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WAVE PROPAGATION IN DRILLING BOREHOLES

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Abstract

A rotary oil well drill-string is primarily composed of drill-pipes and drill-collars. Its role is to convey a rotary downward motion to the drill-bit and to circulate the drilling fluid (drilling mud). A drilling borehole is cylindrically layered waveguide, consisting of mud inside the drillstring, mud outside the drillstring, and the formation. To learn how sound waves propagate in this layered medium, an infinitely long, uniform, three-layered waveguide surrounded by a radially infinite homogeneous formation is analyzed. The first layer is the inner mud, the second is the uniform pipe, and the third is the outer mud layer. The mud as well as the pipe and formation, are assumed to be homogeneous. It is also assumed that the steady mud velocity is slow compared to sound propagation speed in the mud and that static pressure in the mud is low compared to the bulk modulus, allowing their effects to be neglected. In this paper, an analysis of wave propagation in drilling borehole are greatly simplified by assuming axial symmetry and low frequencies with long wavelengths, compared to the borehole radius. Derivation and solution of equations of motion are presented. The non-dimensional parameters for determining the degree of the interactions among the layers are explained. Sound reflection due to the changes in the cross section is discussed. One important application of this analysis is to mud pulse telemetry systems. Finally, the key results are summarized and discussed.

INTRODUCTION

A drilling system consist of a series of hollow cylindrical steel pipes connected to form a long flexible drillstring to which is attached a short heavier segment containing a drillbit at the free end. This segment may contain stabilizing fins and together with the drillbit constitutes the bottom-hole assembly (BHA). The drillstring is driven in a rotary fashion from the top end, often by means of an electric motor and

gearbox, the top-drive, and constrained to pass at a controlled rate through a rotating mass (the rotary) near the surface. Such a drilling system is designed to construct a borehole linking the earth's surface to a reservoir of oil or gas, as shown in Fig. 1. The borehole is lined (usually with steel) and the excess in the diameter of this cavity over the diameter of the drillpipe is referred to below as the over gauge. This annular gap (which in general varies along the borehole) is necessary for the conduction of fluids. During the process of drilling pressurized fluid (mud) is continuously circulated down the centre of the drillstring, out of holes in the drillbit and back to the surface via the space between the rotating drillstring and the surface of the borehole.
The dynamic behavior of a drillstring includes essentially of axial, bending, torsional and whirling motion [1]. Among these, axial vibration has received the most attention in the literature [2-3]. To know how a drillstring, mud, and surrounding formation interact in axial wave propagation, an infinitely-long, uniform and cylindrically three-layered (the inner mud, the uniform pipe and the outer mud, respectively) wave guide is assessed in this paper. In other word, to know how sound wave propagates in the layered medium (drilling bore hole), an infinitely uniform, long, three-layered waveguide surrounded by a radially infinite homogeneous formation is analyzed. Mitchell [4] studied the coupling between the inner and the outer mud pressures through pipe elasticity. Wave propagation in fluid-filled bore hole with one fluid layer is known [5, 6]. The change of sound propagation speed in mud due to depth and viscosity of mud are neglected for simplicity. It is also assumed that the steady mud velocity is slow compared to sound propagation speed in the mud and that static pressure in the mud is low compared to the bulk modulus, allowing their effects to be neglected. In this paper, a study of wave propagation in drilling borehole are greatly simplified by assuming axial symmetry and low frequencies with long wavelengths, compared to the borehole radius.

EQUATIONS OF ELASTIC MOTION

If a circular pipe vibrates in a borehole as shown in Fig. 2, the deformation produced is symmetric about the axis of the cylindrical pipe. also, if the frequency is assumed to be low, then:

$$\omega << 2\pi c_{\phi}/b \ , \ \omega << 2\pi c_{\psi}/b \ , \ \omega << 2\pi c_{x}/b$$
$$\omega << 2\pi c_{o}/c \ , \ \omega << 2\pi c_{x}/c \tag{1}$$

where: ω is circular frequency, c_{ϕ} and c_{ψ} are propagation speed of dilational and shear wave of pipe, respectively, c_x is wave propagation speed in the axial direction, b is outer radius of pipe, c_o is wave propagation speed in mud and c is borehole radius.
With the previous presumptions, the equation of axial motion of a uniform pipe in the borehole can be obtained as follow:

$$\rho_p \frac{\partial^2 u_p}{\partial t^2} = E \frac{\partial^2 u_p}{\partial x^2} + 2\frac{\upsilon}{b^2 - a^2}\left(a^2 \frac{\partial p_i}{\partial x} - b^2 \frac{\partial p_o}{\partial x}\right) \tag{2}$$

where the subscript p, i and o are pipe, inner mud and outer mud respectively, ρ is density, E is the Young's modulus, u is the axial displacement of pipe, and a is the inner radius of pipe.

Also, the equations of axial motions of inner and outer mud are obtained as follow:

$$\rho_m \frac{\partial^2 u_i}{\partial t^2} = K \frac{\partial^2 u_i}{\partial x^2} + K \frac{\partial\left(\frac{2u_{r=a}}{a}\right)}{\partial x} \tag{3}$$

$$\rho_m \frac{\partial^2 u_o}{\partial t^2} = K \frac{\partial^2 u_o}{\partial x^2} + K \frac{\partial\left(\frac{2cu_{r=c} - 2bu_{r=b}}{c^2 - b^2}\right)}{\partial x} \tag{4}$$

where K is bulk modulus of mud. The radial displacements of the boundaries can be obtained in terms of the axial displacements of the pipe and mud pressures. Therefore the equations of motion can be expressed in terms of axial displacements only. In matrix form, they are shown as follow:

$$\left(\frac{\partial^2}{\partial t^2}\begin{bmatrix} \rho_p & \frac{2\upsilon a^2}{b^2 - a^2}\rho_m & -\frac{2\upsilon b^2}{b^2 - a^2}\rho_m \\ \frac{2\upsilon K}{E}\rho_p & \left(1 + \frac{K}{P_a^{'}}\right)\rho_m & -\frac{K}{P_{ab}^{'}}\rho_m \\ -\frac{b^2}{c^2 - b^2}\frac{2\upsilon K}{E}\rho_p & -\frac{K}{P_{ba}^{'}}\rho_m & \left(1 + \frac{K}{P_b^{'}} + \frac{K}{P_c^{'}}\right)\rho_m \end{bmatrix} - \frac{\partial^2}{\partial t^2}\begin{bmatrix} E & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{bmatrix}\right)\begin{Bmatrix} u_p \\ u_i \\ u_o \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{5}$$

where:

$$P_a^{'} = \frac{a}{2}P_a = \frac{E}{2}\left[\frac{b^2 - a^2}{(1-\upsilon)a^2 + (1+\upsilon)b^2}\right] \tag{6}$$

$$P_{ab}^{'} = \frac{a}{2}P_{ab} = \frac{E}{2}\left(\frac{b^2 - a^2}{2b^2}\right) \tag{7}$$

$$P_b^{'} = \frac{c^2 - b^2}{2b}P_b = \frac{E\left(c^2 - b^2\right)}{2b^2}\left[\frac{b^2 - a^2}{(1+\upsilon)a^2 + (1-\upsilon)b^2}\right] \tag{8}$$

$$P_c^{'} = \frac{c^2 - b^2}{2c}P_c \tag{9}$$

$$P_{ba}^{'} = \frac{c^2 - b^2}{2b} P_{ba} = \frac{E\left(c^2 - b^2\right)}{2b^2}\left(\frac{b^2 - a^2}{2a^2}\right) \tag{10}$$

where P_a, P_{ab}, P_b and P_{ba} are the static radial spring constants of the pipe [5], $P_a^{'}$ is the p_i (static inner mud pressure) required to obtain unit area strain of inner mud layer, in the absence of outer mud pressure, $P_{ab}^{'}$ is the p_o (static outer mud pressure) required to obtain unit area strain of inner mud layer, in the absence of outer mud pressure, $P_b^{'}$ is the p_o required to obtain unit area strain of outer mud layer to $u_{r=b}$ only, in the absence of inner mud pressure, $P_c^{'}$ is the p_o required to obtain unit area strain of outer mud layer to $u_{r=c}$ only, in the absence of inner mud pressure, $P_{ba}^{'}$ is the p_i required to obtain unit area strain of outer mud layer, in the absence of outer mud pressure and $P_c (= \frac{2G}{c})$ is the equivalent radial spring constant of the formations and G is shear modulus.

The Eq. (5) is shown a set of coupled equations of the axial motions of the pipe, inner mud and outer mud. If assume a wave-form solution for u_p, u_i and u_o, then the Eq. (5) become an eigenvalue problem. The eigenvalues are squares of the wave propagation speeds in the axial direction. There are three eigenvalue. For each eigenvalue a corresponding eigenvector or mode shape can be obtained from it.

Corresponding mode shapes of related variables such as the axial velocities, axial stress, pressures and radial displacements can be obtained for each mode shape of axial displacement as shown:

$$\begin{Bmatrix} v_p \\ v_i \\ v_o \end{Bmatrix}_n = i\omega \begin{Bmatrix} u_p \\ u_i \\ u_o \end{Bmatrix}_n \tag{11}$$

$$\begin{Bmatrix} \sigma_p \\ p_i \\ p_o \end{Bmatrix}_n = c_{xn} \begin{bmatrix} -\rho_p & 0 & 0 \\ 0 & p_m & 0 \\ 0 & 0 & p_m \end{bmatrix} \begin{Bmatrix} v_p \\ v_i \\ v_o \end{Bmatrix}_n \tag{12}$$

$$\begin{Bmatrix} u_{ra} \\ u_{rb} \\ u_{rc} \end{Bmatrix}_n = c_{xn} \begin{bmatrix} \frac{\upsilon a \rho_p}{E} & \frac{\rho_m}{P_a} & -\frac{\rho_m}{P_{ab}} \\ \frac{\upsilon b \rho_p}{E} & \frac{p_m}{P_{ba}} & -\frac{\rho_m}{P_b} \\ 0 & 0 & \frac{p_m}{P_c} \end{bmatrix} \begin{Bmatrix} v_p \\ v_i \\ v_o \end{Bmatrix}_n \tag{13}$$

where: v_p, v_i and v_o are axial velocity of pipe, inner mud and outer mode, respectively, σ_p, p_i and p_o are axial tensile stress in pipe, pressures in inner mud and outer mode, respectively and n is mode number ($n = 1, 2\, and\, 3$).

Modal analysis

If a harmonic pressure disturbance $\{f\}e^{i\omega t}$ is defined at $x = 0$ in semi-infinite borehole, then the solution can be represented as superposition of the set of modes as follow:

$$\begin{Bmatrix} \sigma_p \\ p_i \\ p_o \end{Bmatrix} = \left(B_1 \begin{Bmatrix} \sigma_p \\ p_i \\ p_o \end{Bmatrix}_1 e^{-\frac{i\omega}{c_{x1}}x} + B_2 \begin{Bmatrix} \sigma_p \\ p_i \\ p_o \end{Bmatrix}_2 e^{-\frac{i\omega}{c_{x2}}x} + B_3 \begin{Bmatrix} \sigma_p \\ p_i \\ p_o \end{Bmatrix}_3 e^{-\frac{i\omega}{c_{x3}}x} \right) e^{i\omega t} \tag{14}$$

The B_1, B_2 and B_3 are determined from the boundary condition: $\{f\} \equiv \begin{Bmatrix} f_p \\ f_i \\ f_o \end{Bmatrix}$ at $x = 0$ as shown:

$$\begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix} = \left[\begin{Bmatrix} \sigma_p \\ p_i \\ p_o \end{Bmatrix}_1 \begin{Bmatrix} \sigma_p \\ p_i \\ p_o \end{Bmatrix}_2 \begin{Bmatrix} \sigma_p \\ p_i \\ p_o \end{Bmatrix}_3 \right]^{-1} \begin{Bmatrix} f_p \\ f_i \\ f_o \end{Bmatrix} \tag{15}$$

A disturbance introduced in the borehole decomposes into three modal components, each propagation at its own speed with its own distribution of pressures in the mud layers and axial stress in the pipe.

Non-dimensional parameters

If neglect the off-diagonal terms in Eq. 5, then three uncoupled equations are obtained. Their eigenvalues and uncoupled speeds are proper in producing non-dimensional parameters obvious the importance of the coupling between the outer and inner mud. The uncoupled speeds become:

$$c_{xp}^{'2} = c_p^2 = \frac{E}{\rho_P} \tag{16}$$

$$c_{xi}^{'2} = c_m^2 \left(1 + \frac{K}{P_a^{'}}\right)^{-1} = \frac{K}{\rho_m}\left(1 + \frac{K}{P_a^{'}}\right)^{-1} \tag{17}$$

$$c_{xo}^{'2} = c_m^2 \left(1 + \frac{K}{P_b^{'}} + \frac{K}{P_c^{'}}\right)^{-1} = \frac{K}{\rho_m}\left(1 + \frac{K}{P_b^{'}} + \frac{K}{P_c^{'}}\right)^{-1} \tag{18}$$

where: c_p is wave propagation speed of a radially-free pipe or bar speed and c_m is wave propagation speed of mud inside a rigid pipe. The uncoupled speeds in the mud can be non-dimensional by dividing by c_m as follow:

$$C_i = \frac{c'_{xi}}{c_m} = \frac{1}{\sqrt{1 + K/P'_a}} \quad (19)$$

$$C_o = \frac{c'_{xo}}{c_m} = \frac{1}{\sqrt{1 + K/P'_b + K/P'_c}} \quad (20)$$

and C' is defined as follow:

$$C' = \frac{c'_{xo}}{c'_{xi}} = \frac{C_o}{C_i} = \frac{\sqrt{1 + K/P'_a}}{\sqrt{1 + K/P'_b + K/P'_c}} \quad (21)$$

If $P'_a >> K$, in other words the pipe is high stiff compared to the bulk modulus of the mud, then there would be no coupling between the inner and outer mud.

APPLICATIONS

An example is presented for a typical drillpipe. The example is for a $0.32\ m$ borehole with a $0.127\, m$ steel drillpipe. Clay was selected for formation. The input parameters used for the Clay are given below:

$G = 199.6596 Mpa$, $c_\phi = 495.6048\, m/\sec$ and $c_\psi = 303.5808\ m/\sec$

and non-dimensional parameters are:

$\frac{K}{P'_a} = 0.164$, $\frac{K}{P'_b} = 0.028$, $\frac{K}{P'_c} = 15.085$, $C_i = 0.927$, $C_o = 0.249$ and $C' = 0.269$

Also the propagation speeds of each mode and uncoupled as follow:

$c_{x1} = 388.3152\ m/\sec$, $c_{x2} = 1444.1424\ m/\sec$, $c_{x3} = 5162.3976\ m/\sec$

and $c'_{xo} = 388.3152\ m/\sec$, $c'_{xi} = 1444.4472\ m/\sec$, $c'_{xp} = 5141.976\ m/\sec$

where $\sqrt{K/\rho_m} = 1558.4424\, m/\sec$.

The pressure mode shapes are obtained as shown below:

1- First mode, $c_{x1} = 388.3152\ m/\sec$: $\begin{Bmatrix} \sigma_p \\ p_i \\ p_o \end{Bmatrix}_1 = \begin{Bmatrix} 0.527 \\ -0.575 \\ 1 \end{Bmatrix}$

2- Second mode, $c_{x2} = 1444.1424\ m/\sec: \begin{Bmatrix} \sigma_p \\ p_i \\ p_o \end{Bmatrix}_2 = \begin{Bmatrix} -5.554 \\ 1 \\ 0.081 \end{Bmatrix}$

3- Third mode, $c_{x3} = 5162.3976\ m/\sec: \begin{Bmatrix} \sigma_p \\ p_i \\ p_o \end{Bmatrix}_3 = \begin{Bmatrix} 1 \\ 0.273 \\ -0.003 \end{Bmatrix}$

In the example $C' = 0.269$ is less than unity, it can be seen that there is little interaction between pressures in the inner mud and the outer mud. The propagation speed of the pipe mode is hardly influenced by the interaction, increasing 0.4% in clay. Therefore, if just the pipe axial vibration due to excitation in the pipe is of interest, then the effect of the mud and formation may usually be neglected.

REFLECTIONS AT DISCONTINUITIES IN CROSS-SECTIONECTION

In a borehole reflection of axial waves may occur at discontinuities in cross-section, in material properties or in both [7, 8]. Two different semi-infinite boreholes joined at $x = 0$ are shown in Fig. 3. Each of three modal components arrives at junction, it will create three transmitted modal components and three reflected components. Each reflected and transmitted modal component will propagate at its own speed. The boundary conditions at the junction require balance of force and continuity of velocity for the pipe [7], and mass conservation and continuity of pressure for the inner and outer mud layers [8]. Velocities can be calculated from pressures. If introduce the reflected and transmitted waves in terms of the modes of the first and second borehole and to take into the boundary conditions, the amplitudes of the reflected and transmitted waves for each mode are determined.

SUMMARY

To explain how drillstring, mud, and formation interact in axial wave propagation, a drilling borehole, which is infinitely–long, uniform, and cylindrically multi–layered waveguide, was analyzed. The results revealed that there are three principal modes of propagation in a drilling borehole, each one travels at its own propagating speed. The fastest was the pipe mode which is dominated by a stress wave in the pipe, and is the mode that one would normally associate with drillstring axial vibration. The mud coupling effect is generally negligible in this pipe mode. The other two were the mud acoustic modes, which are dominated by pressures in the inner mud and \ or outer mud. In these mud modes, the pressure in the inner mud and outer mud interact through the radial flexibility of the pipe. Each mode has a unique mode shape or distribution of pressure in the inner and outer mud and axial stress in the pipe. Significant reflections of propagating modes may occur at discontinuities in borehole diameter or hardness of formation.

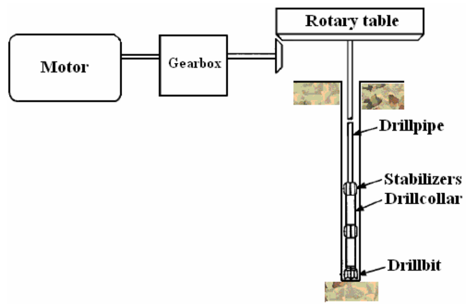

Figure 1– A sketch of the drillstring with the rotary table drive system

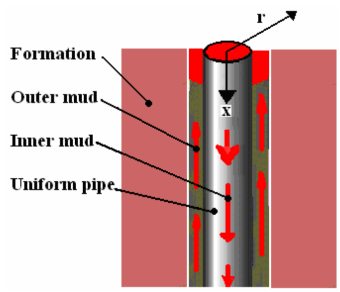

Figure 2 – Uniform pipe in a homogeneous formation

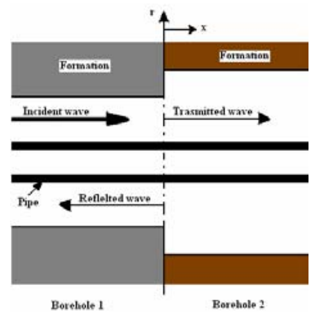

Figure 3 – Discontinuities in borehole cross-section

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