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INTENSITY EFFECTS OF FINITE AMPLITUDE ULTRASONIC WAVES ON THE ABSORPTION COEFFICIENT

H.Khelladi*[1] & H.Djelouah[2]

[1]Faculté d'Electronique & Informatique, USTHB, BP32, El Alia ,16111, Bab Ezzouar, Algérie.
[2]Faculté de Physique, USTHB, BP32, El Alia ,16111, Bab Ezzouar, Algérie.
hassinakhelladi@yahoo.fr

Abstract

For a better understanding of the nonlinear ultrasound interactions with biological media, an analysis based on Goldberg's number is elaborated in order to estimate the amplitude of the generated harmonics in the case of a finite amplitude plane wave that propagates in a dissipative liquid.

INTRODUCTION

Krassilnikov's experimental works show a close dependence of the absorption coefficient with the intensity [1,2]. Because of the mathematical formulation complexity, most authors do not take into account this dependence in their numerical modeling.

In this study, the absorption coefficient dependence versus the intensity is taken into account. An analysis on the validity domain of the fundamental and the second harmonic analytical expressions established on the quasi-linear approximation is elaborated while being based only on Goldberg's number. Moreover, an error analysis is made to consider the deviations resulting from the quasi-linear approximation as compared to the numerical solution of Burgers's equation. This investigation is based on Krassilnikov's experimental data [2], which have been chosen for their precision. These experimental data concern water and glycerin that corresponds respectively to a weakly dissipative liquid approaching the urine or amniotic fluid characteristics and a strongly dissipative liquid having some similarities with soft tissues [3]. According to this investigation, the analytical expressions of the fundamental and the second harmonic analytical expressions established on the quasi-linear approximation can constitute a good approximation of the numerical solution of Burgers' equation for a medium characterized by a Goldberg number very low as compared to the unity, otherwise the analytical expressions of the fundamental and the second harmonic already established on the quasi-linear approximation are not checked and must be redefined.

It should be noted that in this study, all the derivations were entirely developed in the frequency domain in order to reduce the consuming computing time.

THEORY

The nonlinear equation for plane waves in a homogeneous dissipative medium is Burgers' equation, given in dimensionless variables as [4, 5, 6]:

$$\frac{\partial U(\sigma,\theta)}{\partial \sigma} = U(\sigma,\theta)\frac{\partial U(\sigma,\theta)}{\partial \theta} + \Gamma^{-1}\frac{\partial^2 U(\sigma,\theta)}{\partial \theta^2} \qquad (1)$$

By using the characteristic particle velocity U_0 of the liquid, the characteristic time $\frac{1}{\omega_0}$ and the lossless plane wave shock formation distance $l_s = \frac{1}{k\beta M}$ for a sinusoidal source condition , the dimensionless variables are:

$U = \frac{u}{U_0}$, $\theta = \omega_0 \tau$ and $\sigma = \frac{z}{l_s}$.

where $\tau = t - z/c_0$ is the retarded time.

$\Gamma = \frac{k\beta M}{\alpha}$ is the Goldberg number; where k, M, β and α are respectively the wave number, the Mach number, the nonlinearity acoustic parameter and the absorption coefficient.

The Goldberg number Γ [7,8] represents the ratio of attenuation length l_a (the inverse of the absorption coefficient α and corresponds to the beginning of the old age region) to the shock distance l_s at which the waveform would shock if absorption phenomena were absent. The dimensionless parameter Γ measures the relative importance of the nonlinear and dissipative phenomena. Its value provides an indication of the nonlinearity when compared to the unity. Indeed, the nonlinearity and the dissipation are two phenomena in perpetual competition and Goldberg's number is a reliable indicator for any analysis including these two phenomena. An analysis based on Goldberg's number is important since it is an essential step for solving general problems involving ultrasound waves of finite amplitude.

The first term on the right hand side of Eq. (1) is a nonlinear term that accounts for quadratic nonlinearity producing cumulative effects in progressive plane wave propagation.

Nonlinear propagation in dissipative liquids is considered by using Fourier series expansion. By assuming that the solution of Eq. (1) is periodic in time with period $\frac{2\pi}{\omega_0}$, the solution can be written as the sum of the fundamental and the generated harmonics [9,10]. The dimensionless amplitude of the nth harmonic at the dimensionless position $\sigma+\Delta\sigma$ in terms of all harmonics at the preceding dimensionless position σ is given by:

$$U_n(\sigma+\Delta\sigma)=U_n(\sigma)+\frac{1}{2}\left[\sum_{m=1}^{n-1} mU_m(\sigma)U_{n-m}(\sigma)-\sum_{m=n}^{+\infty} nU_m(\sigma)U_{m-n}(\sigma)\right]\Delta\sigma-n^2\Gamma^{-1}U_n(\sigma)\Delta\sigma \tag{2}$$

The first summation term on the right hand side of Eq. (2) represents the contribution of the lower order harmonics to the nth harmonic, while the second is associated with the contribution of the higher order harmonics. According to the sign of each contribution the nth harmonic energy can be enhanced or decreased.

Eq. (2) allows to determine the amplitude of the nth harmonic at the position $\sigma+\Delta\sigma$ in terms of all harmonics at the preceding spatial position σ.

This derivation requires an appropriate truncation of the finite series on the right hand side of Eq. (2), to insure a negligibly small error in the highest harmonic of interest and to maintain some acceptable accuracy [11, 12, 13, 14].

In the hypothesis of the quasi-linear approximation, all the harmonics of higher order than two can be neglected in the numerical solution of Burgers' equation. The experimental determination of the acoustic nonlinearity parameter β, by the finite amplitude method, is based on pressure measurements of the distortion of a finite amplitude wave during its propagation where the growth of the second harmonic pressure amplitude with the distance is determined. For more convenience, it is suitable to establish the analytical expressions of the fundamental pressure and the second harmonic pressure. In terms of the dimensionless notation, the acoustic pressure of the fundamental and the second harmonic component can be expressed as:

$$p_1(\sigma)=P_0 e^{-\alpha_1 l_s \sigma} \tag{3}$$

where P_0 is the characteristic pressure amplitude (the value of the fundamental at the point σ=0).

$$p_2(\sigma)=\frac{1}{2}P_0\left(\frac{e^{-\alpha_2 l_s \sigma}-e^{-2\alpha_1 l_s \sigma}}{(2\alpha_1-\alpha_2)l_s}\right) \tag{4}$$

where $\alpha_1=\alpha_0 f^2$ and $\alpha_2=4\alpha_0 f^2=4\alpha_1$ denote respectively the attenuation coefficients for the fundamental and the second harmonic amplitudes.
In case of $(\alpha_2-2\alpha_1)l_s\sigma<<1$, Eq. (4) becomes:

$$p_2(\sigma)=\frac{1}{2}P_0\sigma e^{-(\alpha_1+\alpha_2/2)l_s\sigma} \tag{5}$$

NUMERICAL RESULTS

In order to determine the validity domain of the fundamental and the second harmonic analytical expressions established in the quasi-linear approximation two analysis have been made, one for water and the other for glycerin, by exploiting Krassilnikov's experimental results. Table 1 lists material properties.

Parameters	Density ρ_0(kg/m^3)	The sound velocity C_0(m/s)	The nonlinearity acoustic parameter β
Water	998	1481	3.48
Glycerin	1260	1980	5.4

Table 1: Material properties.

The experimental data show that the increase of absorption with an increase in intensity going from 0.3 W/cm^2 up to 4.7 W/cm^2 is connected to the change of the ultrasonic wave shape of finite amplitude [2].

All simulations were made with intensities of 0.3-4.7 W/cm^2, which correspond to breast lesion diagnosis [15] (Figure 1). Initially, the ultrasonic wave is taken to be purely sinusoidal with a frequency of 2MHz in the two considered media. Only the fundamental wave exists at the starting point $\sigma = 0$, and the other harmonic modes are generated as the wave propagates from the source. A number of 40 harmonics was retained to simulate the numerical solution of Burgers' equation which has been considered, in the error derivation, as an exact solution.

For a better representation and interpretation of the several graphs, a symbol with a defined shape and type is inserted all the 4096 samples on the graphic layout of the analyzed functions which have been simulated on 2^{15} samples.

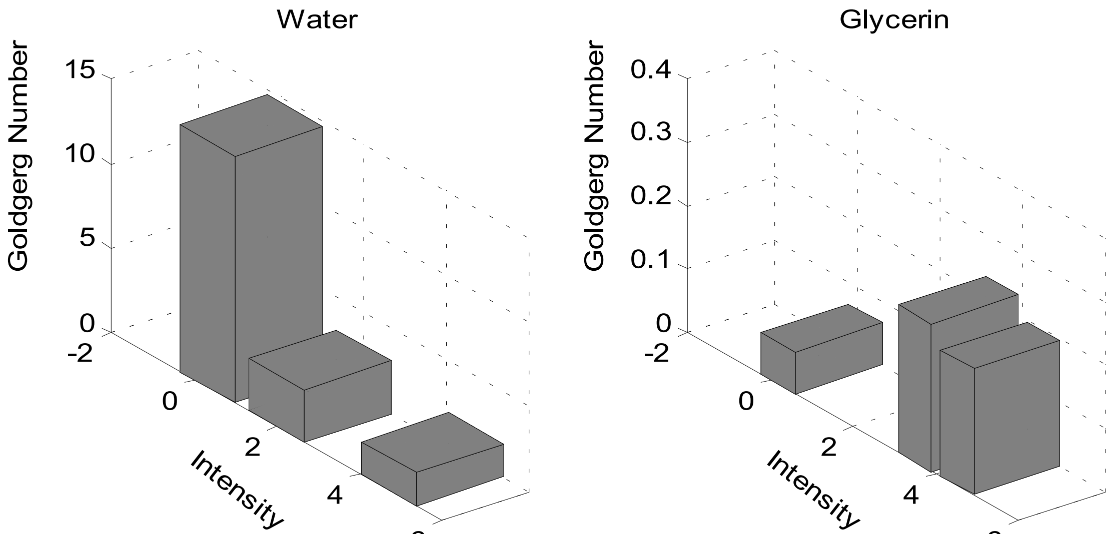

Figure 1: Goldberg numbers for water and glycerin with intensities of 0.3-4.7 W/cm^2 and an insonation frequency of 2 MHz.

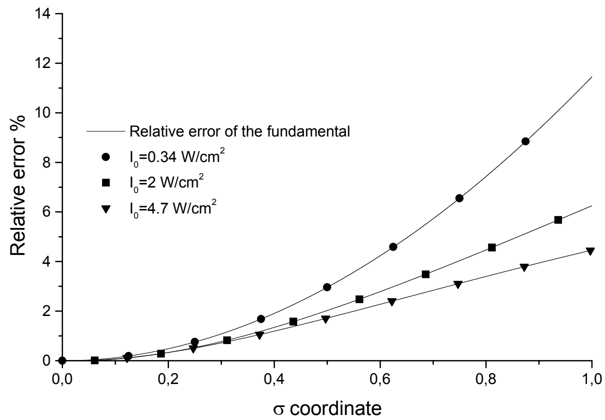

Figure 2: Relative error of the fundamental analytical solution compared to the numerical solution of Burgers' equation versus the σ coordinate: case of water.

Water can generate extreme waveform distortion when compared to glycerin, as indicated by Goldberg's number for water, which is 200 times larger than that of glycerin for an intensity of 0.2W/cm^2 and about 10 times larger than that of glycerin for an intensity of 4W/cm^2 (Figure 1).

The relative error derivation of the analytical solutions as compared to the numerical solution of Burgers' equation is carried out in the following way (Eq. (6)):

$$\mathrm{Error}(\%) = \frac{\left|\text{analytical solution - exact solution(Burgers)}\right|}{\text{exact solution(Burgers)}} \times 100 \tag{6}$$

The relative error, on the selected range, of the fundamental analytical solution (Eq. (3)) as compared to the numerical solution of Burgers' equation is less than 4% for glycerin (Figure 4). As for water, this error is about 12% (Figure 2).

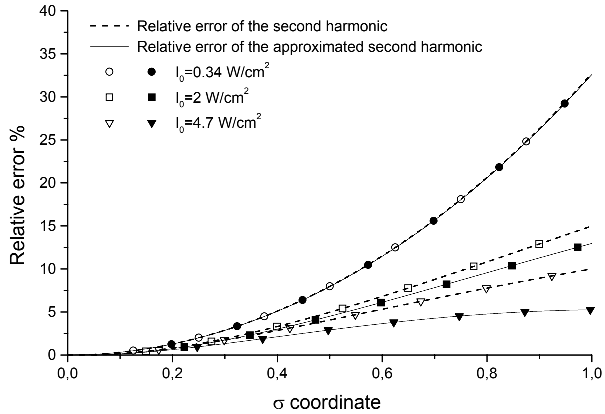

Figure 3: Relative error of the analytical solution of the second harmonic respectively to the approximated second harmonic compared to the numerical solution of Burgers' equation versus the σ coordinate: case of water.

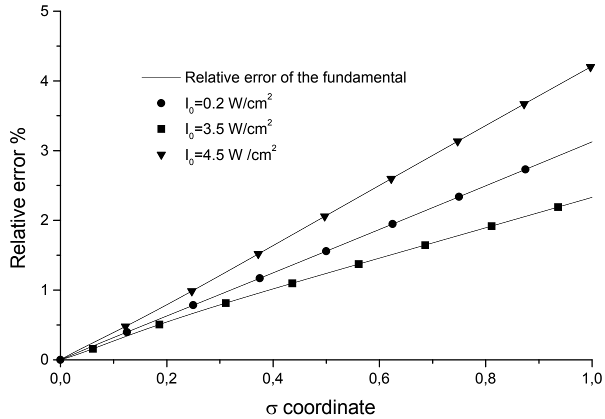

Figure 4: Relative error of the fundamental analytical solution compared to the numerical solution of Burgers' equation versus the σ coordinate: case of glycerin.

In the case of water the relative error, on the selected range, of the approximated second harmonic analytical solution (Eq. (5)) as compared to the numerical solution of Burgers' equation is about 40% for an intensity of $0.34 W/cm^2$ (Figure 3).

For glycerin, the relative error of the second harmonic analytical solution Eq. (4) as compared to the numerical solution of Burgers' equation is much weaker than that resulting from Eq. (5) (Figure 5). As an example, for $\sigma \approx 0.1$ the error obtained from Eq. (4) is lower than 1%, and that produced by Eq. (5) can reach the 40% (Figure 5).

According to this study, the analytical solutions are all the more valid since the measurement is made near the source. In addition, the precision and the choice of the analytical expressions depend essentially on the analyzed medium and on the intensity of excitation I_0.

Moreover for glycerin and with intensity going from $0.3 W/cm^2$ up to $4.7 W/cm^2$, the fundamental and second harmonic expressions (Eq. (3) and Eq. (4)) can constitute a good approximation of the numerical solution of Burgers' equation.

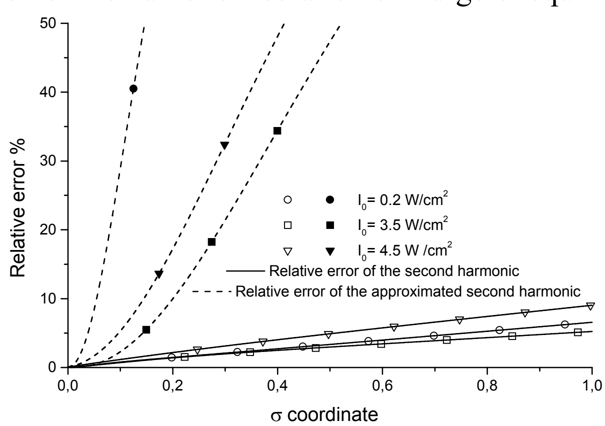

Figure 5: Relative error of the analytical solution of the second harmonic respectively to the approximated second harmonic compared to the numerical solution of Burgers' equation versus the σ coordinate: case of glycerin.

As for water, the analytical expressions used present deviations when compared to the numerical solution of Burgers' equation.

Indeed, Eq. (3) assumes that the fundamental amplitude variation versus the spatial coordinate is only proportional to the product of the absorption coefficient by the fundamental amplitude and that the absorption coefficient is not dependent on the intensity (Eq. (3)), while these hypothesis are not always checked.

CONCLUSION

The validity domain of the fundamental and the second harmonic analytical expressions established on the quasi-linear approximation can be preset while being based only on the derivation of Goldberg's number.

Indeed according to this study, the analytical expressions of the fundamental and the second harmonic established on the quasi-linear approximation can constitute a good approximation of the numerical solution of Burgers' equation for a medium characterized by a Goldberg number very low as compared to the unity (strongly dissipative). On the other hand for a medium characterized by a Goldberg number very large as compared to the unity, the analytical expressions of the fundamental and the second harmonic already established on the quasi-linear approximation are not checked and must be redefined.

A new mathematical formulation of the fundamental and second harmonic for a medium characterized by a Goldberg number large as compared to the unity will be the subject of a forthcoming study.

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