ICSV13 - Vienna

The Thirteenth International Congress on Sound and Vibration

Vienna, Austria, July 2-6, 2006

THE VIBRATIONAL MODES OF THE NOTES OF THE TENOR STEELPAN: A FINITE ELEMENT STUDY

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Abstract

The steelpan is a relatively new percussion instrument developed in the Caribbean country of Trinidad and Tobago in the 1930's. For fullness of sound, it is necessary that its frequency spectrum has a strong sense of pitch. This can be achieved if at least the first three natural frequencies are integer multiples of the fundamental. It is therefore important that the vibration characteristics of the notes be understood. Several parameters affect the vibrational modes of each note: its area, height, aspect ratio and thickness. In this work a finite element study of the notes of the steelpan was conducted. Relationships indicating the variation of the frequency ratios with each parameter were developed. This information can form the basis for a shape optimization scheme for the notes of the steelpan.

INTRODUCTION

The steelpan is a relatively new percussion instrument developed in the Caribbean country of Trinidad and Tobago in the 1930's. There are a number of playing ranges in the steelpan from the tenor pan with 29 notes (from D4 to G6) to the bass pan (G1 to C4). Each pan is constructed from a portion of a 55 gallon oil drum. The flat end of the oil drum is hammered into a shallow well and a portion of the cylindrical part of the drum is retained to form the skirt. The concave ellipsoidal shape forms the playing portion of the drum. The skirt acts as a resonance chamber; it is relatively long for low-pitched instruments such as the bass, cello and guitar steelpans and relatively short for the high-pitched pans such as the tenor, the double tenor and the double second. This work will examine the vibration pattern of the tenor pan which is typically sunk to a depth of about 19 cm and has a skirt about 13 cm in length. A schematic of a tenor pan is shown in Figure 1.

Eds.: J. Eberhardsteiner, H.A. Mang, H. Waubke

After the dome has been sunk, the sections and notes of the pan are marked off and acoustically separated from each other. This may be done by indenting the borders of the notes with a steel punch (grooving) or by making small holes around each note. The steelpan is then ‘tempered’ to more evenly distribute the stresses induced by prior processing of the instrument. The final stage involves careful tuning of the playing surface. The actual note is either flat or slightly convex. Because of the variety of these processes, the note domains differ in geometry, in elastic properties and in initial stresses. Thus, when the note is properly tuned and impacted, a variety of interacting and non-interacting localized modes vibrate based on the natural frequencies.

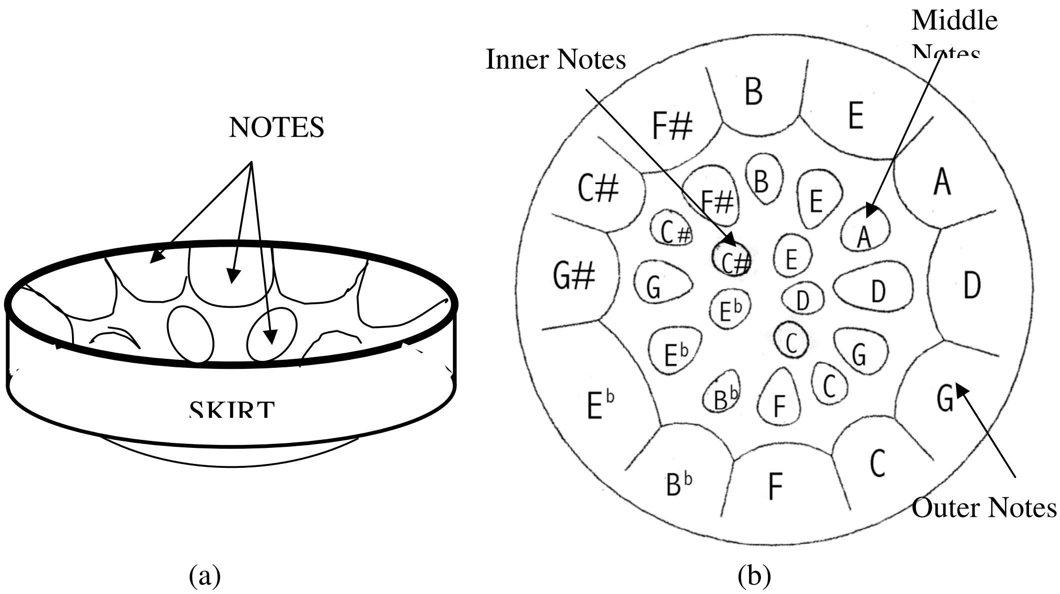


Figure 1: (a) Schematic of a Typical Steelpan.; (b) Notes on the high tenor steelpan

There is still much scope for improvement in the manufacturing and tuning processes of the steelpan. Currently only experienced panmakers can tune pans and it is a relatively slow process. To simplify these processes, it is important that the vibration characteristics of the instrument be better understood. In recent years, several researchers have investigated the steelpan experimentally. Maharaj [1]describes the experimental analysis of the vibrational modes of the tenor steelpan using impact excitation, while Rossing *et. al.* [2] studied these modes using electronic TV holography. Muddeen and Copeland [3] examined the acoustic signatures of the steelpan. Other work has been done on the mathematical/numerical analysis of the steelpan. In Achong [4], the pan was modeled as a system of interconnected non-linear shells and the transient vibration behaviour obtained. Gay [5] examined the bass tenor pan using the finite element method while Mangaroo [6] developed a numerical model of a specific steelpan. This work attempts to identify how the vibration patterns of the steelpan vary with its geometric properties. These relationships may then be used to optimize specific note sizes.

THE FINITE ELEMENT MODEL

The steelpan is thin in comparison with its span and most of its surface is doubly curved, hence its response will be most appropriately modeled using the shell equations. These equations are usually written in local coordinates defined by the principal radii of curvature at a point and the normal to the surface at that point. The geometry of the structure is complex and its principal radii of curvature change direction and magnitude as its surface is traversed. Hence a closed form solution of the equations is not possible.

One of the most important features used to describe the response of the steelpan is its displacement normal to the shell-like structure under impact. This information may then be used to determine the stresses in the system. In general, the excited pan will exhibit bending stresses and membrane stresses (from forces acting tangential to the mid-surface of the pan). The bending stresses become most crucial at those points where the radius of curvature of the shell changes drastically i.e. at the roots of the notes.

Because an exact solution for the steel pan's response cannot be obtained, numerical methods may be used instead. To understand the pan's dynamics, the body is divided into discrete regions (elements) with specified material (mass density, modulus of elasticity) and geometric properties (thickness, curvature, size). The force-balance equations on each element are then used to "connect" the elements, resulting in the matrix equation:

$$[\boldsymbol{M}]\ddot{\vec{\boldsymbol{u}}} + [\boldsymbol{C}]\dot{\vec{\boldsymbol{u}}} + [\boldsymbol{K}]\vec{\boldsymbol{u}} = \vec{\boldsymbol{F}}$$

from which an overall solution (for variables such as displacement, stress, rotation) may be found. Here [$\boldsymbol{M}$], [$\boldsymbol{C}$] and [$\boldsymbol{K}$] are, respectively, the mass, damping and stiffness matrices. In this work, the damping in the system is ignored since the damping ratio for steel (from which the instrument is made) is typically low.

Experiments have shown that the steelpan has localized modes of vibration i.e. most of the vibrational energy in a steelpan is constrained to the single note that is struck. To approximate this behaviour, the note area was considered to be surrounded by a rigid internote which minimized the transmission of energy away from the note. Hence in the model used, each note was separately analysed, neglecting its effect on its neighbours.

General Modelling Considerations

Most of the surface of the steelpan is doubly curved and the structure is thin in the direction normal to the playing surface. Hence shell elements were used to model the motion of the instrument. The stress through the thickness is neglected. Reissner/Mindlin assumptions are made, i.e. material particles on a straight line perpendicular to the midsection of the shell remain on a straight line during deformations but shear deformations are included so the resulting line need not remain perpendicular to the midsection. Each shell element is flat and has a stiffness matrix obtained by superimposing plate bending stiffness and plane stress membrane stiffness. The consistent mass matrix formulation was used for the dynamic analysis. The ADINA software package [7] was used in the analysis. The material properties used in the model are those of mild steel i.e. Modulus of Elasticity 200 GPa, Poisson's ratio 0.3 and Density 7850 kg/m^3.

NUMERICAL INVESTIGATION OF THE SINGLE NOTE

The tenor steelpan consists of 29 notes. There are 12 playing notes in the outer ring (D to C#, frequencies 294 to 554 Hz), 12 playing notes in the middle ring (D to C#, frequencies 587 to 1109 Hz) and 5 notes (D, Eb E, C, C#, frequencies 1175 to 2218 Hz) in the inner ring. This work will examine the middle notes. The general shape of the note is assumed to be as shown in Figure 2. Physical measurements of the notes on the tenor steelpan reveal that, for the middle ring of notes, the major axis length varies between 60 and 100 mm while the minor axis length varies between 47 and 70 mm. The aspect ratio varies between 1.1 and 1.4. The variation of the height of the note was assumed to lie between 0 and 5% of the minor axis length. The thickness of the pan is measured as 0.25 mm for the middle notes. All these parameters are important to the vibration characteristics of each note and the complete pan.

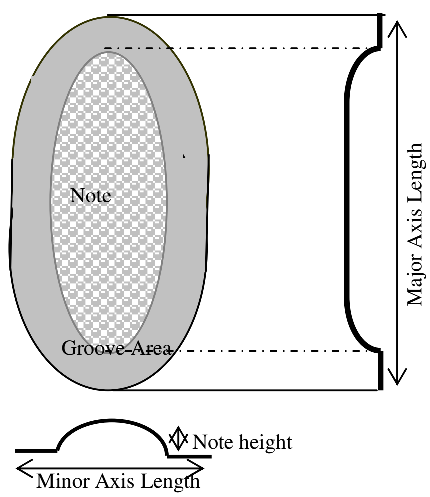


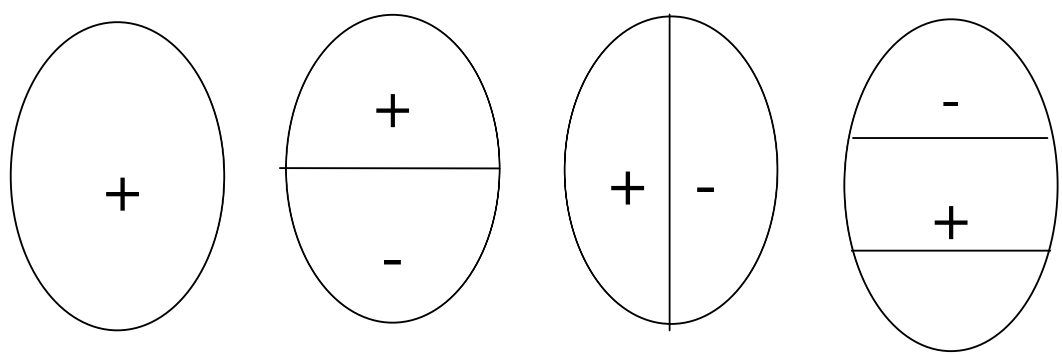
Figure 2: Schematic of a steelpan note

A generic note was developed to determine the qualitative effect of varying geometric parameters of the pan. The generic note had a major axis length of 80 mm, a height of 1.25 mm, an aspect ratio of 1.25 and a thickness of 0.25 mm. The projection of the dent area is assumed to be 95% of the area enclosed by the groove. Each of these parameters is in turn varied to examine its effect on the overall note performance.

t	The thickness of the plate
h	The height of the note
a	The length of the major axis
AR	The aspect ratio of the note

Boundary Conditions of the Single Note

The steelpan exhibits localized vibration when struck i.e. most of the energy is concentrated in the note played. This suggests that some basic features of how the overall pan behaves may be determined from examining the individual notes. The effect of various geometric parameters on the vibration pattern of the system could then be assessed. The grooved section of each note acts as an inflexible connector between the note and the rest of the steelpan; consequently, the model chosen for analyzing each steelpan note assumes rigid body motion of the groove. This condition is less restrictive than an assumption of clamped conditions but more constrained than the free boundary conditions. For the generic node the normal modes of vibration are as shown in Figure 3.

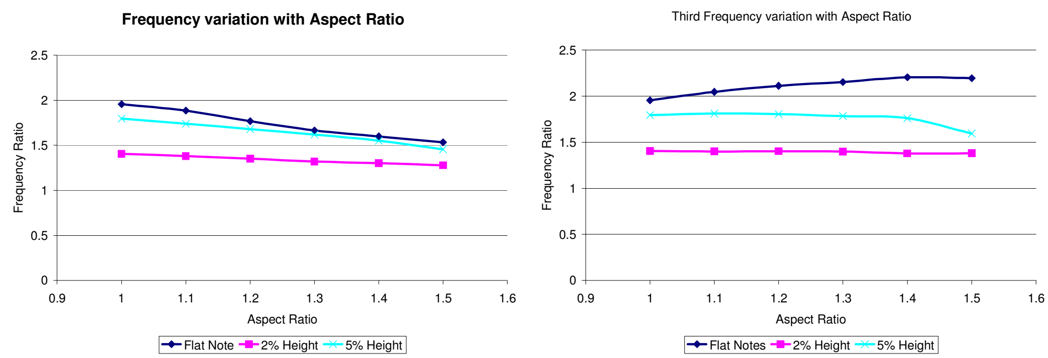


(a) (b) (c) (d)

Figure 3 The first four mode shapes of the generic vibrating note corresponding to natural frequencies of 771 Hz, 1512 Hz, 2115 Hz and 2650 Hz.

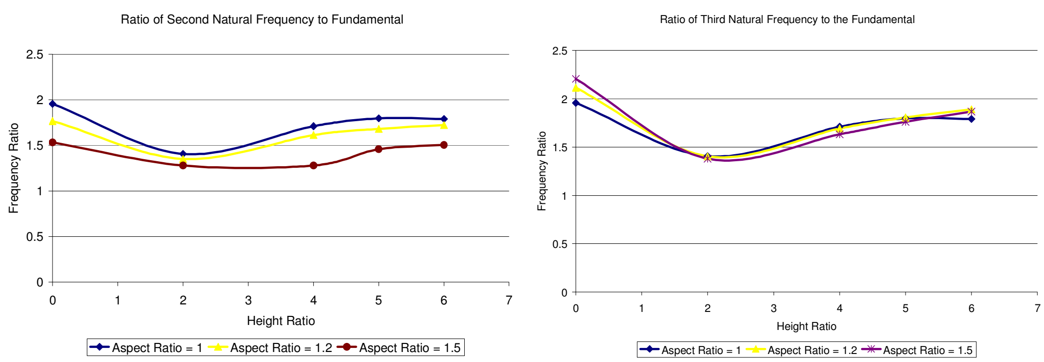
Effect of Aspect Ratio

One of the easiest geometric properties of the note to visualize is the aspect ratio of the elliptical notes. In the initial study the aspect ratio of the grooved area was changed without changing any of the other parameters. The area enclosed by the groove was also held constant. The fundamental frequency of the note decreased as the aspect ratio was increased, as did the second and third overtones. Note also that the ratio of the second natural frequency to the fundamental decreases as the aspect ratio is increased, while the third natural frequency ratio increases with aspect ratio. Figures 4(a) and (b) show how aspect ratio affects the ratio of the second and third natural frequencies with respect to the fundamental frequency. For the generic note described above, the maximum second frequency ratio is 1.96 and occurs with a flat note with an aspect ratio of 1. Thus the aspect ratio alone cannot be used to achieve equally spaced harmonics.



(a) (b)

Figure 4 Change in the ratio of the (a) second and (b) third natural frequencies with respect to the fundamental frequency as the aspect ratio is varied.



(a) (b)

Figure 5 Change in the ratio of the (a) second and (b) third natural frequencies with respect to the fundamental frequency as the dent height is varied.

Effect of Note Height

Another key factor affecting the note vibration is the height of the note. As the note height increases the fundamental frequency and the first two partials increase. Figure 5 shows the variation in the frequency ratios as the note height is varied. The ratio of the first three partials to the fundamental frequency also increases. This means that one is able to use the note height only to achieve the desired frequency ratios between the fundamental and the first partial. However, when the first partial is an integer, the third partial is not. This means that another means of changing the frequency ratio is needed.

Effect of Thickness

As the thickness of the plate decreases the frequency increases. The thickness affects both the stiffness of the plate (order $\boldsymbol{t^3}$) and the mass per unit area of the plate (order $\boldsymbol{t}$). Hence the net effect will be an increase in frequency as the thickness increases. For a flat note there is a linear relationship between the thickness and the natural frequencies; therefore, changing the thickness alone cannot ensure that the second natural frequency is a harmonic of the first.

For the raised note most of the vibration of the system takes place in the flat portion of the note. The resulting frequency is dependent on the effective boundary conditions provided by the root of the note and hence depends on the relative geometry of this system. The ratio of the radius of curvature of the crown root interface and the thickness will therefore be constantly varying. Hence there is not a strict linear relationship between frequency and thickness. In general, as the thickness decreases the ratio of $\boldsymbol{f_n/f_1}$ increases. Figure 6 shows the variation for a dent height of 5%.

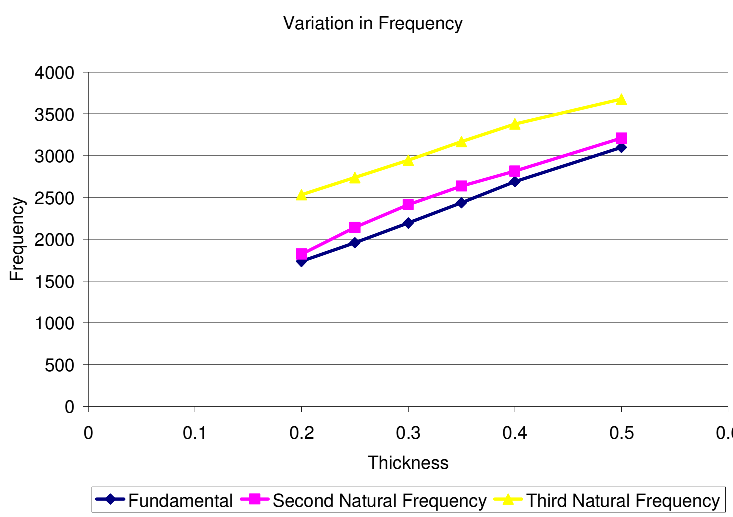


Figure 6 Variation of the Natural Frequencies with thickness

Effect of Major Axis Length

The ratio between $\boldsymbol{f_n/f_1}$ is a constant for any two values of major axis length. If the aspect ratio is held constant, then one can normalize all the lineal dimensions with respect to the major axis length and obtain a normalized shape which does not differ for the system described above.

With all the other parameters fixed, as the major axis length increases the frequency decreases; this is consistent with observations of the steelpan. In fact the relationship is of the form, $\boldsymbol{r^2 f_n = c}$, where $\boldsymbol{r}$ is the major axis length. Since the tenor pan notes are each separated by a ratio of $2^{1/12}$, the ratio of the effective length of two consecutive notes in each section of the pan (outer, middle, inner) should be approximately 0.97 (i.e. $2^{1/24}$).

CONCLUSIONS

The steelpan is a musical instrument; for fullness of sound, it is necessary that its frequency spectrum has a strong sense of pitch. This can only be achieved if at least the first three natural frequencies are integer multiples of the fundamental. It is therefore important that the vibration characteristics of the notes be understood as a function of the geometric properties of the pan. This will enable the manufacturing and tuning processes of the instrument to be simplified. In this work, several theoretical relationships were determined.

It was observed that the assumption that the groove acts as a rigid body connector allows one to investigate the localized vibration of each note, while keeping some of the characteristics exhibited in the general vibration of the overall steelpan. It was noted that for a fixed thickness, the optimal relationship between lengths of successive notes is 1: 0.97. The frequency increased with thickness in a nonlinear manner for the curved notes. This was mainly due to the large change in radius of curvature at the root of the note. As the note height is increased each frequency increases. However, the ratio of the partials to the fundamental frequency decreases, until it reaches a minimum at about 2% of the minor axis length and then begins to increase again. These observations will form the basis for an effort to optimize the shape of each note.

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