ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

SOURCE RECONSTRUCTION USING INVERSE SPHERICAL WAVE SPECTRUM FITTING

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Abstract

The inverse sound source reconstruction addressed in this paper seeks, given a transmission matrix, to reconstruct the monopole distribution on the source surface using measured sound pressure data. Here the focus is on the transmission matrix. Conventional inverse methods use transmission matrices with point to point transfer functions, that is, transfer functions from volume velocity in discrete source positions to local acoustic field variables. The resulting transmission matrix is ill-conditioned and needs regularisation, with consequent loss of accuracy. Unlike the conventional methods, the new method proposed in this paper uses transfer functions from source positions to global field variables. These global variables, the components of the spherical wave spectrum, are obtained through expansion of the pressure field in terms of spherical harmonics. In this paper the performance of this new formulation is compared with that of the conventional inverse reconstruction scheme.

INTRODUCTION

From an acoustical point of view any operating source may be substituted by the non-operating source covered by a sufficient number of substitution monopoles displaying a well defined volume velocity distribution and mutual phase relationship. The inverse sound source reconstruction addressed in this paper seeks, given a transmission matrix, to reconstruct this monopole distribution using measured sound pressure data. Unfortunately, sound pressure measurements are always contaminated with measurement errors. Whereas systematic errors may be eliminated by careful calibration, the burden of the random noise superimposed on the signals remains. These random errors may lead to large spurious volume velocities unless the transmission matrix is sufficiently well conditioned or else the measurement errors are sufficiently small.

Conventional inverse methods use transmission matrices with point to point transfer functions, that is, transfer functions from volume velocity in discrete source positions to local acoustic field variables. In general these transmission matrices are ill-conditioned and need regularisation in order to prevent the occurrence of large spurious volume velocities. Unlike the conventional methods, the new method proposed in this paper uses transfer functions from source positions to global field variables. These global variables, the components of the spherical wave spectrum, are obtained through expansion of the pressure field in terms of spherical harmonics.

The idea behind this alternative formulation of the inverse problem is that the spherical harmonic transform is expected to reduce, through the surface integration, the effects of random pressure measurement errors. This is especially true for the low order spectrum components representing spatial frequencies which are much lower than those introduced by the random noise. In this paper the new inverse formulation is presented and its performance compared with that of the conventional inverse reconstruction scheme through numerical experiments.

THEORY

Spherical harmonics

For an exhaustive treatment of spherical harmonics the reader is referred to [1]. A brief overview is given here. Any arbitrary pressure distribution $p(\theta,\phi)$ on a sphere can be expanded in terms of spherical harmonics $Y_n^m(\theta,\phi)$ of order *n* and rank *m*:

$$p(\theta,\phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} c_{nm} Y_n^m(\theta,\phi). \tag{1}$$

The spherical harmonics are angle functions defined as

$$Y_n^m(\theta,\phi) = \sqrt{\frac{(2n+1)}{4\pi} \cdot \frac{(n-m)!}{(n+m)!}} \cdot P_n^m(\cos\theta) \cdot e^{im\phi}, \tag{2}$$

where P_n^m are associated Legendre functions, θ is the polar angle ($-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$) and ϕ is the azimuthally angle ($0 \le \phi \le 2\pi$). Owing to the orthonormality of the spherical harmonics, the complex coefficients c_{nm}, also referred to as the spherical wave spectrum, may be obtained from

$$c_{nm} = \int_S p(\theta,\phi) Y_n^m(\theta,\phi)^* \, d\Omega \,, \tag{3}$$

where the integration is carried out over the surface S of an unit sphere.

A new formulation for the acoustical inverse problem

Consider a non operating source structure. Assume that N monopoles are chosen in arbitrary positions on its surface and that their volume velocity is given by a vector $\boldsymbol{u}$. The resulting sound pressure distribution on a spherical surface containing the source structure is expanded in terms of spherical harmonics. The first M spherical wave spectrum components resulting from this expansion are stored in vector $\boldsymbol{c}$. By invoking the superposition principle (the spherical wave spectrum of the total field may be obtained by the sum of the spherical wave spectra of each monopole), this radiation problem may be written as

$$\boldsymbol{c} = [\boldsymbol{H}]\boldsymbol{u} \,, \tag{4}$$

where $[\boldsymbol{H}]$ is a $M \times N$ transmission matrix of complex frequency response functions relating the source strength of each single monopole to its spherical wave spectrum components. When written in full equation (4) becomes

$$\begin{bmatrix} c_{0,\,0} \\ c_{1,-1} \\ c_{1,\,0} \\ c_{1,\,1} \\ c_{2,-2} \\ \vdots \\ c_{n,\,m} \end{bmatrix} = \begin{bmatrix} H^1_{0,\,0} & H^2_{0,\,0} & \cdots & H^N_{0,\,0} \\ H^1_{1,-1} & H^2_{1,-1} & \cdots & H^N_{1,-1} \\ H^1_{1,\,0} & H^2_{1,\,0} & \cdots & H^N_{1,\,0} \\ H^1_{1,\,1} & H^2_{1,\,1} & \cdots & H^N_{1,\,1} \\ H^1_{2,-2} & H^2_{2,-2} & \cdots & H^N_{2,-2} \\ \vdots & \vdots & & \vdots \\ H^1_{n,m} & H^2_{n,m} & \cdots & H^N_{n,m} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}, \tag{5}$$

where the superscript of the transmission matrix components refers to the source position, whereas the subscripts refer respectively to order and rank of the spherical harmonics involved.

As mentioned before, the inverse sound source reconstruction addressed in this paper seeks, given a transmission matrix, to reconstruct the monopole distribution on the source surface using measured sound field data of the operating source. In order to do so the complex sound pressure field due to the actual source is measured on the sphere and expanded in terms of spherical harmonics. The spherical wave spectrum components obtained from these measured sound pressure data are stored in vector $\hat{\boldsymbol{c}}$. The optimal estimate of the source strength vector $\boldsymbol{u}_0$ may now be obtained as

$$\boldsymbol{u_0} = [\boldsymbol{H}]^+ \hat{\boldsymbol{c}} \,, \tag{6}$$

where $[\boldsymbol{H}]^{+}$ is the pseudo-inverse (Moore-Penrose) of the matrix $[\boldsymbol{H}]$, resulting in a "least square" solution for the complex source strengths sought.

Discrete spherical harmonic transform

In practice, the sound field is sampled only at a finite number of discrete positions around the source and the wave spectrum integral (3) is approximated by a double summation. The angular discretisation steps were chosen $\Delta\theta = \Delta\phi = \pi / 9$, corresponding with 146 microphone positions. The results reported in this work were achieved using a very simple zero-order integration scheme. Both the spherical harmonics and the pressure field were evaluated in the microphone positions and considered constant within each integration step.

In order to prevent aliasing effects due to the spatial sampling, the number of computed wave spectrum components has been limited (equivalent Nyquist criterion). For the present sample distribution the series expansion was truncated at order n = 6 (see [2]), corresponding with 49 harmonics.

NUMERICAL EXPERIMENTS

Description of the experiments

The source to be reconstructed consists of a cluster of monopoles in free space. The sound pressure is calculated in the 146 above mentioned discrete points, the "microphone" positions, on a spherical surface enclosing the monopoles.

The conventional transmission matrix was simply obtained by assigning unitary volume velocity to each single monopole, one by one, and calculating the sound pressure in all microphone positions. The spherical transmission matrix was obtained by applying the discrete spherical harmonic transform to the sound field of each unitary monopole.

Next the sound pressure vector was calculated for the particular source strength distribution to be reconstructed and random noise was added. This corrupted sound pressure vector represented the "measured" source field to be used straightforwardly for reconstruction with the conventional transmission matrix and after the discrete spherical harmonic transform, for the reconstruction with the novel transmission matrix. The errors in the transmission matrix are due to round-off errors and are negligible as compared to the errors in the pressure vector.

Results

The source configuration used for the numerical experiments consists of N = 20 monopoles distributed randomly within the space between two concentric spheres of R = 0.4 and R = 0.5

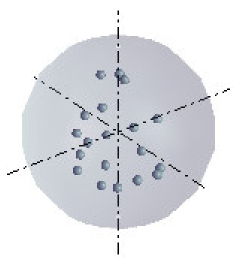

Figure 1 - Source configuration.

(see Figure 1). The calculated pressure vector was contaminated with an error varying randomly between $\pm 1\,\text{dB}$ and ± 3 degrees. This random error is identical at all frequencies.

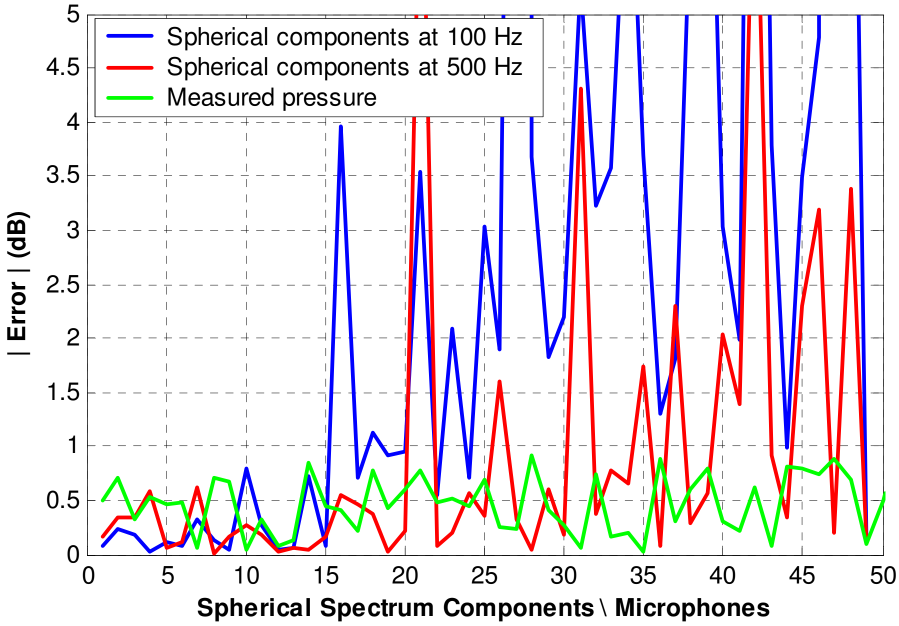

Figure 2 – Pressure vector errors (50 of 146) and resulting spherical wave spectrum errors at 100 and 500 Hz.

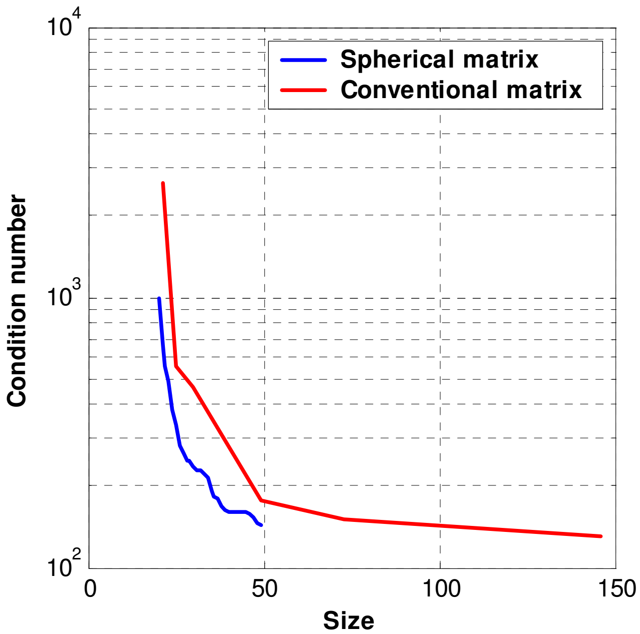

Figure 3 – Condition number of spherical and conventional matrices at 100 Hz.

Before proceeding to the inversion, the optimal number M of wave spectrum components to be included in the analysis has to be assessed. In any case this number must be larger than N = 20 in order to assure an unique solution. The optimal M-value depends on two factors: the condition number of the matrix and the error in the wave spectrum vector. In order to gain more insight in this matter, the behaviour of these two parameters was studied at two frequencies: 100 and 500 Hz.

At 100 Hz the errors in the wave spectrum vector $\hat{\boldsymbol{c}}$, indeed, turn out to be reduced as compared to the errors introduced into the original pressure vector, but only for the first 6 spectral components (see Figure 2). From the 6th until the 15th component the errors are roughly comparable. Above the 15th component the error in the spherical wave spectrum grows rapidly. This is due to the fact that at 100 Hz the source generates a very smooth pressure distribution which contains almost exclusively low order spherical wave spectrum components. The high order wave spectrum contents mainly describe the noise and not the signal, resulting in large errors. The condition number at 100 Hz is presented in Figure 3 both for the conventional and the spherical transmission matrices. In this figure the abscissa represents the number of wave spectrum components (for the spherical case), or alternatively the number of microphone positions (for the conventional case) included in the analysis. The conventional system shows an almost monotonically decreasing condition number, as a result of growing overdimensioning of the inverse problem. The size of the conventional system was varied by choosing microphone configurations according to $\Delta\theta = \Delta\phi = \pi/k$, with $k = 4,5,\cdots,9$. Since the error is constant the best results are obtained using all 146 microphones. The spherical system, on the other hand, exhibits a strongly decreasing condition number while the error is increasing. The condition number turns out to dominate and the best result is again obtained including all 49 components of the spherical spectrum.

Conventional and spherical reconstructions are compared in Figure 4, before and after regularisation (Tikhonov regularisation with L-curve parameter selection). The two inverse formulations turn out to give equivalent results.

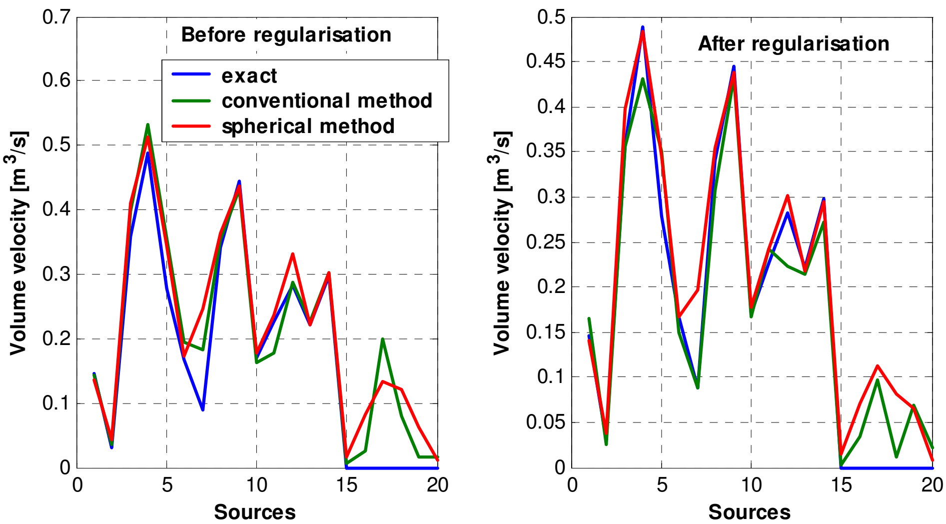

Figure 4 – Source vector reconstructions at 100 Hz.

It must however be noted that the size of the inversion problem has been reduced by switching to spherical harmonics, in our case with approximately a factor three. Since matrix inversion is typically an M^3 process, this operation has been accelerated by a factor of roughly 27. On the other hand, however, it takes more time to assemble the field vector and the transmission matrix associated with the spherical formulation, as they involve a discrete spherical harmonic transform. For the code used in this study, the new spherical wave spectrum based method resulted in a reduction of the calculation time with a factor of 2.

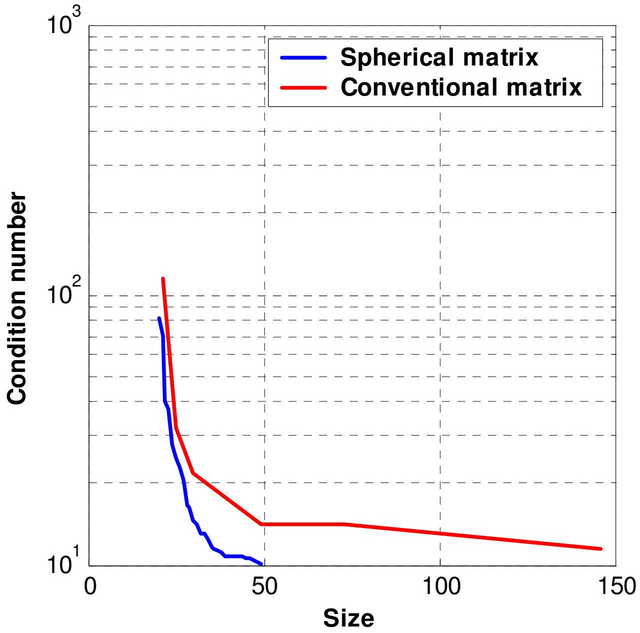

Figure 5 - Condition number of spherical and conventional matrices at 500 Hz.

It should also be noted that, as such, the spherical matrix features a much better condition, which is only equalled or improved by the conventional system trough massive overdimensioning. This is however an intrinsic advantage of the classical approach as there are always much more microphones than spherical spectrum components.

At 500 Hz the situation of the wave spectrum vector $\hat{\boldsymbol{c}}$ improves for higher orders, showing smaller errors than the 100 Hz case (see Figure 2). All in all, however, the spherical harmonic transform turned out to have a negative effect on the error rather than being beneficial. The condition of the spherical matrix, on the other hand, again turned out better than its conventional counterpart.

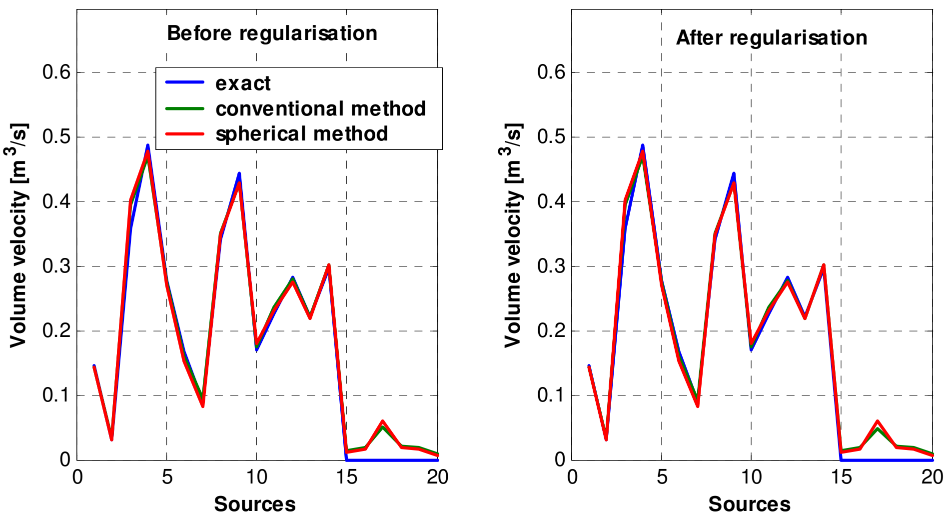

Figure 6 – Source vector reconstruction at 500 Hz.

Again the best condition was obtained for maximum overdimensioning, this time, however, the spherical matrix reached a slightly lower final condition number than the conventional one (Figure 5).

In Figure 6 the reconstructed volume velocity vector is compared to the original one. The quality of the two reconstruction methods, conventional and spherical, again proves equivalent. Regularisation has almost no effect due to the excellent initial matrix conditions.

It must be emphasised that in the presented case study the positions of the substitution monopoles correspond with the source monopoles. The solution space therefore contains the actual source configuration. In practice, however, the source is unknown and the substitution monopoles must therefore be chosen arbitrarily resulting in an incomplete solution space (see [3]). The effect of an incomplete solution space on the performance of the different inverse formulations has not yet been investigated.

SUMMARY

The discrete inverse source reconstruction problem could be reformulated by expanding the pressure field in terms of spherical harmonics. The performance of this new formulation was evaluated through numerical experiments. In particular the analysis concentrated on the random errors in the field vector and the condition of the transmission matrix. The following trends have been observed:

- the discrete spherical harmonic transform of noisy pressure data results in error amplification, especially for high order spectral components,
- the “spherical” transmission matrix is generally better conditioned than the conventional one (and overdimensioning helps a lot),
- reconstruction results appear to be equivalent,
- the spherical problem is smaller and therefore faster.

REFERENCES

[1] E. G. Williams , “Fourier Acoustic”, Academic Press, San Diego, 1999.

[2] D. M. Healy Jr, D. Rockmore and P. J. Kostelec, “FFTs for the 2-Sphere – Improvements and Variations”, Proc. ICASSP-96, vol.3, pp. 1323-1326, 1996.

[3] L. Chiesa, E.J.M. Nijman, “Improved regularisation criteria for inverse sound source recontruction”, ICSV13 – Vienna, 2006.