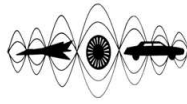


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OPTIMIZATION OF A COUPLED FLUID-STRUCTURE SYSTEM USING A MODAL APPROACH

Sébastien Besset^{*1} and Louis Jézéquel¹

¹Laboratoire de Tribologie et Dynamique des Systèmes, École Centrale de Lyon
36, avenue Guy de Collongue, 69130 Ecully, France
sebastien.besset@ec-lyon.fr

Abstract

Modal analysis methods have long been studied because the use of generalized coordinates makes it possible to reduce calculation costs. The modal analysis method that we will use deals with coupled fluid-structure systems. Complex systems, including structural and acoustic parts, are described using generalized coordinates. Thus, the systems we will study will be made of fluid cavities, structural plates and hollow parts. Hollow parts will be analyzed using a “modal finite element” method that we will explain in the paper. Mass and stiffness matrices of the complete system are used to optimize structural parts of the system, in order to reduce the pressure level in the acoustic part. This optimization aims at modifying the geometry of the hollow parts of the structure. Criteria used for the optimization allow to consider two kinds of vibration transmission, that can be separately optimized. In this paper, we optimize the structure using both of these criteria.

INTRODUCTION

Using matrices resulting from the modal analysis of a structure in order to optimize this structure has multiple advantages. Once the structure has been analyzed, optimization criteria become very easy to compute. Secondly, it is possible to link the optimization of the structure and the modal matrices coming from the modal analysis. Thus, it is possible to find the causes of the problems to solve. The criteria we will use in this paper have been developed by P. Lemerle [1], using the Craig & Bampton method to analyze a structure.

The modal analysis we will propose first leads on “double modal synthesis” proposed by Jézéquel [2, 3]. Complex structures often include hollow part and stiffeners, which must be quite precisely analyzed to give good results. Indeed, in the case of complex structures like

cars, stiffeners and formed steel constituting the skeleton of the structure are mostly responsible for the behavior of the whole structure. To analyze these elements, we will use a method we proposed in [4]. We will also study the acoustical parts of the coupled fluid-structure system using acoustic modes through a “triple modal synthesis method”. An example of modal analysis of a coupled fluid-structure system can be found in [5].

The modal analysis of the structure will lead to modal mass and stiffness matrices that will be used to obtain effective modal parameters. These modal parameters will lead to criteria that will allow to optimize the structure. These criteria will take into account the value of the pressure of points located in the acoustic parts of the system – inside a car for example – in function of an excitation point located on a hollow part of the structure – that can be a spar near the engine of a car.

MODAL ANALYSIS OF THE FLUID-STRUCTURE SYSTEM

The structure we consider in this paper is a complex structure including hollow parts and plates. It is made of formed steels constituting its skeleton, as shown in figure 1. Plates are fixed on this skeleton, and a fluid cavity is represented inside the structure. The geometry of the structure is near from the geometry of a car, in order to show the methods we propose are able to be used in an industrial context.

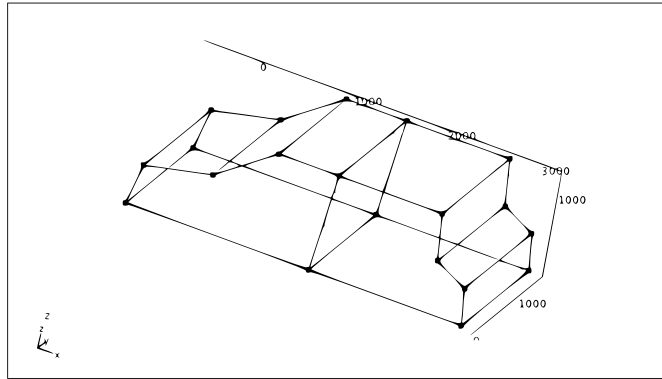


Figure 1: Structure to optimize

The acoustical part of the system is analyzed through a modal synthesis method using cavity modes. Hollow parts of the structure – that can be stiffeners for example – are analyzed using a model we proposed in [4]. These elements constitute the skeleton of the structure. This modelization leads to modal matrices that can be assembled like finite elements matrices. The main characteristic of this method is to produce matrices including only generalized degrees of freedom. There may remain nodal degrees of freedom in order to assemble the hollow parts with other structures, but boundaries between the substructures constituting the hollow part only comprise generalized degrees of freedom. The assembling of these substructures is possible because of the choice of the modes used for the modal analysis of the substructures.

Plates are assembled with the skeleton of the structure through the nodal degrees of freedom remaining from the modal analysis of the hollow parts of the structure. In this paper, only one plate will be used, situated on the top of the structure. The whole structure is then analyzed using “double modal synthesis” method proposed by Jézéquel [2, 3]. This method uses “branch modes” to describe the behavior of the boundaries between substructures. In this paper, will used these “branch modes” to describe the behavior of the skeleton of the structure.

Mass and stiffness matrices of the structure can be split into degrees of freedom concerning the fluid, degrees of freedom concerning plates and degrees of freedom concerning hollow parts of the structure. Degrees of freedom concerning hollow parts include generalized degrees of freedom resulting from the modal analysis of the substructures constituting the skeleton, that will be denoted \mathbf{q}_{Hc} , and generalized degrees of freedom resulting from the double modal synthesis, which will be denoted \mathbf{q}_{Hb} . We denote $\mathbf{q}_H = \begin{Bmatrix} \mathbf{q}_{Hc} \\ \mathbf{q}_{Hb} \end{Bmatrix}$. Hence the motion equation:

$$\left(-\omega^2 \begin{bmatrix} \mathbf{M}_{HH} & \mathbf{M}_{HP} & \mathbf{M}_{HA} \\ \mathbf{M}_{PH} & \mathbf{M}_{PP} & \mathbf{M}_{PA} \\ \mathbf{M}_{AH} & \mathbf{M}_{AP} & \mathbf{M}_{AA} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{HH} & \mathbf{K}_{HP} & \mathbf{K}_{HA} \\ \mathbf{K}_{PH} & \mathbf{K}_{PP} & \mathbf{K}_{PA} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{AA} \end{bmatrix} \right) \begin{Bmatrix} \mathbf{q}_H \\ \mathbf{q}_P \\ \mathbf{q}_A \end{Bmatrix} = \begin{Bmatrix} \bar{\mathbf{f}}_H \\ \bar{\mathbf{f}}_P \\ \bar{\mathbf{f}}_A \end{Bmatrix} \quad (1)$$

Generalized degrees of freedom used in the motion equation 1 are linked to nodal degrees of freedom through the following equations:

$$\begin{cases} \mathbf{p} = \Phi_A \mathbf{q}_A \\ \mathbf{u}_P = \Phi_P \mathbf{q}_P + \Psi_P \mathbf{u}_{Hb} + \Psi_{Pe} \mathbf{u}_{He} \\ \mathbf{u}_{Hc} = \Phi_{Hc} \mathbf{q}_{Hc} + \Psi_{Hc} \mathbf{u}_{Hb} + \Psi_{He} \mathbf{u}_{He} \\ \mathbf{u}_{Hb} = \Phi_{Hb} \mathbf{q}_{Hb} \end{cases} \quad (2)$$

Matrix Φ_A is the modal matrix of the acoustical modes of the system.

Matrix Φ_P is the modal matrix of the fixed modes of the plates. Ψ_P is the matrix of the static modes of the plates, as for Craig & Bampton method [6].

Matrices Φ_{Hc} and Ψ_{Hc} are modal matrices resulting from the analysis of the hollow part described in [4].

Matrix Φ_{Hb} is the matrix of the “branch” modes of the structure [2, 3].

Excitation points are not analyzed. The degrees of freedom corresponding to these points remain nodal, and will be denoted \mathbf{u}_{He} in the following.

Thus, the motion equation 1 becomes:

$$\begin{aligned}
& \left(-\omega^2 \begin{bmatrix} \mathbf{M}_{EE} & \mathbf{M}_{EHc} & \mathbf{M}_{EHb} & \mathbf{M}_{EP} & \mathbf{M}_{EA} \\ \mathbf{M}_{HcE} & \mathbf{M}_{HcHc} & \mathbf{M}_{HcHb} & \mathbf{M}_{HcP} & \mathbf{M}_{HcA} \\ \mathbf{M}_{HbE} & \mathbf{M}_{HbHc} & \mathbf{m}_{Hbk} & \mathbf{M}_{HbP} & \mathbf{M}_{HbA} \\ \mathbf{M}_{PE} & \mathbf{M}_{PHc} & \mathbf{M}_{PHb} & \mathbf{m}_{Pk} & \mathbf{M}_{PA} \\ \mathbf{M}_{AE} & \mathbf{M}_{AHc} & \mathbf{M}_{AHb} & \mathbf{M}_{AP} & \mathbf{m}_{Ak} \end{bmatrix} + i\omega \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{c}_{Hck} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{c}_{Hbk} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{c}_{Pk} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{c}_{Ak} \end{bmatrix} \right. \\
& \left. + \begin{bmatrix} \mathbf{K}_{EE} & \mathbf{K}_{EHc} & \mathbf{K}_{EHb} & \mathbf{K}_{EP} & \mathbf{K}_{EA} \\ \mathbf{K}_{HcE} & \mathbf{K}_{HcHc} & \mathbf{K}_{HcHb} & \mathbf{K}_{HcP} & \mathbf{K}_{HcA} \\ \mathbf{K}_{HbE} & \mathbf{K}_{HbHc} & \mathbf{k}_{Hbk} & \mathbf{K}_{HbP} & \mathbf{K}_{HbA} \\ \mathbf{K}_{PE} & \mathbf{K}_{PHc} & \mathbf{K}_{PHb} & \mathbf{k}_{Pk} & \mathbf{K}_{PA} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{k}_{Ak} \end{bmatrix} \right) \begin{Bmatrix} \mathbf{u}_{He} \\ \mathbf{q}_{Hc} \\ \mathbf{q}_{Hb} \\ \mathbf{q}_P \\ \mathbf{q}_A \end{Bmatrix} = \begin{Bmatrix} \bar{\mathbf{f}}_E \\ \bar{\mathbf{f}}_{Hc} \\ \bar{\mathbf{f}}_{Hb} \\ \bar{\mathbf{f}}_P \\ \mathbf{0} \end{Bmatrix} \quad (3)
\end{aligned}$$

where matrices $[\mathbf{m}_{Ak}]$, $[\mathbf{m}_{Hbk}]$, $[\mathbf{m}_{Pk}]$, $[\mathbf{k}_{Hbk}]$, $[\mathbf{k}_{Pk}]$, $[\mathbf{c}_{Hbk}]$, $[\mathbf{c}_{Pk}]$ and $[\mathbf{c}_{Hck}]$ are diagonal matrices.

OPTIMIZATION CRITERIA USED IN THE PAPER

Using the last line of equation 3, we obtain the following equation:

$$\begin{aligned}
& -\omega^2 \left(\mathbf{M}_{AE}^k \mathbf{u}_{He} + \mathbf{M}_{AHc}^k \mathbf{q}_{Hc} + \mathbf{M}_{AHb}^k \mathbf{q}_{Hb} + \mathbf{M}_{AP}^k \mathbf{q}_P + \mathbf{m}_{Ak} \mathbf{p}^k \right) \\
& + i\omega \mathbf{c}_{Ak} \mathbf{q}_A^k + \mathbf{k}_{Ak} \mathbf{q}_A^k = \mathbf{0} \quad (4)
\end{aligned}$$

where \mathbf{M}_{AE}^k , \mathbf{M}_{AHc}^k , \mathbf{M}_{AHb}^k and \mathbf{M}_{AP}^k are the k^{th} lines of matrices \mathbf{M}_{AE} , \mathbf{K}_{AE} , \mathbf{M}_{AHc} , \mathbf{M}_{AHb} and \mathbf{M}_{AP} .

Equation 4 leads to:

$$\mathbf{q}_A = \frac{\omega^2 (\mathbf{M}_{AE}^k \mathbf{u}_{He} + \mathbf{M}_{AHc}^k \mathbf{q}_{Hc} + \mathbf{M}_{AHb}^k \mathbf{q}_{Hb} + \mathbf{M}_{AP}^k \mathbf{q}_P)}{-\omega^2 \mathbf{m}_{Ak} + i\omega \mathbf{c}_{Ak} + \mathbf{k}_{Ak}} \quad (5)$$

Let Φ_A^k be the k^{th} column of Φ_A . Equation 5 becomes:

$$\begin{aligned}
\mathbf{p} &= \sum_k \Phi_A^k \mathbf{q}_A \\
&= \sum_k \left[\frac{\omega^2 \Phi_A^k \mathbf{M}_{AE}^k}{-\omega^2 \mathbf{m}_{Ak} + i\omega \mathbf{c}_{Ak} + \mathbf{k}_{Ak}} \right] \mathbf{u}_{He} \\
&\quad + \sum_k \left[\frac{\omega^2 \Phi_A^k \mathbf{M}_{AHc}^k}{-\omega^2 \mathbf{m}_{Ak} + i\omega \mathbf{c}_{Ak} + \mathbf{k}_{Ak}} \right] \left(\tilde{\Phi}_{Hc} (\mathbf{u}_{Hc} - \Psi_{Hc} \mathbf{u}_{Hb} - \Psi_{He} \mathbf{u}_{He}) \right) \\
&\quad + \sum_k \left[\frac{\omega^2 \Phi_A^k \mathbf{M}_{AHb}^k}{-\omega^2 \mathbf{m}_{Ak} + i\omega \mathbf{c}_{Ak} + \mathbf{k}_{Ak}} \right] \tilde{\Phi}_{Hb} \mathbf{u}_{Hb} \\
&\quad + \sum_k \left[\frac{\omega^2 \Phi_A^k \mathbf{M}_{AP}^k}{-\omega^2 \mathbf{m}_{Ak} + i\omega \mathbf{c}_{Ak} + \mathbf{k}_{Ak}} \right] \left(\tilde{\Phi}_P (\mathbf{u}_P - \Psi_P \mathbf{u}_{Hb} - \Psi_{Pe} \mathbf{u}_{He}) \right) \quad (6)
\end{aligned}$$

Equation 6 provides modal criteria that will be used to optimize the coupled fluid-structure system:

$$C_D = \max_k \left| \Phi_A^k \mathbf{M}_{AE}^k - \Phi_A^k \mathbf{M}_{AHc}^k \tilde{\Phi}_{Hc} \Psi_{He} \right| \quad (7)$$

$$C_P = \max_k \left| \Phi_A^k \mathbf{M}_{AP}^k \Psi_{Pe} \right| \quad (8)$$

C_D corresponds to the direct path between the excited points \mathbf{u}_{He} and the pressure level in the cavity. C_P corresponds to the vibration propagation through the plate. Thus, two different paths can be separately treated. Figure 2 shows the correspondence between the criteria and the vibration propagation.

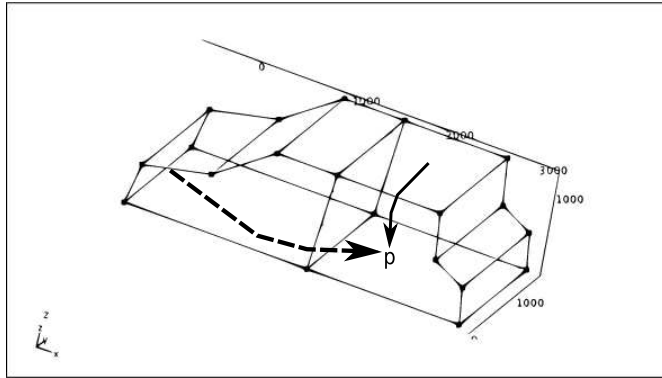


Figure 2: Correspondence between criteria and vibration propagation (- - -: C_D —: C_P)

ANALYSIS OF THE CRITERIA

In this section, the criteria developed in the previous section are analyzed. It is possible to find which of the criteria C_D and C_P is the most influent for the vibration propagation. In this section, we will minimize the both criteria through an optimization method based on the K  n & Tucker conditions. We introduce a third criteria C_M linked to the mass of the structure.

The optimization problem can then be written:

$$\text{Minimize } C_M(\boldsymbol{\alpha}) \text{ assuming } \begin{cases} \tilde{C}_D(\boldsymbol{\alpha}) \leq 0 \\ \tilde{C}_P(\boldsymbol{\alpha}) \leq 0 \end{cases} \quad (9)$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_p]$ are the parameters to optimize (in this paper, we choose to optimize the geometry of the hollow parts). \tilde{C}_D and \tilde{C}_P are based on C_D and C_P . The method used in this paper need the criteria to be derivate, which is not possible with C_D and C_P . Criteria used in this section are defined as follow:

$$\tilde{C}_D = \frac{1}{4} \log \sum_k \left(\Phi_A^k \mathbf{M}_{AE}^k - \Phi_A^k \mathbf{M}_{AHc}^k \tilde{\Phi}_{Hc} \Psi_{He} \right)^4 - c_d \quad (10)$$

$$\tilde{C}_P = \frac{1}{4} \log \sum_k \left(\Phi_A^k \mathbf{M}_{AP}^k \Psi_{Pe} \right)^4 - c_p \quad (11)$$

where c_d and c_p are the objectives of the criteria.

\tilde{C}_D and \tilde{C}_P have almost the same minima and extrema as C_D and C_P . The optimization method we use in this paper is based on the K  hn & Tucker conditions, which can be written as follow:

$$\begin{aligned} \exists (\lambda_1(\boldsymbol{\alpha}), \lambda_2(\boldsymbol{\alpha})), \quad \lambda_1(\boldsymbol{\alpha}) \nabla \tilde{C}_D(\boldsymbol{\alpha}) + \lambda_2(\boldsymbol{\alpha}) \nabla \tilde{C}_P(\boldsymbol{\alpha}) + \nabla C_M(\boldsymbol{\alpha}) = 0 \\ \text{and } \begin{cases} \lambda_1(\boldsymbol{\alpha}) \geq 0 \\ \lambda_2(\boldsymbol{\alpha}) \geq 0 \end{cases} \end{aligned} \quad (12)$$

The problem we propose needs equation 12 to be written as follow:

$$\forall i = 1, 2, 3, \dots, p \quad \lambda_1(\boldsymbol{\alpha}) \frac{\frac{\partial \tilde{C}_D}{\partial \alpha_i}(\boldsymbol{\alpha})}{\frac{\partial C_M}{\partial \alpha_i}(\boldsymbol{\alpha})} + \lambda_2(\boldsymbol{\alpha}) \frac{\frac{\partial \tilde{C}_P}{\partial \alpha_i}(\boldsymbol{\alpha})}{\frac{\partial C_M}{\partial \alpha_i}(\boldsymbol{\alpha})} = -1 \quad (13)$$

In this equation, we suppose $\frac{\partial C_M}{\partial \alpha_i}(\boldsymbol{\alpha}) \neq 0$. A relaxation parameter γ is then introduced. Equation 13 becomes:

$$\alpha_i = \gamma \alpha_i - \left[(1 - \gamma) \left(\lambda_1(\boldsymbol{\alpha}) \frac{\frac{\partial \tilde{C}_D}{\partial \alpha_i}(\boldsymbol{\alpha})}{\frac{\partial C_M}{\partial \alpha_i}(\boldsymbol{\alpha})} + \lambda_2(\boldsymbol{\alpha}) \frac{\frac{\partial \tilde{C}_P}{\partial \alpha_i}(\boldsymbol{\alpha})}{\frac{\partial C_M}{\partial \alpha_i}(\boldsymbol{\alpha})} \right) \right] \alpha_i \quad (14)$$

This equation leads to the following recurrence equation:

$$\alpha_i^{k+1} = \gamma \alpha_i^k - \left[(1 - \gamma) \left(\lambda_1(\alpha) \left. \frac{\frac{\partial \tilde{C}_D(\alpha)}{\partial \alpha_i}}{\frac{\partial \tilde{C}_M(\alpha)}{\partial \alpha_i}} \right|_{\alpha_i = \alpha_i^k} + \lambda_2(\alpha) \left. \frac{\frac{\partial \tilde{C}_P(\alpha)}{\partial \alpha_i}}{\frac{\partial \tilde{C}_M(\alpha)}{\partial \alpha_i}} \right|_{\alpha_i = \alpha_i^k} \right) \right] \alpha_i^k \quad (15)$$

γ must be chosen close from 1 if we want to be sure the algorithm converge. The smaller γ will be, the faster the algorithm will converge, but it may not converge at all if γ is too small...

For each step of the algorithm, $\lambda_i(\alpha)$ is being computed. The method used to obtain these $\lambda_i(\alpha)$ is explained by P. Lemerle [1]. Let consider $\Delta\alpha_i$ such as:

$$\begin{cases} \tilde{C}_D(\alpha_1 + \Delta\alpha_1, \alpha_2 + \Delta\alpha_2, \dots, \alpha_p + \Delta\alpha_p) = 0 \\ \tilde{C}_P(\alpha_1 + \Delta\alpha_1, \alpha_2 + \Delta\alpha_2, \dots, \alpha_p + \Delta\alpha_p) = 0 \end{cases} \quad (16)$$

This equation allows to express $\Delta\tilde{C}_D(\alpha_1, \alpha_2, \dots, \alpha_p)$ as follow:

$$\begin{aligned} \Delta\tilde{C}_D(\alpha_1, \alpha_2, \dots, \alpha_p) &= \tilde{C}_D(\alpha_1 + \Delta\alpha_1, \alpha_2 + \Delta\alpha_2, \dots, \alpha_p + \Delta\alpha_p) - \tilde{C}_D(\alpha_1, \alpha_2, \dots, \alpha_p) \\ &= -\tilde{C}_D(\alpha_1, \alpha_2, \dots, \alpha_p) \\ &= \sum_{i=1}^p \frac{\partial \tilde{C}_D}{\partial \alpha_i} \Delta\alpha_i \end{aligned} \quad (17)$$

Replacing $\Delta\alpha_i$ in equation 17, \tilde{C}_D can be written as follow:

$$\tilde{C}_D(\alpha_1, \alpha_2, \dots, \alpha_p) = (1 - \gamma) \sum_{i=1}^p \frac{\partial \tilde{C}_D}{\partial \alpha_i} \left[1 + \lambda_1 \frac{\frac{\partial \tilde{C}_D}{\partial \alpha_i}}{\frac{\partial \tilde{C}_M}{\partial \alpha_i}} + \lambda_2 \frac{\frac{\partial \tilde{C}_P}{\partial \alpha_i}}{\frac{\partial \tilde{C}_M}{\partial \alpha_i}} \right] \alpha_i^k \quad (18)$$

In order to simplify the notations, we denote $\lambda_1 = \lambda_1(\alpha)$, $\lambda_2 = \lambda_2(\alpha)$ and $\frac{\partial \tilde{C}_j}{\partial \alpha_i} = \left. \frac{\partial \tilde{C}_j}{\partial \alpha_i}(\alpha) \right|_{\alpha_i = \alpha_i^k}$. The same equation can be written for \tilde{C}_P , which allows to write the following equation:

$$\begin{bmatrix} \sum_{i=1}^p \frac{\left(\frac{\partial \tilde{C}_D}{\partial \alpha_i} \right)^2}{\frac{\partial \tilde{C}_M}{\partial \alpha_i}} \alpha_i^k & \sum_{i=1}^p \frac{\partial \tilde{C}_D}{\partial \alpha_i} \frac{\frac{\partial \tilde{C}_P}{\partial \alpha_i}}{\frac{\partial \tilde{C}_M}{\partial \alpha_i}} \alpha_i^k \\ \sum_{i=1}^p \frac{\partial \tilde{C}_P}{\partial \alpha_i} \frac{\frac{\partial \tilde{C}_D}{\partial \alpha_i}}{\frac{\partial \tilde{C}_M}{\partial \alpha_i}} \alpha_i^k & \sum_{i=1}^p \frac{\left(\frac{\partial \tilde{C}_P}{\partial \alpha_i} \right)^2}{\frac{\partial \tilde{C}_M}{\partial \alpha_i}} \alpha_i^k \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \lambda_2 \end{Bmatrix} = \begin{Bmatrix} \frac{\tilde{C}_D}{1 - \gamma} - \sum_{i=1}^p \frac{\partial \tilde{C}_D}{\partial \alpha_i} \alpha_i^k \\ \frac{\tilde{C}_P}{1 - \gamma} - \sum_{i=1}^p \frac{\partial \tilde{C}_P}{\partial \alpha_i} \alpha_i^k \end{Bmatrix} \quad (19)$$

This equation allows to obtain (λ_1, λ_2) for each step of the optimization.

CONCLUSION

The method we propose in this paper allow to optimize a coupled fluid-structure system using modal criteria. We give an example of optimization for parameters that correspond to the geometry of the hollow parts of the structure, their section S and inertia I . There are 16 parameters (the hollow parts of the structure are split into 8 parts). Figure 3 shows the evolution of the criteria during the optimization. Figure 4 show the pressure level in the fluid. It is quite the same after the optimization, but the mass of the structure decreased.

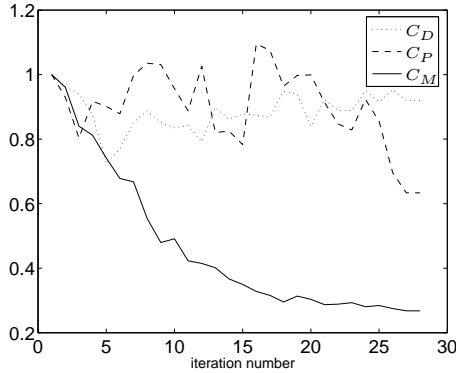


Figure 3: Evolution of the criteria

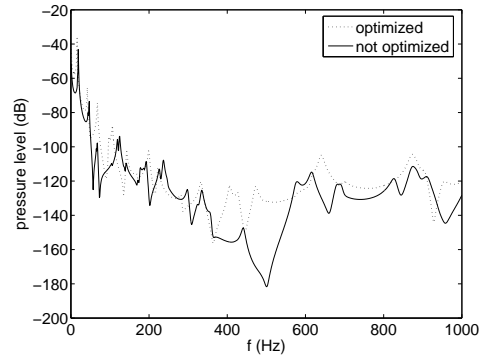


Figure 4: Pressure level in the cavity

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