

# TITLE – STABILITATION OF A NON-COLLOCATED ACTIVE STRUCTURAL ACOUSTIC CONTROL ON A CLAMPED-FREE BEAM

Aitziber Aizpuru\*<sup>1</sup> and José M. Abete<sup>1</sup>

<sup>1</sup>Mechanical Engineering Department, Mondragon University Loramendi 4, 20500 Arrasate (Gipuzkoa), Spain <u>aaizpuru@eps.mondragon.edu</u>

#### Abstract

Nowadays noise pollution has become an important environmental concern due to uncomforting and health problems it causes. Noise control can promise solutions to this problem, but because of its complexity it constitutes an important technological challenge. The aim of this work is to reduce the sound radiation of a cantilever beam using a non-collocated "Active Structural Acoustical Control" (ASAC) system. This configuration limits the gain of the controller since it produces instability of the non-controlled modes. Therefore, a new strategy has been analysed to stabilize the destabilized modes involving another sensor in a collocated or nearly-collocated configuration with the actuator.

# **INTRODUCTION**

This paper presents a strategy to reduce the sound radiation by means of a modal damping, using a non-collocated actuator and sensor pair. It is well known that the collocated configuration of the actuator and sensor pair guarantees the closed-loop stability in a broad class of controllers. The collocated control also provides a robust design against system parameter uncertainties and some dynamic nonlinearities of sensors and actuators. But the use of a collocated configuration is not always possible or practical and in these cases it is necessary to use a non-collocated configuration. However, the non-collocation has some drawbacks: the closed-loop stability is sensitive to parameter uncertainties and the controller design is quite complex.

There are some important studies in the field of non-collocated controls whose aim is to understand the effect of non-collocation in control systems and to stabilize the closed-loop behavior. Miu [1] presented the physical interpretation of the transfer function zeros for control systems. It shows that the zeros are related to the energy propagation. On the one hand, the complex zeros near the imaginary axis correspond to the propagation of the energy from the actuator, but this will be absorbed by the structure so that no displacement will appear at the sensor. On the other hand, the real zeros, which cause the non-minimum phase of the system, are related to the nonpropagation of the energy.

Other works [2] about non-collocated systems demonstrates that, in some cases, conservative systems may have complex zeros. The presence of these complex zeros seems to decrease the control system robustness with respect to parameter variations. If such complex zeros occur inside the bandwidth of the control systems, it will produce the system instability, even with small variations of the structural model.

Other researches [3] conclude that an accurate model of the structure is very critical in a non-collocated control design: small inaccuracies in the model can produce system instability.

In the last years there have been several researches to stabilize non-collocated controllers. Two of those techniques to reduce the effect of the destabilization of non-collocated systems are the *time delay method* and the *passivity-based method*.

The time delay control [4] can make the system robust with respect to uncertainties in large-scale system parameters and in the implementation of the time delays themselves. The latter method [5] consists of transforming non-passive systems in passive ones using a suitable compensation.

The disadvantage that those methods present is the necessity of a good model of the system. The aim of this paper is to stabilize non-collocated systems without an accurate model of the system.

### **MODEL DESCRIPTION**

The geometric and mechanical properties of the clamped-free beam can be seen in the Figure 1. A piezoelectric stack type actuator was selected (PPA40M, Cedrat Technologies), and placed in parallel to the neutral axis of the controlled beam to induce bending moments. An accelerometer was used as the error sensor due to its natural way to measure vibrations.

The optimum actuator and sensor placements were calculated with an algorithm which consists of evaluating the  $H_2$  norm for each mode, actuator and sensor combination [6]. The actuator optimum placement to control the first two modes is at the clamped tip of the beam, being the sensor placement at the free tip of the beam. On the other hand, the vertical offset of the actuator was calculated in 15mm considering the thickness of connects and the total displacement that the chosen actuator can support.



Figure 1 - Clamped-free beam, actuator and sensors

In order to control the second mode a single-input single-output control strategy was implemented using an "Acceleration Feedback Control" (AFC). The Figure 2 shows the block diagram of the control.



Figure 2 – Block diagram of AFC

$$\begin{bmatrix} \mathbf{M} \end{bmatrix} \{ \ddot{\mathbf{y}} \} + \begin{bmatrix} \mathbf{C} \end{bmatrix} \{ \dot{\mathbf{y}} \} + \begin{bmatrix} \mathbf{K} \end{bmatrix} \{ \mathbf{y} \} = -\{ \mathbf{X}_{act} \} \{ \mathbf{G} \}^T \begin{bmatrix} \Omega_c \end{bmatrix} \{ \nu \} + \{ \mathbf{F} \}$$

$$\{ \ddot{\nu} \} + \begin{bmatrix} \Delta_c \end{bmatrix} \{ \dot{\nu} \} + \begin{bmatrix} \Omega_c \end{bmatrix} \{ \nu \} = \{ \mathbf{X}_{acc} \}^T \{ \ddot{\mathbf{y}} \}$$

$$(1)$$

Where:

[M], [C] and [K] are the mass, the damping and the stiffness matrices, respectively. {y} and {v} are the nodal displacements and the compensator coordinates, respectively. {F} and {G} are the excitation force and the gain row vector, respectively. [ $\Delta_c$ ] and [ $\Omega_c$ ] are the compensator damping and the natural frequency matrices, respectively.

The design of the compensator parameters was carried out by the "cross-over point" method [7]. The aim of the design of these parameters is to reduce the effect of the resonant peaks without creating new ones near the controlled mode. The cross-over point technique chooses the operation point in which the pole of the structure and the controller crosses. The "cross-over point" of the second mode of the system is shown in the Figure 3.



Figure 3 – Root locus of the second controlled mode

The stability of a non-collocated controller is sensitive to the uncertainties of the model parameters. In order to avoid this effect, in this work the parameters of the controller filter were calculated experimentally with the residue of the transfer function between the actuator and the sensor of the mode that it was wanted to control.

#### STRUCTURAL RESULTS

In this study a control design of the second mode was performed with a noncollocated configuration. This configuration limits the gain of the controller since it produces certain instability of the non-controlled modes. Therefore, a new strategy has been analysed to stabilize the destabilized modes involving another sensor in a collocated or nearly-collocated configuration with the actuator. The Figure 4 shows the non-collocated (a) and nearly-collocated (b) transfer functions between the actuator and the sensor 1 and the sensor 2, respectively, of the modelled system (Figure 1).



Figure 4 – Transfer functions of the actuator and the sensors

#### Non-collocated control of the second mode

The second mode was controlled with the compensator parameters designed by the cross-over point. The Figure 5a shows the transfer functions of the system, with and without control. As it is shown, the amplitude of the transfer function is reduced in the second mode frequency. Therefore, the controlled system has smaller amplitude in the second mode, but out of this frequency the two curves coincide. However, the control excites the third non-controlled mode due to the gain that was chosen.



Figure 5 – Transfer functions and root locus of the non-collocated system

The Figure 5b confirms the stability of the control, but it is also possible to see that if the gain is increased the system can be destabilized. To avoid the instability of the third mode, this was controlled with the second sensor in a nearly-collocated configuration with the actuator. This nearly-collocated sensor was used to avoid the instability of the other non-controlled modes.

#### Collocated control of the third mode

The third mode was controlled with the parameters of the compensator designed by the cross-over point. The Figure 6a shows the transfer functions of the system, with and without control. In this case, the amplitude of the transfer function is reduced in the third mode frequency. But in this case, unlike the previous section, there is less risk the control to excite the other non-controlled modes (Figure 6b). The reason to use the nearly-collocated controller to stabilize the destabilize modes is that this type of controller has less risk to destabilize the non-controlled modes.



Figure 6 – Transfer functions and root locus of the nearly-collocated system

In the next part a single-input multiple-output (SIMO) control is carried out with two sensors, and the stability of the SIMO control with two compensators is studied.

#### SIMO control of multiple modes

The second and the third modes of the system were controlled with two accelerometers (non-collocated and nearly-collocated) and with one actuator. The Figure 7a shows the transfer function between the actuator and the sensor 1 with and without control.

A sufficient condition for the stability of a closed-loop system is that the poles of the closed-loop transfer function between the points  $y_r$  and  $f_{exc}$  have negative real parts for any sensor position and excitation position of the system. In this case those considered points were the sensor 1 and the excitation force in the actuator locations.

$$G_{r,exc} = \frac{\ddot{y}_{r}}{f_{exc}} = \frac{G_{r,exc} + \sum_{i=1}^{2} H_{fi} \left( G_{r,act} G_{i,exc} - G_{r,exc} G_{i,act} \right)}{1 + \sum_{i=1}^{2} H_{fi} G_{i,act}}$$
(2)

Where:

 $G_{r,exc}$ ,  $G_{r,act}$  are the open-loop transfer function between the response point and, the excitation force and the actuator force, respectively.  $G_{i,exc}$ ,  $G_{i,act}$  are the open-loop transfer function between one of the sensors and, the excitation force and the actuator force, respectively.  $H_{fi}$  is the compensator of the i-th sensor,



Figure 7 – Transfer functions and root locus of the sensor 1 and the actuator

The Figure 7a shows the transfers function between the sensor 1 and the actuator with and without control. The Figure 7b shows the poles of the closed-loop transfer function between the sensor 1 and the actuator with two controlled modes with two accelerometers. In this case, the poles at the operating gains show less instability risk. The third mode in the previous case (with only one sensor) was at the limit of the instability, but in this case it has less risk to destabilize because the pole has a greater negative real part than before (Figure 5b). The control system changes some poles, but with the gains that were chosen the system have less instability risk.

#### **Experimental results**

The experimental set up is shown in the Figure 8. A cantilever beam with one piezoelectric stack (PPA40M) and two accelerometers was used to prove the method. A digital control system, which consists of the MATLAB/SIMULINK modelling software and a dSPACE DS1104 R&D Controller, was developed. The SIMULINK software was used to programme the control block diagram, and the Real-Time Interface to generate the required real-time code together with the Real-Time Workshop from the SIMULINK. Once the model was implemented, Real-Time Interface downloads and executes this code on the dSPACE hardware. The IDEAS spectrum analyser was used to obtain the frequency response between the acceleration of the sensor 1 (m/s<sup>2</sup>) and the random noise Vi (V), as it is shown in the Figure 8. The control structure in MATLAB/SIMULINK is shown in the Figure 9.



Figure 8 – Experimental clamped-free beam



Figure 9 – Controller system in MATLAB/SIMULINK

The experiments were performed to damp the second mode with the sensor 1 in order not to destabilize the third mode, which was damped with the sensor 2. The Figure 10 shows the transfer functions with and without control. The parameters of the controller filter were calculated experimentally with the transfer function residue between the actuator and the sensor of the mode that it was wanted to control. The experimental method reduces the sensitivity to parameter uncertainties; therefore the method reduces the instability risk.



Figure 10 – Transfer function of the sensor 1 and the random noise with and without control

The Figure 7 and the Figure 10 confirm clearly the improvement due to the nearly-collocated control of the third mode with the second sensor. The root locus of the closed-loop in the Figure 7b shows that the third mode is more damped and that

the risk of instability is less than without having controlled the third mode with a nearly-collocated sensor.

## CONCLUSION

The aim of this work is to stabilize the destabilized modes by a non-collocated control. To improve the stability a new strategy has been analysed involving another sensor nearly-collocated with the actuator. A SIMO control system was used to control the third mode with the non-collocated sensor and a nearly-collocated sensor was used to stabilize the destabilized mode.

In this work two modes were damped with two compensators, but the design of the each compensator was performed independently. This approach, called "selective design", supposes that the influence of the other modes is negligible, although this could induce the loss of the "cross-over point".

As it is known, the stability of non-collocated controls is sensitive to model parameter uncertainties. In order to avoid this, in this work the parameters of the controller filter were calculated experimentally with the transfer function residue of the actuator and the sensor of the mode that it was wanted to control.

The next step will be to reduce the sound radiation of the beam with a noncollocated configuration, whilst the stability of the system will be improve with the method described in this paper.

#### REFERENCES

[1] Miu, D.K., Physical Interpretation of Transfer Function Zeros for Simple Control Systems with Mechanical Flexibilities *Journal of Dynamic Systems, Measurement and Control,* Vol.113, pp.419-424 (1991).

[2] Loix, N. Kozanek, J. and Foltete, E. On the complex Zeros of Non-Colocated Systems *Jouranl of Structural Control* Vol. 3 pp. 79-87 (1996).

[3] Spector, V.A., Flahner, H. Modeling and Design Implications on Noncollocated control in Flexible Systems *Journal of Dynamic Systems, Measurement and Control* Vo.112 pp.186-193 (1990).

[4] Udwadia, F.E., von Bremen, H.F., Kumar, R. and Hosseini, M. Time Delayed Control of Structural Systems *Earthquake Engineering and Structural Dynamics* Vol.32 pp.495-535 (2003)

[5] Gosavi, S.V. and Kelkar A.G. Modelling, Identification, and Passivity-Based Robust Control of Piezo-actuated Flexible Beam *Journal of Vibration and Acoustic* Vol.126 pp. 260-271 (2004)

[6] Aizpuru, A. and Abete, J.M. Active Structural Acoustic Control of Clamped-free Beam by means of Modal Reduction *Proceedings of the*  $6^{th}$  *International conference on Structural Dynamics*, Vol. 2, pp.1239-1244 Paris, France, 4-7 September (2005).

[7] Bayon de Noyer, M.P. and Hanagud, S.V. Single Actuator and Multi-Mode Acceleration Feedback Control Adaptive Structures and Material Systems, *ASME, AD*, Vol.54 pp.227-235 (1997).