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APPLICATION OF ACOUSTIC HOLOGRAPHY TO THE SOUND FIELD WITH UNIFORM FLOW

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Abstract
Acoustic holography uses Kirchhoff-Helmholtz integral equation and Green's function which satisfies Dirichlet boundary condition. Applications of acoustic holography mainly have been taken to the sound field neglecting the effect of flow. The existence of the uniform flow, however, changes sound field, the governing equation, and associated Green's function. Thus the conventional method of acoustic holography should be changed for the sound field in which the effect of flow cannot be neglected. In this research, one possibility to apply acoustic holography to the sound field with uniform flow is introduced. Green's function that copes with the effect of uniform flow is introduced. One easy way to find this will be explained for plane wave with uniform flow in a duct. This method can be also expanded to two dimensional sound field, which is verified by numerical computation.

INTRODUCTION

Acoustic holography[1~2], which predicts sound field by using acoustic pressure at the boundary surface, essentially assumes sound field to be coherent. That is, coherence between measurement points should be close to 1. Otherwise, the prediction results of sound field may differ from the correct sound field which we want to have. There are several cases in which sound field is poorly coherent, for example, sound field due to multiple incoherent sources, and the case that measurement noise is not negligible, and so on. Nam and Kim[3] showed that the error due to low coherence can be reduced by using multiple reference microphones. Especially, this theory can be applicable to the sound field accompanied by flow since this causes measured signals to be poorly coherent[4]. In this case, however, another problem is that the air flow changes the whole sound field governed by wave equation[5]. Since acoustic holography uses wave equation and Green's function satisfying homogeneous Dirichlet boundary condition[6] on the assumption that the effect of flow is negligible, general approach itself should be changed. To overcome this, there have been researches on sound field prediction with uniform flow for two and three dimensional sound fields[7~8]. In these researches, boundary integral formulation was derived by using convective wave equation. Green's function used in those researches, however, does not satisfy homogeneous Dirichlet boundary condition, which makes it difficult to predict sound field by using only acoustic pressure at boundary surfaces. Recently, Ruhala and Swanson[9] tried to develop one method of acoustic holography in uniform flow. They mainly dealt with change of Green's function and filter in wave number domain directly, which doesn't provide the tangible physical meaning.

For the easy understanding of acoustic holography in uniform medium, the process how Green's function satisfying homogeneous Dirichlet boundary condition can be found should be accompanied mathematically and physically. For this, we start with plane wave sound field in a duct with uniform flow.

EXPRESSION OF ONE DIMENSIONAL SOUND FIELD WITH UNIFORM FLOW

Convective wave equation

Since Green's function is driven from the wave equation, the convective wave equation, which is the wave equation in consideration with uniform flow, should be introduced. The whole process to derive convective wave equation is based on reference [5], which says,

$$\frac{d^2 p(x,t)}{dx^2} = \frac{1}{c^2}\left(\frac{\partial}{\partial t} + U\frac{d}{dx}\right)^2 p(x,t), \tag{1}$$

where $p(x,t)$ represents sound pressure at position x and time t, c the wave speed when there is no flow and U the mean flow speed. Note that equation (1) becomes the conventional wave equation when $U = 0$. If we assume simple harmonic motion, i.e., $p(x,t) = P(x)e^{-j\omega t}$, and define a dimensionless factor $M = U/c$, eq. (2) can be rewritten as

$$\frac{d^2 P(x)}{dx^2} + \left(k + jM\frac{d}{dx}\right)^2 P(x) = 0, \tag{2}$$

where k means wave number. Note that we are dealing with the subsonic case and incompressible flow[10], that is, $0 \le M < 0.3$.

One dimensional Green's function

To derive Green's function with uniform flow, consider that there exists an impulsive excitation at $x = x_0$:

$$\frac{d^2 G(x\,|\,x_0)}{dx^2} + \left(k + jM\frac{d}{dx}\right)^2 G(x\,|\,x_0) = -\delta(x - x_0). \tag{3}$$

Any kinds of solution satisfying eq. (3) can be Green's function. Integrating eq. (3) and assuming the form of Green's function gives one solution:

$$G(x\,|\,x_0)=\begin{cases}-\dfrac{1}{j2k}e^{j\frac{k}{1+M}(x-x_0)}, x>x_0\\ -\dfrac{1}{j2k}e^{-j\frac{k}{1-M}(x-x_0)}, x<x_0\end{cases}. \quad (4)$$

Figure 1 shows the shape of Green's function w.r.t. position. The dotted line shows Green's function when $M=0$, the solid line when $M=0.2$. As shown in figure, the wavelength can be changed w.r.t. direction and magnitude of uniform flow. When the direction of propagation is the same with that of uniform flow, wave propagates farther with flow than wave without flow. Otherwise, the reverse phenomenon occurs.

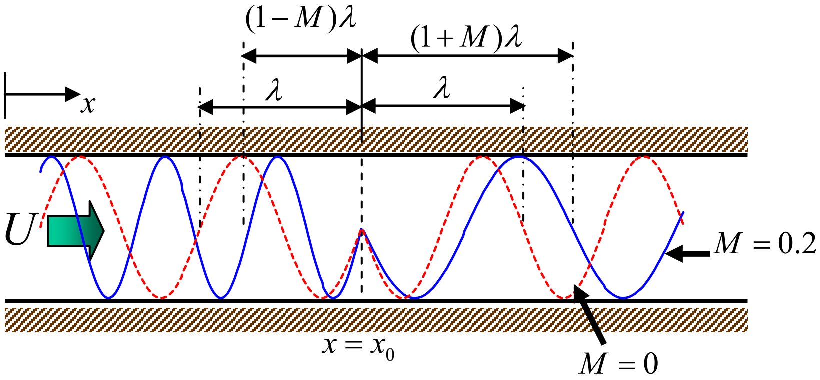


Figure 1- one dimensional Green's function in a duct with uniform flow, where M means Mach number, x_0 the excitation point, λ the wavelength without flow.

ONE WAY TO FIND GREEN'S FUNCTION SATISFYING HOMOGENEOUS DIRICHLET BOUNDARY CONDITION

One dimensional Kirchhoff-Helmholtz integral equation

Using Green's function satisfying eq. (3) and using eq. (2), we can get the following form:

$$\frac{d}{dx}\left(G\frac{dP}{dx}-P\frac{dG}{dx}\right)+\frac{j2Mk}{1-M^2}\left(G\frac{dP}{dx}-P\frac{dG}{dx}\right)=\frac{1}{1-M^2}P\delta(x-x_0), \quad (5)$$

where independent variables such as x, x_0 are temporarily omitted for simplicity. By utilizing one integral technique in reference [11], and neglecting sound pressure when x goes to infinity, we can get the following form for the region $[0,\infty]$:

$$P(x_0)=(1-M^2)\left[P(x)\frac{dG(x\,|\,x_0)}{dx}-G(x\,|\,x_0)\frac{dP(x)}{dx}\right]e^{j\frac{2Mk}{1-M^2}(x-x_0)}\Bigg|_{x=0}. \quad (6)$$

Eq. (14) says that acoustic pressure at $x = x_0$ can be predicted using pressure and gradient of pressure, and Green's function satisfying eq. (4) at $x = 0$.

Green's function which satisfies homogeneous Dirichlet boundary condition

In many cases of acoustic measurement problem, it is easier to measure sound pressure than gradient of sound pressure. Hence, it is recommended to use Green's function which becomes zero at $x = 0$, which does not need the information on gradient of sound pressure. When there is no flow, such Green's function can be found just by putting impulsive excitation at $x = -x_0$ with opposite phase to Green's function in eq. (4), that is,

$$G_D(x \mid x_0) = G(x \mid x_0) - G(x \mid -x_0)\,^{[12]}, \tag{7}$$

where the subscript D denotes homogeneous Dirichlet boundary condition. This simple method can't be applied to the case when mean flow exists, since the wavelengths of left going wave and right going wave differ each other. Figure 2(a) shows this phenomenon graphically.

One simple way to make $G_D(x \mid x_0)$ is to change the excitation position of $G(x \mid -x_0)$ into some other value in consideration with its changed wavelength. This is possible since $G(x \mid -x_0)$ is homogeneous solution of eq. (3). Since the ratio between wavelength of left and right going wave is $(1+M)/(1-M)$, we can change $x = -x_0$ into $x = -\frac{1+M}{1-M}x_0$, that is,

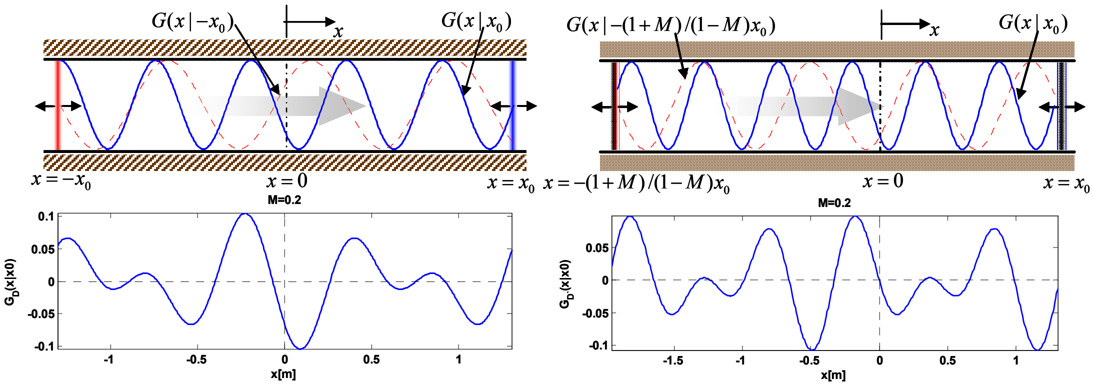
$$G_D^{'}(x \mid x_0) = G(x \mid x_0) - G(x \mid -\frac{1+M}{1-M}x_0)\,, \tag{8}$$

which is Green's function satisfying homogeneous Dirichlet boundary condition considering the effect of uniform flow. Using this function, we can make Green's function zero at $x = 0$ even if there is uniform flow (see figure 2(b)).

Substituting eq. (4), (8) into eq. (6), we can get the acoustic pressure at $x = x_0$:

$$P(x_0) = P(0)e^{j\frac{k}{1+M}x_0}\,. \tag{9}$$

Since there is no acoustic source within the region $[0, \infty]$, only right going wave propagates within this region with the wave number $k/(1+M)$, which gives a clear explanation on the result of eq. (9) physically. Through this, we can see that the proposed Green's function can be applied to the one dimensional sound field.



(a) *(b)*

Figure 2- Graphical expression of $G(x\,|\,x_0)$, $G(x\,|-x_0)$, *and (a)* $G_D(x\,|\,x_0)$, *and (b)* $G_D^{'}(x\,|\,x_0)$ *when* $M=0.2$. *The solid line means* $G(x\,|\,x_0)$, *and the dotted line* $G(x\,|-x_0)$ *and* $G(x\,|-\dfrac{1+M}{1-M}x_0)$ *in upper figures, and the lower figures show* $G_D(x\,|\,x_0)$ *and* $G_D^{'}(x\,|\,x_0)$ *each.*

Application to the two or three dimensional sound field

For more general and applicable case, two or three dimensional sound field, the above way to find Green's function can be applied. See figure 3, which depicts three dimensional sound field due to sources within the region $x<0$ when uniform flow exists along the x-axis. Sound pressure when x or y go to infinity is assumed to be zero, which enables to apply Sommerfeld radiation condition[13]. In this case, sound pressure at arbitrary point on the surface $x=x_0$ is determined by integrating sound pressure at $x=0$. In this case, Green's function can be written as $G(x,y,z\,|\,x_0,y_0,z_0)$, which means sound pressure at (x,y,z) when there exists a monopole source at (x_0,y_0,z_0). Using this, Green's function satisfying homogeneous Dirichlet boundary condition can be written as

$$G_D^{'}(x,y,z\,|\,x_0,y_0,z_0)=G(x,y,z\,|\,x_0,y_0,z_0)-e^{-j\frac{2Mk}{1-M^2}x_0}G(x,y,z\,|-x_0,y_0,z_0)\,. \tag{10}$$

At the second term of right hand side in eq. (10), $e^{-j\frac{2Mk}{1-M^2}x_0}$ and $-x_0$ plays a role controlling phase which enables $G_D^{'}(x,y,z\,|\,x_0,y_0,z_0)$ to become zero at $x=0$. For the explicit form of $G(x,y,z\,|\,x_0,y_0,z_0)$ and two or three dimensional Kirchhoff-Helmholtz integral equation with uniform flow, see reference [7] and [8].

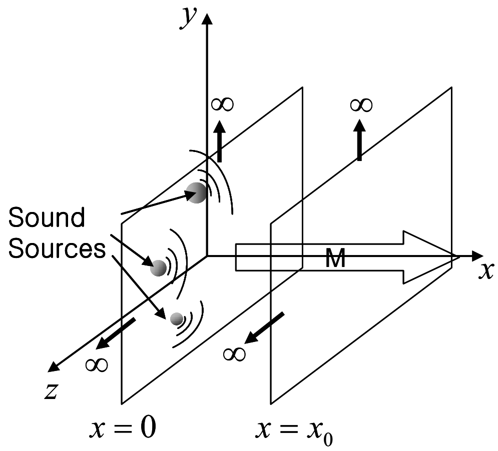


Figure 3- Three dimensional sound field due to sound sources within the region $x < 0$

NUMERICAL COMPUTATION

To verify the applicability of the proposed Green's function, numerical simulation for the two dimensional sound field is conducted for the two dimensional sound field generated by monopole source. The measurement line, where the measurement point lies, is x_H apart from the prediction line where the monopole source lies. The distance between measurement points is d, and the aperture length is L. Since this paper deals mainly with the applicability of sound prediction method to the sound field where uniform flow cannot be neglected, the prediction results w. r. t. the variation of uniform flow is observed and compared with the exact solution. Also the prediction results are compared with those by using the conventional method.

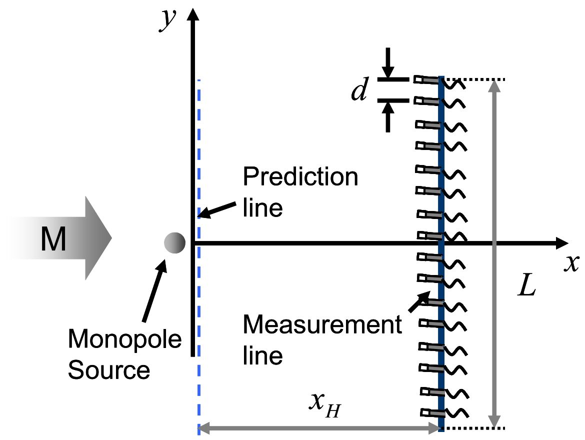


Figure 4- Illustration of variables for numerical computation. d *means the distance between adjacent measurement points,* L *the aperture size,* x_H *the distance between measurement line and prediction line.*

Figure 5 shows the computation results compared with the conventional method and the exact solution.
Prediction results shows that when $M = 0$, the three results matches well except for the point at $y = 0$, which seems to be a strong singular point near the monopole source. As the flow speed increases, results using the conventional methods go further from the exact solution. Results using the proposed Green's function, on the other hand, match well regardless of the flow speed.

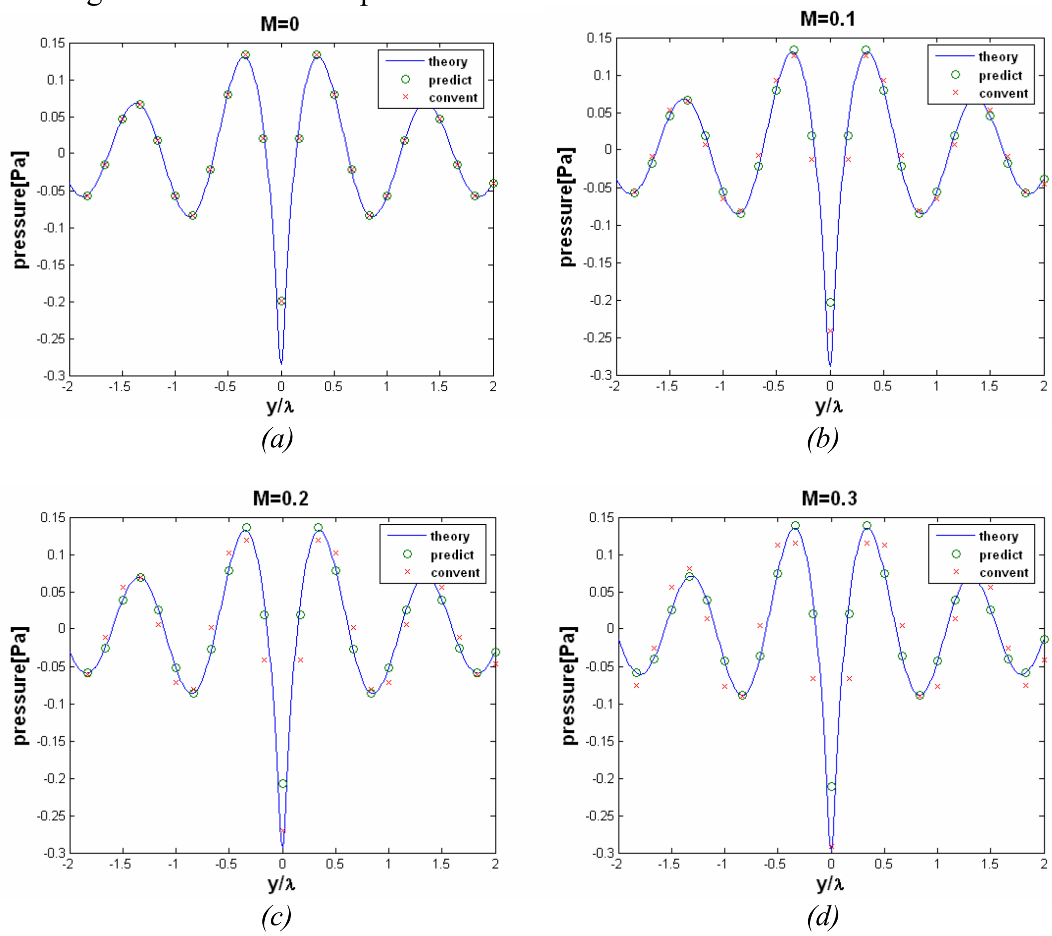


Figure 5- Numerical computation results for $\lambda = 0.34m, d = \lambda / 6, L = 6\lambda$, $x_H = 0.1m$ *and (a)* $M = 0$, *(b)* $M = 0.1$, *(c)* $M = 0.2$, *and (d)* $M = 0.3$. *The solid line depicts the exact solution, circles mean prediction by using the proposed Green's function, x marks mean prediction by using the conventional method.*

SUMMARY

We have derived the sound field prediction process by obtaining Green's function satisfying homogeneous Dirichlet boundary condition when there exists uniform flow. The idea to obtain such Green's function stems from one dimensional sound field, which easily explains how to change Green's function compared with the conventional method in which the effect of flow is neglected. Using these, we have shown that the sound field can be predicted without knowing the gradient of boundary pressure with uniform flow. Computation results for the two dimensional sound field also show the applicability of this idea to the sound field with uniform flow.

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