



## Normal Incidence Transmission Loss of Sandwich Structures in a Plane Wave Tube

P. Bonfiglio<sup>\*1</sup>, F. Pompoli<sup>1</sup>, and A.T. Peplow<sup>2</sup>

<sup>1</sup>Department of Engineering, G Saragat 1, University of Ferrara, 44100 Ferrara, Italy.

<sup>2</sup>Marcus Wallenberg Laboratory for Sound and Vibration Research, Department of Aeronautics & Vehicle Engineering, KTH, S-100 44, Stockholm, Sweden.

[paolo.bonfiglio@unife.it](mailto:paolo.bonfiglio@unife.it)

### Abstract

The *Transmission Loss* of different kind of layered structures is investigated by means of a four microphone technique in a plane wave tube. By using the decomposition technique, incident, reflected and transmitted contributions are separated and transmission coefficient is easily calculated. In this paper a single measurement approach based on transfer matrix, taking into account reflection contribution from the end termination, is presented. Results are discussed and compared with similar techniques and with the two-load method.

### INTRODUCTION

Several experimental techniques have been developed to test normal incidence acoustical performances of both homogeneous and layered systems. All these approaches are based on the decomposition of the plane wave field into an incident, a reflected and a transmitted wave. It is worth emphasizing that in the case of non-symmetric systems, it can be proved that a complete solution for the normal incidence reflection and transmission problem requires two measurements, for instance by using different sources or end terminations [1]–[2].

By the way in literature different methods requiring a single measurement for evaluating transmission properties of materials can be found. One of the first techniques was proposed by Chung and Blaser [3] and it was widely used in the past. The limitation of this method is the assumption of a perfectly anechoic termination. Afterwards, Song and Bolton [4] proposed a transfer matrix technique for measuring transmission coefficient for homogeneous and isotropic materials. Recently Liu *et al.* [5] proposed a revision of the Chung and Blaser's formula, taking into account the reflection from the end termination. Several tests were carried out by Pispola *et al.* [6] in order to compare all these experimental procedures.

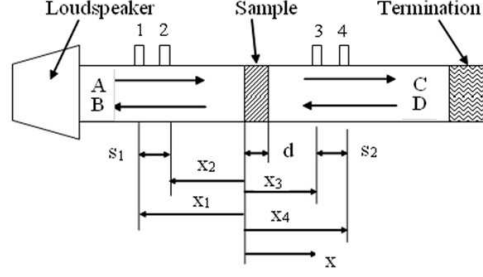


Figure 1: Sketch of the measurement set-up

In this paper an approach for determining complex transmission coefficient and *Transmission Loss*, regardless the end termination, of layered structures is presented. It takes origin from the transfer matrix technique presented in [4], rewritten taking into account reflection contribution from the not perfectly anechoic termination. In the paper comparison between these methods are reported and discussed. Furthermore simplified finite element models for the tested systems are developed.

## THEORETICAL BACKGROUND

By considering the measurement set-up shown in Figure 1 and by using the decomposition technique, the contributions of the incident and reflected acoustical waves at both sides of the material can be calculated by measuring complex pressures at the different microphone positions, as follow:

$$A = \frac{j(P_1 e^{kx_2} - P_2 e^{kx_1})}{2 \sin k(x_1 - x_2)} \quad (1)$$

$$B = \frac{j(P_2 e^{kx_1} - P_1 e^{kx_2})}{2 \sin k(x_1 - x_2)} \quad (2)$$

$$C = \frac{j(P_3 e^{kx_4} - P_4 e^{kx_3})}{2 \sin k(x_3 - x_4)} \quad (3)$$

$$D = \frac{j(P_4 e^{kx_3} - P_3 e^{kx_4})}{2 \sin k(x_3 - x_4)} \quad (4)$$

The complex amplitudes can be related in a matrix formulation as follows:

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \tau & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix} \quad (5)$$

The (5) is a general formulation and the elements of the matrix depend only on the physical properties of the system. For instance the complex transmission loss is proved to be equal to  $\tau^{-1}$ . As previously mentioned, according to Chung and Blaser approach it has to be imposed simply  $D=0$  and then the complex transmission coefficient is:

$$T_{c-B} = \frac{C}{A} \quad (6)$$

In reality, it is extremely difficult to realize a perfectly absorbing termination, hence a correction of the (6) has been necessary in order to evaluate the correct propagation of the waves in the duct. Therefore Liu *et al.* have provided a revised formula for the normal incidence transmission coefficient, that is:

$$T_{Liu} = \frac{AC - BD}{A^2 - D^2} \quad (7)$$

It has to be emphasized that these method presumes the validity of the conditions of symmetry and reciprocity that is to say reflection and transmission coefficients are assumed to be the same at both directions of incidence.

A different transfer matrix formulation was proposed by Song and Bolton, which related sound pressure  $P$  and particle velocity  $V$  at both surfaces of the tested sample by means of the following expression:

$$\begin{pmatrix} P \\ V \end{pmatrix}_{x=0} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} P \\ V \end{pmatrix}_{x=d} \quad (8)$$

Even if their work was mainly concerned on the determination of the complex characteristic properties of homogeneous and isotropic materials, they proposed some formulation based on transfer matrix elements for calculating the transmission coefficient of these systems. In fact starting from the (8) the authors have proved that complex transmission coefficient can assume following form:

$$T_{s-B} = \frac{2e^{jkd}}{T_{11} + (T_{12}/\rho_0 c_0) + (\rho_0 c_0 T_{21}) + T_{22}} \quad (9)$$

It is interesting notice that it is not possible to determine each of the matrix elements  $T_{ij}$  and two additional equations are required. An indirect way to obtain these relations it is to repeat measurements with two different end terminations. It can proved that results obtained by using this approach are identical to those obtained using the standard two load method.

Still by using this formulation and applying symmetry and reciprocity condition it is possible obtain, through a single measurement, both effective transmission and reflection coefficients by solving the system:

$$\begin{cases} R = R_{eff} + T_{eff}R_1T \\ T = T_{eff} + R_{eff}R_1Te^{2jkd} \end{cases} \quad (10)$$

where  $R=B/A$ ,  $T=C/A$  and  $R_1=D/C$ .

It can be easily proved that complex transmission coefficient can be calculated by using following formula:

$$T_{ENDIF} = \frac{T(1 - RR_1e^{2jkd})}{1 - (R_1T)^2e^{2jkd}} \quad (11)$$

Of course it has to be emphasized that the (11) is rigorously valid for homogeneous and isotropic materials. By the way it will be showed that it provides results more reliable than (7) if compared with the two load method or equivalently with (9).

## MATERIALS AND METHODS

The experimental set-up consists in a 100 mm diameter plane wave tube equipped with four microphones positions and the material under test in the middle. A perfectly rigid and an high absorbing end terminations were used in the proposed test. The absorption coefficient for the last termination lies between 0.6 and 0.9 for frequencies between 100 Hz and 700 Hz, and 1 for higher frequencies. In Figure 2 the measurement device and some details are shown.

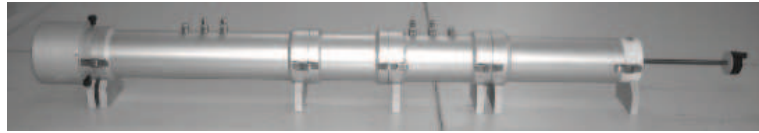


Figure 2: Experimental set-up

The sound pressures at the four positions are measured by impulse responses, obtained by exponential sine sweep method. The experimental set-up consist of 4 (1/4 inches) condenser microphones, a Laptop and a 4 channels audio sound card. For the signal generation and acquisition the software *Adobe Audition*® was used. The post-processing was carried out by using *Matlab*® codes. Measurements were carried out in the frequency range between 100 Hz and 1600 Hz.

## EXPERIMENTAL RESULTS

Different both homogeneous and layered structures have been investigated and comparison between the above-mentioned approaches are shown. In Figure 3 curves of *Transmission Loss*

are reported for an homogeneous polyester fiber material ( $40 \text{ kg/m}^3$ , 40mm thick). It is possible to notice that results of the approach here proposed are good if compared with the two load method. On the contrary Chung and Blaser and Liu results show some differences mainly below 700-800 Hz, where the end termination is not perfectly anechoic. It has to be strongly emphasized that since an homogeneous material was used, all the results should have been similar because of the validity of the symmetry and reciprocity conditions in such particular case.

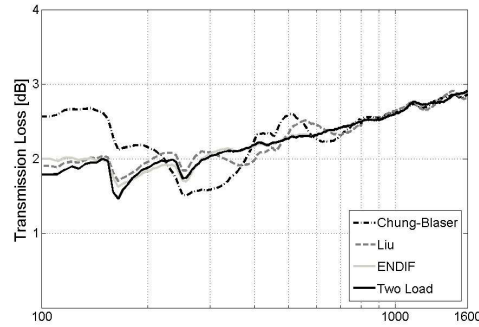


Figure 3: Transmission Loss for an homogeneous material

In Figure 4 results for a symmetric layered structure are shown. The system is made of a thin steel laminate (1mm thick), melamine foam ( $10 \text{ kg/m}^3$ , 20mm thick), open cell synthetic rubber ( $24 \text{ kg/m}^3$ , 68mm thick) and again melamine foam and laminate respectively.

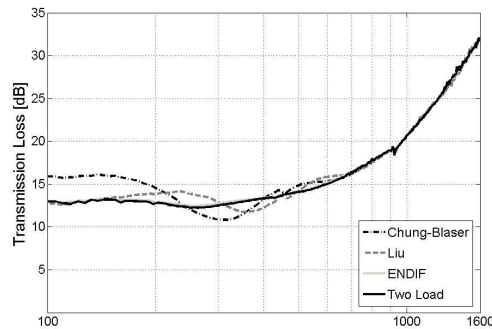


Figure 4: Transmission Loss for a symmetrically layered structure

In Figure 5 the same comparison is reported for an asymmetric system obtained by coupling previous homogeneous system and layered structure (total thickness was 0.106mm).

In both case it can be notice that results obtained through the proposed formulation are closer to the two load formulation than those obtained by using (6) and (7). It can be

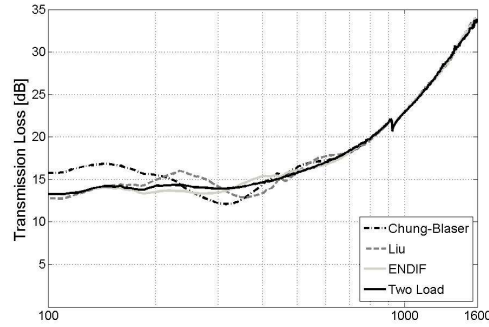


Figure 5: Transmission Loss for an asymmetrically layered structure

underlined that no appreciable differences can be noticed for frequencies higher than 700 Hz, because in this frequency range the end termination can be considered perfectly anechoic; on the contrary at lower frequencies the reflections (in terms of energy and phase) from the end termination are relevant; from the analysis it appears that the proposed formulation takes better into account the effect of the wave contribution from the back. To this end in Figure 6 the comparison for the amplitude and angle of complex transmission coefficient related to the symmetric structure above-mentioned is reported. It can be noticed that the proposed formula is more reliable for evaluating transmission performances of the examined system.

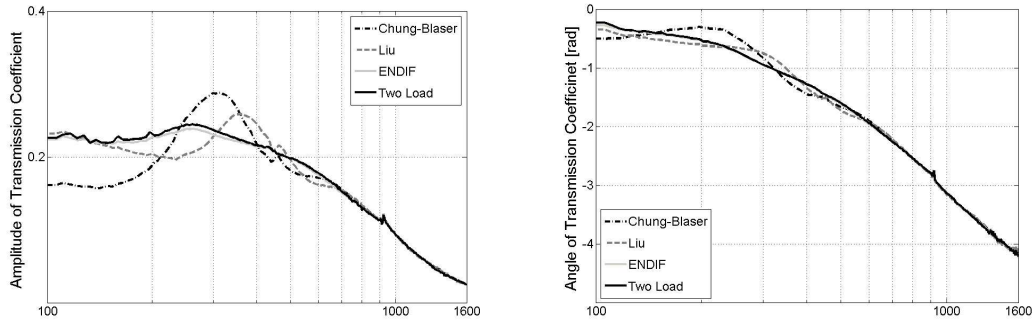


Figure 6: Amplitude and Angle of the Transmission Coefficient for the symmetrically layered structure

Finally the comparison between experimental curves of complex transmission coefficient (Amplitude and Phase) and finite element models are shown. Three different systems were investigated and results are shown in Figure 7. It has to be mentioned that real diameter of laminate was 98mm and the same size was used in the FEM models. The description of these systems is reported in Table 1.

In order to model the fibrous (polyester made) and porous (rubber made) materials used in the simulations, the equivalent fluid model was used [7]. The equivalent complex density

Name	Description
S1	Polyester fiber
S2	Steel Laminate - Polyester fiber
S3	Steel Laminate - Polyester fiber - Open Cell Synthetic Rubber

Table 1: Description of tested systems.

and sound velocity were calculated by using characteristic impedance and complex wave number measured experimentally by means of the transfer matrix method [4]. The Young modulus and the loss factor of the steel laminate were calculated by using a procedure similar to the one proposed in [8], based on the measurement of the eigenfrequencies for a free-free suspended beam. The value for elasticity modulus was 200 GPa and loss factor equal to 0.001. In the model laminate and materials were coupled by using a general fluid-structure interaction. From the figure it is possible to notice that comparison between experimental and numerical curves is quite satisfying and small differences could be due to the mounting of fibrous and porous materials.

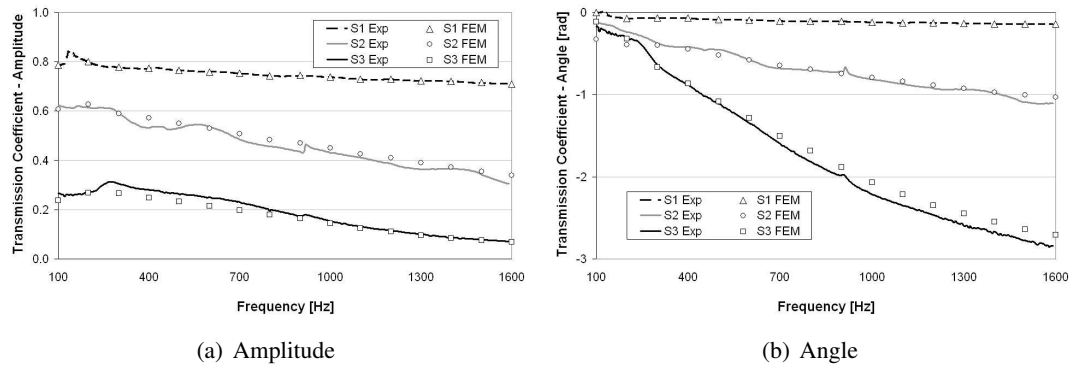


Figure 7: Comparison between Experimental and FEM : Amplitude and Angle of the transmission coefficient

## CONCLUDING REMARKS

In this paper different approaches requiring a single measurement for determining normal incidence complex transmission coefficient and *Transmission Loss* of homogeneous and layered structures have been investigated and results have been compared with the two load method. A new formulation requiring a single measurement has been proposed and it has been shown that provided results were more reliable if compared with the exact formulation. From the present study it can be ended that the reliability of results is strongly dependent on the correct evaluation of the reflected wave from the end termination. Furthermore simplified finite element models have been developed in order to predict transmission coefficient of tested systems. It has been shown that these models can be able to describe the vibro-acoustical

behavior of both homogeneous and layered structures. Future developments of this numerical analysis will involve the modeling of elastic frame materials and the implementation of a more efficient numerical model.

### ACKNOWLEDGEMENTS

This work has been developed within the laboratory of research and technology transfer LAV (Laboratory of Acoustics and Vibration) realized through the contribution of *Regione Emilia Romagna - Assessorato Attività Produttive, Sviluppo Economico, Piano telematico, PRRIITT misura 3.4 azione A - Obiettivo 2*

### References

- [1] M. L. Munjal, A. G. Doige, "Theory of a Two Source-location Method for Direct Experimental Evaluation of the Four-pole Parameters of an Aeroacoustic Element", *Journal of Sound and Vibration*, **141** (2), 323-333.
- [2] T. Y. Lung, A. G. Doige, "A Time-averaging Transient Testing Method for Acoustic Properties of Piping Systems and Mufflers", *J. Acoust. Soc. Am.*, **73**, 867-876. (1983)
- [3] J. C. Chung, D. A. Blaser, "Transfer function method of measuring in-duct acoustic properties. I. Theory ", *J. Acoust. Soc. Am.*, **68**(3), 907-916 (1980)
- [4] B. H. Song, J. S. Bolton, "A transfer matrix approach for estimating the characteristic impedance and wave numbers of limp and rigid porous materials", *J. Acoust. Soc. Am.*, **107** (3), 1131-1152 (2000)
- [5] K. Liu, Z. Qijun, T. Feng, "The measuremet theory and experiment investigation of transmission loss of material- four transducer method", *Proc. of 11<sup>st</sup> Int. Cong. on Sound and Vibration*, St. Petersburg (Russia), **107** (3), 1585-1592 (2004)
- [6] G. Pispola, K. V. Horoshenkov, "Consolidated granular media for sound insulation: performance evaluation through different methods", *Proc. of 12<sup>st</sup> Int. Cong. on Sound and Vibration*, Lisbon, (2005)
- [7] J.F. Allard , "Propagation of sound in porous media", Elsevier Applied Science, (1993)
- [8] ISO 6721-3: "Plastics – Determination of dynamic mechanical properties" (1994)