate elastic properties and of the wave propagation model allows the calculation of the direct arrival acquired by any receiver (*simulated or recalculated data*): the transmitted signal can be *forward propagated* of the exact distance between source and receiver.

Since we want to simulate the complete elastic wave propagation in the plate, in order to compare the observed response with the calculated one, we have to take into account the edge reflections. A fast beam tracer [5, 6] can be used to achieve this purpose. We can therefore compute the complete board response as the result of the sum of the signals due to the direct arrival and to the most energetic reflected rays.

Let us consider an experiment on a MDF plate, whose configuration is shown in Fig. 3. We calculate the direct arrival, corresponding to the ray directly linking the source with the receiver (bold line) and the first four delayed arrivals, corresponding to the path of the rays reflected only once by the borders of the plate and linking the source with the receiver (solid lines).

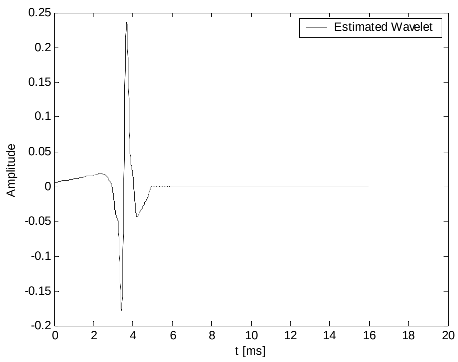

Figure 2 – Estimated signature of the finger touch in a MDF plate.

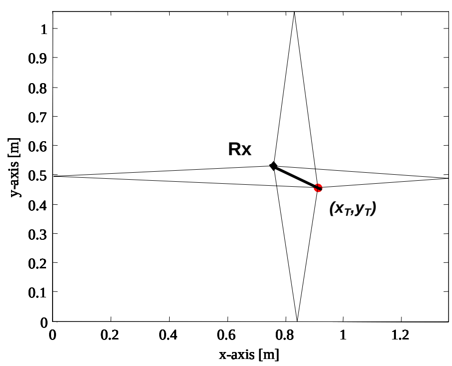

Figure 3 - Direct arrival and the first four reflected rays.

First we compare the observations with the simulated data only considering the direct arrival (Fig. 4). There is a good agreement before the arrival of the reflected waves (about *6 ms*). Then we add the information about the reflections and the more reflected rays we consider in the computation of the simulated signal response, the more the agreement between observations and calculated data is good. In Fig. 5 we compare the observed data with the calculated one, considering the direct arrival and the most energetic reflected rays.

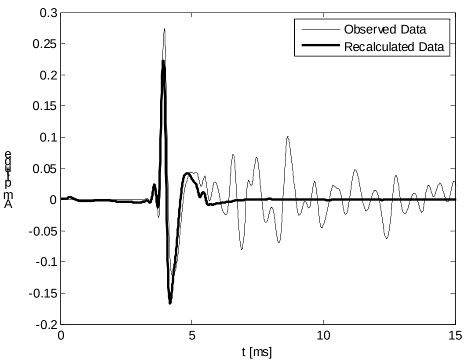

Figure 4 - Comparison of the observations with the simulated data only considering the

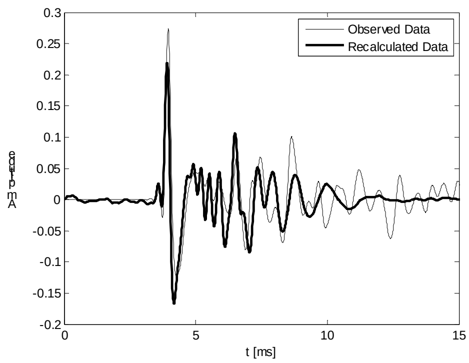

Figure 5 - Comparison of the observations with the simulated data considering the

direct arrival. *direct arrival and four reflected rays.*

SOLUTION FOR REVERBERATING AND WEAKLY ATTENUATIVE FLEXURAL PLATES

When dealing with metal plates of relatively small size, another solution has been developed, that takes into account diffraction by the borders of the plate. Except for plates with simply supported edges, no analytical solution for the free vibration of plates exist [7]. Therefore, the wave propagation equation is numerically solved, taking into account the boundary conditions. We present here the case of clamped boundaries, but the method could be extended to the case of free boundaries as well, on condition that one uses a more precise numerical scheme because of the end resonance of the plate.

Propagation Equation Used

A finger knock given at the surface of a plate gives birth to flexural waves in the audible range. In other words, the created wave is the first antisymmetric Lamb mode, noted a_0. Actually, when the product frequency by plate thickness is small compared to one, the transverse displacement component *w* does not depend on the thickness coordinate any more, and the classical flexural plate theory is valid. According to it, *w* obeys the following equation [8]:

$$\Delta^2 w + \frac{\rho h}{D}\frac{\partial^2 w}{\partial t^2} = S(x,y,t) \qquad (2)$$

where D is equal to $\frac{Eh^3}{12(1-\nu^2)}$, E being the Young's modulus, h the plate thickness, ν the Poisson ratio, ρ the density (kg/m^3), and $S(x,y,t)$ the source term. Now in an isotropic plate, it is easy to show that this equation is equivalent to:

$$\Delta^2 w + \frac{12}{v_P^2 . h^2}\frac{\partial^2 w}{\partial t^2} = S(x,y,t) \qquad (3)$$

with v_P the plate velocity defined as $2v_\beta\sqrt{1-\frac{v_\beta^2}{v_\alpha^2}}$, v_β being the shear velocity and v_α the longitudinal velocity. Therefore, to compute the wave propagating in a plate, one will need to measure the bulk velocities and the plate thickness.

Now suppose that the boundary situated at x equal to 0 is clamped. The well-known clamped condition on the transverse displacement w is the following:

$$\left.\frac{\partial w}{\partial x}\right|_{x=0} = \left.w\right|_{x=0} = 0 \qquad (4)$$

Numerical Scheme And Stability Criterion

We chose an explicit finite differences scheme centred in space and in time. We will note $A = \frac{{v_p}^2 h^2}{12}$, and $w_{i,j}^n$ the displacement component at position *x* equal to $i\Delta x$, *y* equal to $j\Delta y$, and at time step *n*. Then the sampled equation is:

$$A(\Delta^2 w)^{n-1} + \frac{w_j^n - 2w_j^{n-1} + w_j^{n-2}}{(\Delta t)^2} = S(i\Delta x, j\Delta y, n\Delta t) \qquad (5)$$

where $(\Delta^2 w)^{n-1}$ is the double Laplacian operator applied to *w* at time step *n-1*, the simple sampled Laplacian operator being :

$$\Delta w = \frac{w_{i+1,j} - 2w_{i,j} + w_{i-1,j}}{(\Delta x)^2} + \frac{w_{i,j+1} - 2w_{i,j} + w_{i,j-1}}{(\Delta y)^2} \qquad (6)$$

Obviously, Eq. 6 allows to compute the displacement at time step *n* using the displacements at previous time steps.
Thanks to the Fourier stability analysis [9], the stability criterion is the following : for a given sampling step in space Δ (in the case where we took the same in both the *x* and *y* direction), which is imposed by the smallest wavelength one wants to simulate, the sampling step in time must obey the relation :

$$\Delta t < \frac{(\Delta)^2}{4\sqrt{A}} \qquad (7)$$

Validation With Experiments Found In Literature

The advantage of this simulation is that it is expected to reproduce the eigen modes of a free vibrating clamped plate. Hazell [10] measured the eigen frequencies of a *1.83mm* by *30.5cm* by *30.5cm* aluminium plate. From data given in this article, we could deduce that the shear velocity was *3000m/s* and the longitudinal velocity was *5874.4m/s*. Using these parameters in our simulation, with a sampling step in space of *2mm*, the propagation of a pulse during *50µs* was computed. Fig. 6 shows the spectrum of the simulated wave at a certain point, compared with the eigen frequencies experimentally measured by Hazell : there is a good agreement between both, especially at low frequencies, as expected by the theory.

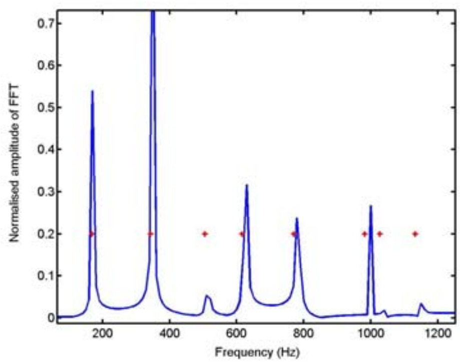

Figure 6 – Solid blue line: Normalized amplitude of FFT of simulated signal in an aluminum plate; red markers: experimental eigen-frequencies measured in the same plate in literature.

EXPERIMENTAL DETERMINATION OF THE BULK VELOCITIES

Pulse-Echo Method In The Ultrasonic Range

The first method that can be used to measure the bulk velocities in the plate is a simple pulse-echo measurement. To this end, one needs first a shear transducer, and then a longitudinal one. The transducer is glued at the plate surface, and sends a pulse; then the same transducer measures the reflected echoes. If the pulse is short enough, two successive echoes are well separated, and the time between them allows to deduce the velocity.

However, this technique will give good results with weakly attenuating materials only. Indeed, for example in wood, attenuation is so high that in a *5mm* thick plate, no echo comes back for a *2MHz* central frequency pulse. In intermediate situations, given that attenuation increases with frequency, the high frequencies contained in the pulse can vanish when travelling through the thickness, and the measured echo, containing only the lowest frequencies, will arrive with an additional delay: the velocity may be underestimated.

As a conclusion, the pulse-echo method is a good one for weakly attenuating media such as metal plates; for other media, another method was developed, as presented in the following section.

Active Method In The Low Frequency Range

The transducer T (Fig. 1) generates sinusoidal bursts ranging from *1500 Hz* to *5000 Hz* for the MDF, with a step of *250 Hz*. At these frequencies only a_0 Lamb mode can be excited. The observed phase velocities are obtained by averaging several realizations (also varying Δx and x_0) to reduce the effect of noise. For each frequency, the difference between the theoretical $v_{a,cal}$ and the experimental $v_{a,obs}$ phase velocities is computed and optimal values for *P-wave* and *S-wave* velocities, v_α and v_β, can be obtained by minimizing the data residual R_d:

$$R_d(v_\alpha, v_\beta) = \left\| v_{a,obs} - v_{a,cal}(v_\alpha, v_\beta) \right\|^2 \qquad (8)$$

The minimization problem is solved with an exhaustive search approach and the estimated elastic properties for the MDF board are $v_\alpha = 2900\ m/s$ and $v_\beta = 1600\ m/s$. These values are in a good agreement with the corresponding ones found in literature. Observed data and calculated data are shown in the range of frequencies of the measurements in Fig. 7.

CONCLUSIONS

A study of the elastic wave propagation in thin plates has been conducted in order to develop a method for predicting the board response.

We presented two complementary approaches: the former excels with large and weakly reverberating flexural plates, while the latter with reverberating and weakly attenuative ones. Both methods need to know the excitation signal and the elastic parameters of the plate to simulate the propagation in the panels. The signature transmitted by a finger touch is estimated through an inverse propagation technique, while the elastic properties (bulk velocities) of the board can be recovered from the acquired data with two different techniques, based on the estimation of the bulk velocities respectively in the low frequency range and in the ultrasonic range.

We compared the predictions of different plate responses with the observation, showing a good agreement on a wide range of situations.

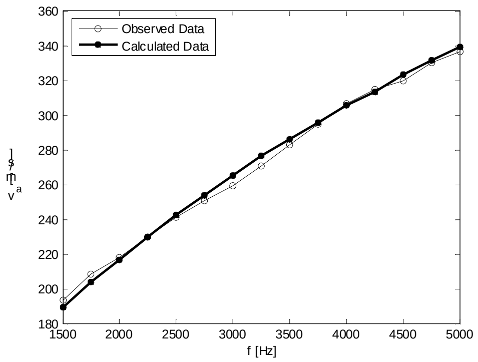

Figure 7 - Phase velocities of the a_0 mode in the MDF board: measured and calculated data.

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