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IMPROVED REGULARISATION CRITERIA FOR INVERSE SOUND SOURCE RECONSTRUCTION

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Abstract

Inverse sound source reconstruction seeks, given a transmission matrix, to reconstruct the volume velocity distribution on the source surface using sound pressure data. In practice this discrete inversion problem turns out to be ill-conditioned and needs regularisation in order to produce stable results. The core issue remains the determination of the appropriate degree of regularisation, such as to guarantee a stable solution with a minimal loss in spatial resolution for the reconstructed source. In this paper a new optimisation criterion is presented for the Tikhonov regularisation parameter. This new method uses errors expressed in decibels instead of the conventional squared pressure differences to estimate the accuracy of the reconstructed field. Such procedure allows to make a better use of the information carried by sensors with a relatively low signal. Laboratory experiments have been carried out and the performance of the new criterion has been compared with those of the conventional L-curve and cross validation techniques.

INTRODUCTION

The inverse problem considered is that of reconstructing the source strengths of a group of substitution monopoles given the sound field of the original source in a number of so called "indicator positions" and given the transmission matrix from the monopoles to these indicators. In particular the hybrid inverse methods, i.e. inverse methods combining calculated transmission matrices with measured sound pressure data, have proven accurate ([1], [3]). A major drawback of these methods is however the large number of indicator microphones needed. This fact not only makes the experimental work arduous and unattractive but basically limits the method to stationary applications.

Here a restriction is therefore imposed on the size of the discrete inverse problem, meaning that the total amount of measurement positions must be limited to the number of acquisition channels available, typically 150. The number of substitution monopoles is consequently limited to less than 150 as well (a unique solution is desired).

Conventional methods choose the indicators in the near-field of the source for two reasons. In the first place the near-field is unique and unlike the far-field contains all the detailed source characteristics. Complete reconstruction of the near-field assures correct reconstruction of the source, complete reconstruction of the far-field doesn't. In the second place the inverse problem appears to become best conditioned when the indicators are positioned in the near-field [4].

As long as the spatial density of the chosen substitution monopole distribution is high enough to allow for a proper description of the velocity distribution on the source surface all sensors should consequently indeed be chosen in the near-field. If, in other words, the solution space contains the exact solution than the measured near-field should indeed lead to the only true source configuration. This paper, however, addresses those cases where the substitution sources are far too few to accurately describe the velocity distribution on the source surface, especially in the higher frequency range. In such case the near-field can only be approximated, which may cause considerable source reconstruction errors. The idea is therefore to move part of the sensors of the inversion problem from the near-field to the far-field in order to force the system to reproduce both the near-field and the far-field radiation pattern, albeit both in an approximate manner. Although this rearrangement of the sensors is likely to deteriorate the condition of the inverse problem, it is also expected to add propagating field information to the inverse problem.

It is well-known that measurement noise and modelling errors may cause large deviation in the source reconstruction especially when the inversion problem is ill-conditioned, which is generally the case. Regularisation is therefore indispensable. The Tikhonov regularisation [2] has become the established method and will be used throughout this work. The success of this regularisation strategy depends on the appropriate choice of the Tikhonov regularisation parameter β. Different criteria exist for the selection of the optimal Tikhonov regularisation parameter, such as the "cross validation" technique [4] and the "L-curve" method [2].

In this paper the hybrid inverse substitution monopole source reconstruction is further investigated, above all in view of applications with limited amount of indicator microphones. Special attention is paid to the microphone positions and to regularisation criteria. In particular, an improved cross validation criterion has been developed which also proves beneficial for conventional applications.

TIKHONOV REGULARISATION PARAMETER SELECTION: A CLOSER LOOK TO THE CROSS VALIDATION TECHNIQUE

The Conventional Cross Validation Technique

The cross validation technique is based on the “leaving-one-out” process: having M indicator microphones, the inversion problem is in principle solved M times, omitting successively a microphone and evaluating the pressure prediction in the “left-out” position by comparison with the measured value. This procedure is repeated for varying Tikhonov regularisation parameter: the optimal β value is the one which leads to minimisation of the cross validation function

$$V(\beta)=\sum_{k=1}^{M}\| \hat{p}_k - p_k(\beta)\|^2 , \tag{1}$$

where $\hat{p}_k$ is the pressure measured in the k^{th} position and $p_k(\beta)$ is the pressure calculated in this position when it is omitted from the inversion process. In principle the calculation of the cross validation function in equation (1) would require M matrix inversions for each value of β. Fortunately it may be shown (see [4]) that equation (1) can be rewritten in a more convenient form requiring only one single matrix inversion. For the ordinary cross validation this formulation becomes:

$$V(\beta)=\| \boldsymbol{C}(\boldsymbol{I}-\boldsymbol{B}(\beta))\hat{\boldsymbol{p}}\|_e^2 . \tag{2}$$

In this expression $\| \|_e$ denotes the Euclidean norm, $\boldsymbol{I}$ represents the identity matrix, and $\hat{\boldsymbol{p}}$ the measured complex pressure vector. The “influence matrix” $\boldsymbol{B}(\beta)$ is given by

$$\boldsymbol{B}(\beta)=\boldsymbol{H}\left(\boldsymbol{H}^H\boldsymbol{H}+\beta\boldsymbol{I}\right)^{-1}\boldsymbol{H}^H , \tag{3}$$

where $\boldsymbol{H}$ is the matrix of acoustic transfer functions and the superscript H indicates the Hermitian transpose (complex conjugate transpose). The matrix $\boldsymbol{C}$ is a diagonal matrix which is assembled from elements of the influence matrix:

$$C_{kk}(\beta)=\frac{1}{1-B_{kk}(\beta)} . \tag{4}$$

Cross validation in the presence of both near- and far-field sensors

The selection of the Tikhonov regularisation parameter turns out not to be straightforward when the inversion sensors are distributed over both near- and far-field as is the case in the present situation. The problem is that the far-field error, being much lower than the near-field error in an absolute sense, hardly participates in the cross validation function in equation (1). An adapted more general formulation of the cross validation scheme, hereafter referred to as “logarithmic cross validation”, has therefore been developed. This formulation is not restricted to applications with only near-field sensors but may also be used in the presence of both near- and far-field microphones. In order to do so, the following modified cost function has been introduced:

$$V_L(\beta) = \sum_{k=1}^{M} \left| log \left\| \frac{\hat{p}_k}{p_k(\beta)} \right\| \right| . \tag{5}$$

The β value which minimises this new cross validation function corresponds with minimising the sum of the absolute pressure level differences in decibels between measured field and the field calculated in omitted positions. The advantage of this function as compared with the conventional one in equation (1) is that it is expected to have the same sensitivity for near- and far-field errors. The modified cross validation function may again be written in a compact format needing only a single matrix inversion:

$$V_L(\beta) = \sum_{k=1}^{M} \left| log \left\| \frac{\hat{p}_k}{e_k(\beta)} \right\| \right| , \tag{6}$$

where

$$\boldsymbol{e}(\beta) = \left(\boldsymbol{I} - \boldsymbol{C}(\boldsymbol{I} - \boldsymbol{B}(\beta)) \right) \hat{\boldsymbol{p}} . \tag{7}$$

Using the β value which minimises this new cross validation function assures that the field of the Tikhonov regularised source distribution fits best with the measured field in both near- and far-field positions which are not included in the inversion process.

A similar procedure may be applied to the generalised cross validation regularisation scheme.

EXPERIMENTS

In order to test the different source reconstruction procedures, experiments were carried out in a laboratory environment. The source used for the experiments consisted of a rectangular plywood box (520 x 490 x 250 mm, wall thickness 25 mm) which was positioned, a large face down, on the reflecting surface of the semi-anechoic chamber. Four of the five exposed faces feature a circular orifice with a diameter of 36 mm. Inside the box, tubes of different lengths connect the orifices to a single central sound source. Four coherent sources were thus obtained exhibiting different amplitude and frequency characteristics. The orifices were equipped with internal microphones in order to experimentally asses their volume velocity. The sound emission of the box due to vibration of the walls was way below the orifice emissions and could be neglected.

Ninety near-field quarter inch microphones, were mounted on a steel wire mesh structure surrounding the box-source at an average distance of 140 mm. The far-field indicator microphones, 30 of them, were positioned on randomly placed steel wire mesh panels with their distance from the source varying between 1 m and 3 m.

The position of the substitution monopole sources to be reconstructed, 40 in all, were chosen in a regular pattern on the surface of the box, 5 for each lateral face and 20 for the larger top face. Four of the chosen monopoles correspond with the actual source orifices.

The complete transfer function matrix was obtained by a BEM calculation. The model of the box-source features about 4800 nodes and consisted of a regular grid of rectangular elements with an average length of 15 mm suited for frequencies up to 3500 Hz. The 40 substitution monopoles were chosen at nodes, and modelled by the four corresponding elements vibrating in phase and with the same amplitude. In the model the box is placed on a perfectly reflecting plane and radiating into a free, fluid filled half-space. The transfer functions were obtained by assigning a unitary velocity to the four elements of a single monopole and calculating the pressure response in the 120 indicator microphone positions (90 near- and 30 far-field positions) using the indirect boundary element method (indirect BEM). This calculation was repeated for each of the 40 monopoles, all in all 4800 transfer functions.

SOURCE RECONSTRUCTION: RESULTS

The monopole volume velocity vector $\boldsymbol{q}$ was obtained using the Tikhonov regularised solution

$$\boldsymbol{q} = \left(\boldsymbol{H}^{H}\boldsymbol{H} + \beta\boldsymbol{I}\right)^{-1}\boldsymbol{H}^{H}\hat{\boldsymbol{p}} \,. \tag{8}$$

In this expression $\boldsymbol{H}$ represents the calculated transfer function matrix, β the regularisation parameter and $\hat{\boldsymbol{p}}$ the measured pressure vector.

The measured and calculated orifice volume velocity spectra presented in this section are displayed (up to 3000 Hz) and compared with the “residual source strength”. This “residual source strength” consists of the average calculated source strength of all remaining sources (which don’t correspond with an orifice), and shall in principle be zero. This residual source strength is caused by measurement and modelling errors and may be interpreted as a background noise level limiting the dynamic range of the inverse reconstruction process.

Effects of incomplete solution space

Initially all 40 monopoles were included in the inversion process, meaning that the solution space contained the exact source distribution sought. The conventional reconstruction of the orifice source spectra, i.e. using only near-field indicators (90 sensors), are shown in Figure 1. The box-source is well reconstructed indicating high quality measurements. Regularisation, as expected in this case, hardly improves the accuracy but slightly improves the dynamic range of the calculation at low frequencies. The performance of the different regularisation schemes seems roughly equivalent.

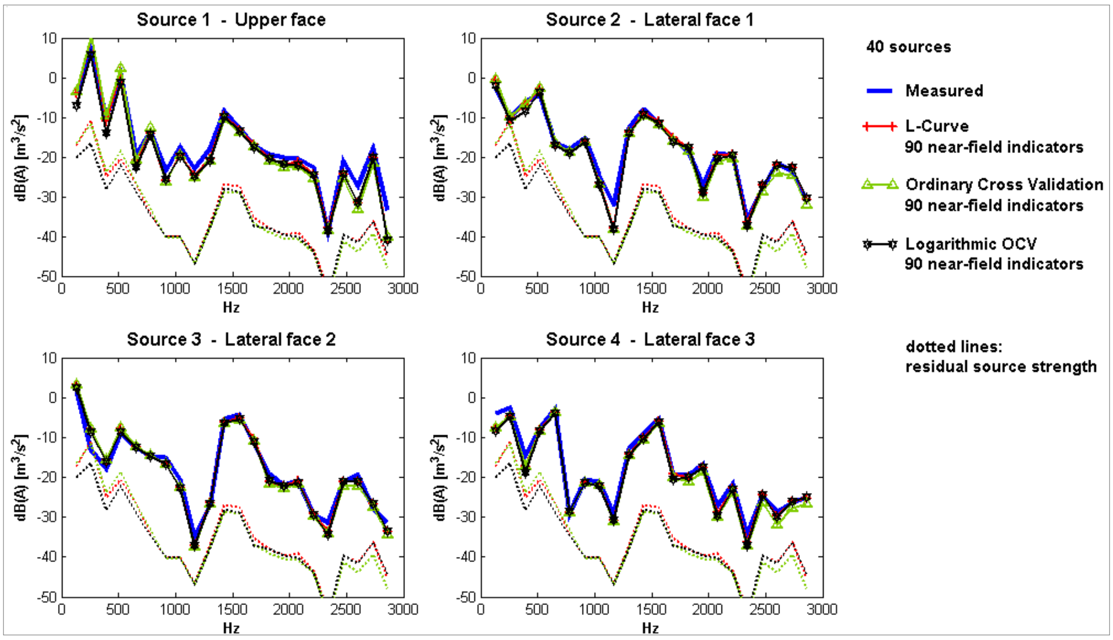

Figure 1 – Regularised solution with complete solution space, only near-field indicators

Next the reconstruction was repeated using an incomplete solution space with 36 monopoles, that is, removing the 4 substitution monopole sources corresponding with the orifices from the inversion problem. All indicators are still kept in the near-field. The measured source strength is now compared with the algebraic sum of the calculated source strengths of the monopoles surrounding the orifice. After all it is essential that the method distributes the source strengths over the closest available substitution monopoles. Large errors may be observed in the reconstruction for sources 3 and 4 with the conventional cross validation technique (Figure 2). The new logarithmic cross validation regularisation criterion, although specifically designed to allow for far-field indicator microphones, turns out to be also favourable in the conventional near-field indicator case performing as well as the L-curve method.

Introduction of far-field indicator microphones

The same incomplete solution space with 36 sources was also solved with a mixed set of near-field and far-field indicator microphones. In the presented case 60 indicator microphones were chosen in the near-field and 30 microphones in the far-field, resulting in a deterioration of the condition number (Figure 4). The results of the

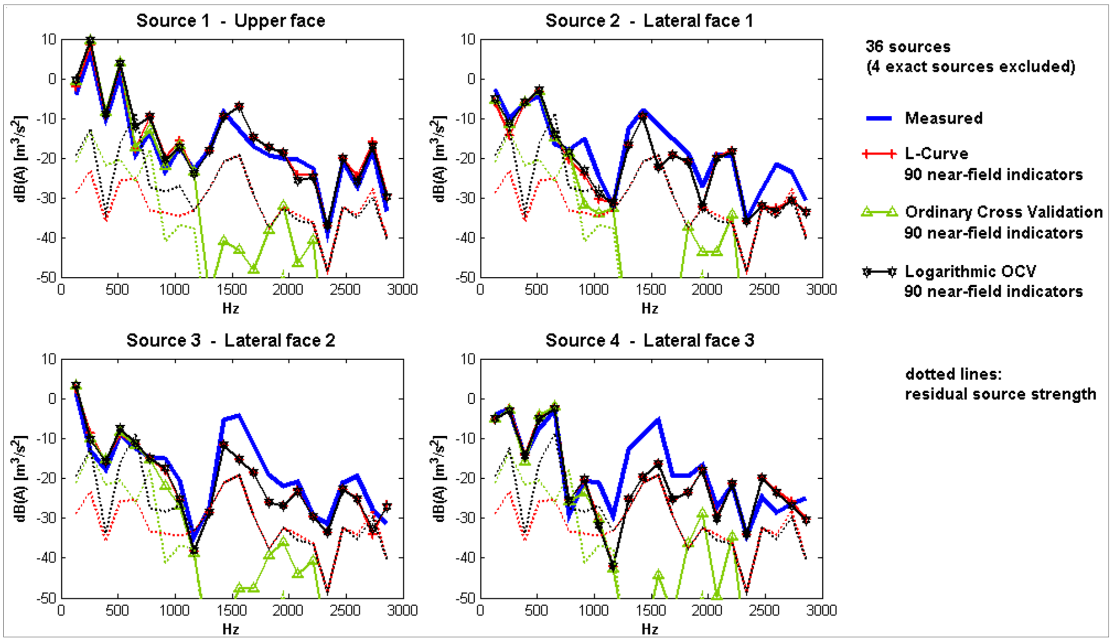

Figure 2 – Regularised solution with incomplete solution space, only near-field indicators

reconstruction are shown in Figure 3. Again both the L-curve and the logarithmic cross validation regularisation schemes perform well. For the source under investigation the introduction of far-field indicators had no clear effects on the reconstruction (compare figures 2 and 3). Far-field indicators are however expected to be beneficial for sources with a more reactive near-field.

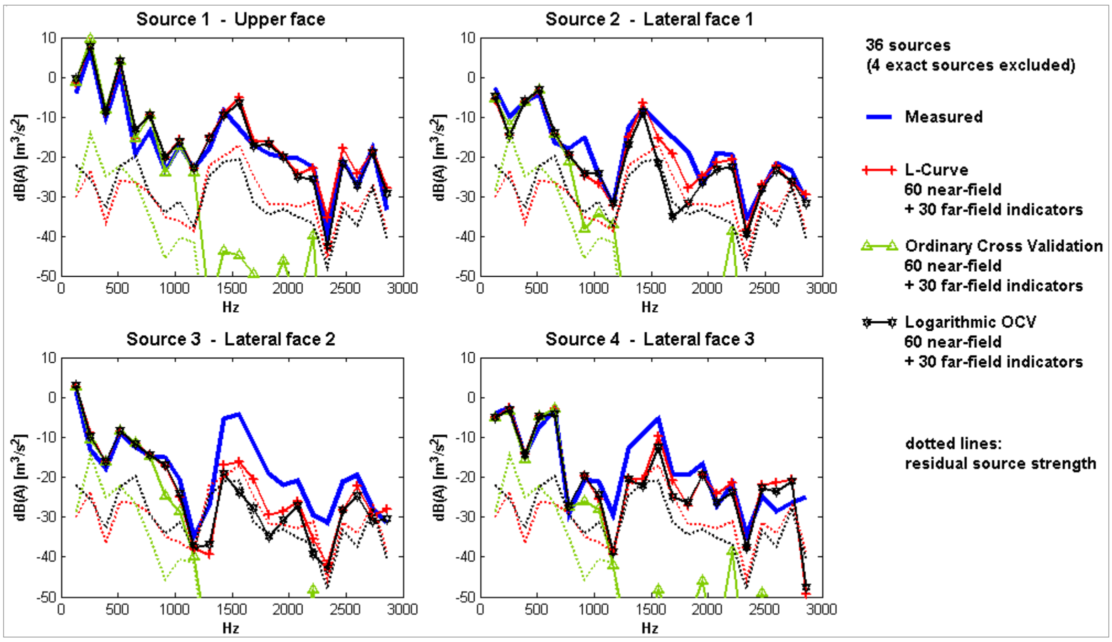

Figure 3 – Regularised solution with incomplete solution space, near- & far-field indicators

CONCLUSIONS

It may be concluded that in the absence of suitably positioned substitution sources the inversion method indeed correctly re-distributes the source strengths over the closest available substitution monopoles.

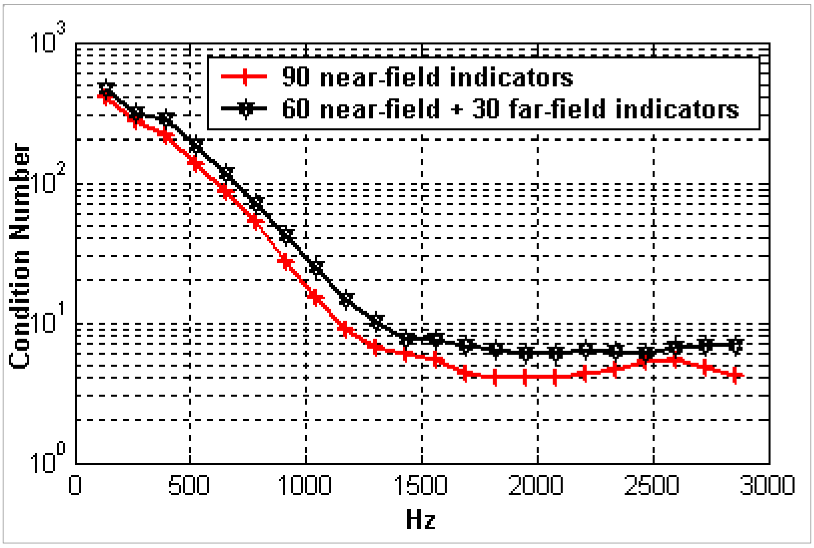

Figure 4 – Condition Number: different sets of indicator microphones with 36 sources

The improved cross validation criterion developed in this paper, the logarithmic cross validation, seems to perform as good as the well established L-curve criterion.

For the investigated source the reconstruction was neither improved nor deteriorated by the introduction of far-field indicator microphones in the inversion process. Further investigations on sources featuring reactive near-fields are needed in order to reach reliable conclusions.

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