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Spectral element and experimental analysis of lightweight sandwich structures

P. Bonfiglio*[1], F. Pompoli

Department of Engineering, G Saragat 1, University of Ferrara, 44100 Ferrara, Italy.
pbonfiglio@ing.unife.it

A. T. Peplow, A. C. Nilsson

Marcus Wallenberg Laboratory for Sound and Vibration Research, Department of Aeronautics & Vehicle Engineering, KTH, S-100 44, Stockholm, Sweden.

Abstract

The dynamic response of the vibrating structures are studied with the standard finite element method against the more computationally efficient spectral finite element method. First a simple beam structure is modelled with the standard method and newly developed spectral elements; which has the advantage that dispersion relations for all beam structures may be developed. Some numerical examples are given to illustrate and validate the developed method and studies of the measured responses of structures that may be used for vehicle panels are compared.

INTRODUCTION

In this work the transmission and propagation of vibrations through simple laminated sandwich structures is investigated theoretically and experimentally. A number of computational methods, in particular the finite element method is a robust technique to solve dynamic responses with complex geometrical and material structures. However, with high frequency excitation , many structures of interest require extremely large computational models. Spectral methods are now an established alternative to finite difference and finite element methods to solve elliptic partial differential equations. Spectral methods are naturally chosen to solve problems in regular rectangular, cylindrical or spherical regions.

The spectral finite element method has been applied, in various forms, to a number of acoustic and structural waveguide problems. In particular the method used in [1]–[2] can be viewed as a merger of the dynamic stiffness method and the finite element method. The principle of the method is based on a variational formulation for non–conservative motion in the frequency domain. The SFEM has also been used to study vibration in beam frame–works [1], for fluid–filled pipes [2], and recently for acoustic ducts [3].

Consider the cross–section sandwich laminate illustrated in Fig.1. The laminate may consist of an arbitrary number of solid linear viscoelastic and elastic layers and possesses the same properties as a layered waveguide. The current analysis assumes that the layers are planar and isotropic. Uniform properties along the x-direction are assumed in the waveguide formulation described here. The Lagrangian is minimised for the true motion of the system, subject to the boundary conditions. Displacements, stresses and strains in a particular rectangular region, may be denoted by $\mathbf{u}(\mathbf{x}, t)$, $\boldsymbol{\sigma}(\mathbf{x}, t)$, and $\boldsymbol{\varepsilon}(\mathbf{x}, t)$ respectively and $\mathbf{x} = (x, z)$. To represent a wave propagating in the direction of the waveguide, displacements of the form: $\mathbf{u}(\mathbf{x}, t) \;=\; \mathrm{Re}\left\{\mathbf{U}(z)e^{i(\omega t - \kappa\, x)}\right\}$, where $\mathbf{n}$ represents a cross–sectional mode shape, κ the propagation constant and ω the angular frequency.

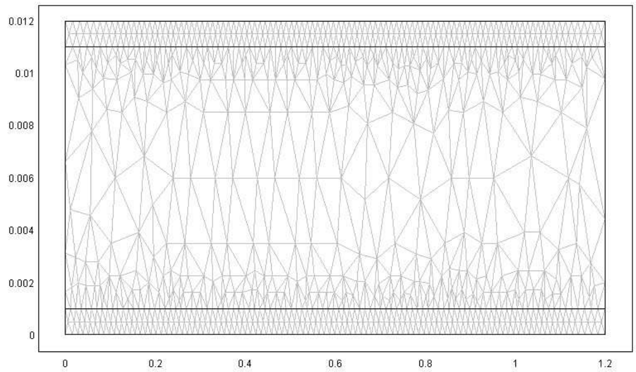

Figure 1: Finite element mesh (not to scale) for sandwich beam, FEMLAB [4], using 1593 *cubic triangular elements totalling* 14978 *degrees of freedom for each frequency. Not the inhomogeneous mesh design featured in the software package.*

NUMERICAL MODELLING

In elastic waveguides with constant geometrical cross–section, the solutions of the governing equations of motion are a combination of exponential terms describing the propagation with cross–sectional modes. Thus, the axial x–dependence is separable from the cross–sectional z–dependencies. In this case, the application of the finite element method, the displacement field of the cross–section of a rectangular element may be represented by high–order polynomials, $N_j(z)$, $j = 1, \ldots, M$, reference [3]. Within one *spectral finite element*, the in–plane displacement field may be represented in the co–ordinate system such that a dimensionless $2 \times 2m$ real–valued matrix $[\ \mathbf{N}(z)\ ]$ with the given co–ordinate functions as elements, and a $2m \times 1$ vector $\hat{\mathbf{u}}(x)$ have the dimension of length,

$$\mathbf{u}(x, z) \ = \ [\ \mathbf{N}(z)]\ \{\hat{\mathbf{u}}(x)\} \tag{1}$$

where the displacement function, $\mathbf{u}(x, z) = [u(x, z), w(x, z)]^T$, using M piecewise polynomial shape functions, compactly supported over each interval in the vertical direction.

Partial integration in the x–direction yields a system of equations possessing constant matrix coefficients in the form of square $(m \times m)$ matrices $[\mathbf{K}_2]$, $[\mathbf{K}_1]$, $[\mathbf{K}_0]$, and $[\mathbf{M}]$. Hence, the solutions of the linear homogeneous system, $\{\hat{\mathbf{u}}\}_l = (U(x), W(x))_l^T$, may be written as :

$$\begin{aligned} \{U(x)\,\}_l &= c_l\,\{\,\Phi\,\}_l\, e^{i\zeta_l x}, \quad l = 1, \ldots, m, \\ \{W(x)\,\}_l &= d_l\,\{\,\Phi\,\}_{m+l}\, e^{i\zeta_{m+l} x}, \quad l = 1, \ldots, m \end{aligned} \tag{2}$$

where $\{\ \Phi\ \}_l$ is a vector representing the nodal amplitudes $(\Phi_{1,l}, \Phi_{m,l})$, $(\Phi_{1+m,l}, \Phi_{2m,l})$, and interior amplitudes $(\Phi_{j,m}, 2 \le j \le m-1)$, $(\Phi_{j+m,m}, 2 \le j \le 2m-1)$ and c_l, d_l are arbitrary constants. Under this assumption a non–linear eigenvalue problem arises, $\mathbf{K}(\zeta)\ \mathbf{\Phi} = \mathbf{0}$, which may be cast, simply, as a polynomial eigenvalue problem,

$$[\,\mathbf{K}(\zeta)\,]\,\mathbf{\Phi} \quad = \quad \left\{\zeta^2[\,\mathbf{K}_2\,] - \ i\zeta[\,\mathbf{K}_1\,] - [\,\mathbf{K}_0\,] + \omega^2[\,\mathbf{M}\,]\right\}\ \mathbf{\Phi} \ = \ \mathbf{0}, \tag{3}$$

of order m for the variable ζ.

The eigenvalue problem relates the wavenumber ζ to the angular frequency ω, one of them being given and the other being the eigenvalue to be sought. In the latter case, the eigenvalues are generally complex–valued and correspond to evanescent waves. For real values of the wavenumber eigenvalues, ζ, corresponding to propagating waves, it is possible to describe dispersion relations and is discussed further in Section . The eigenvalue problem may be reduced to first–order form by introducing an auxiliary parameter, $\gamma = i\zeta$ and variable $\{\hat{\mathbf{v}}\} = \gamma\,\{\hat{\mathbf{u}}\}$. With this change of variables the polynomial eigenvalue problem (3) may be recast as a generalized $(4m \times 4m)$ linear eigenvalue problem known as a *second companion* form in Tisseur and Meerbergen, [5].

Final construction of the set of basis functions, Eq. (1) and local dynamic stiffness matrix, defined over a full region is standard and and is not described here. However evaluation of the dynamic stiffness matrix applies to a single spectral finite element. The matrix is full, complex and symmetric. For a combination of finite elements, describing a general elastic domain, the corresponding dynamic stiffness matrix has a block diagonal structure. The description above applies to a layered waveguide where a cross–section is discretized by locating appropriate interface nodes. Wave functions are subsequently found for the composed layers, by imposing displacement continuity at the interface, thus increasing the size of the matrix eigenvalue problem. Construction of stiffness matrices follows as with construction of stiffness matrices for a single layer waveguide. Hence it is possible to solve multi–layered sandwich panel problems using super–spectral elements bearing in mind the large generalised matrix eigenvalue problems to be solved.

Point forces may be easily included into the model if defined at nodal points. The complete element formulation, for an arbitrary element length, element, including assembling element matrices, solving dispersion relations, and evaluating the dynamic stiffness matrix required only 0.351 seconds on a 1.3 GHz PC for a single layer eleven degree–of–freedom element.

RESULTS

The use of the finite element scheme is used in the following examples. First a description of the measurements taken is given.

MEASUREMENTS

Two different structures were tested in the present work: a steel beam and a non–symmetric ceramic panel. The physical parameters of the samples used are listed in Table 1. The experimental procedure used in this work was simply based on the measurement of frequency response of an accelerometer mounted on the tested beam excited by an impact hammer. The set-up consisted of an accelerometer Bruel & Keir (B&K) type 4393 (2.4 grams), a B&K type 8202 impulse hammer, two charge amplifiers B&K type 2635, and a two channel signal analyser HP 3562A. The samples were suspended, by means of rubber strings, with both ends free. In order to obtain values for the mechanical parameters of the homogeneous steel beam (*i.e.* Young modulus) and the sandwich structures components (*i.e.* Young modulus of the laminates and shear modulus of the core), measurements on the beams with accelerometer and impact source were as close to the ends as possible were carried out. For the steel beam Bernoulli-Euler beam theory, giving the bending stiffness as functions of the eigenfrequencies, was employed. For the sandwich structures an inverse strategy for the estimation of material properties was used, based on sixth order theory.

Material property	Steel Panel	Ceramic Panel Laminate	Ceramic Panel Core
Density, ρ (kg/m^3)	7850	1580	101
Young's modulus (N/m^2)	2.05×10^{11}	8.70×10^{9}	1.40×10^{8}
Shear modulus (N/m^2)	7.80×10^{10}	3.34×10^{9}	5.37×10^{7}
Poisson ratio	0.3	0.3	0.3
Loss factor	0.0015	0.001	0.04
thickness (h_1,h_2) (mm)	-	2.5/2.5	-
core thickness (h) (mm)	1.0	-	50.0
Length (m)	1.0	1.65	-

Table 1: Physical parameters for panels.

DISPERSION RELATIONS

The eigenvalues, ζ, also referred as wavenumbers, are solved numerically for a given frequency once the equations of motion have been assembled through solutions of the quadratic eigenvalue problem 3

The effective isotropic properties of a soft foam core with ceramic laminates was used in the sandwich example and are given by: E=140×10^6 N/m^2, ν=0.3, ρ=101 kg/m^3. The skins are taken to be aluminum with isotropic properties given by: E=8700×10^6 N/m^2, ν=0.3, ρ=1580 kg/m^3. A symmetric sandwich construction is considered with 2.5 mm thick skins and a 50 mm thick core. The dispersion curves of the propagating wave types of the sandwich panel were calculated using the spectral element approach and are plotted in Fig. 2. Below approximately 4.0 kHz, two propagating waves exist corresponding to flexural and shear waves. Around 4.0 and 6.0 kHz two additional propagating waves cut on which involve an out-of-phase motion of the skins of the panel in the in-plane direction. At higher frequencies it is evident that additional waves cut on.

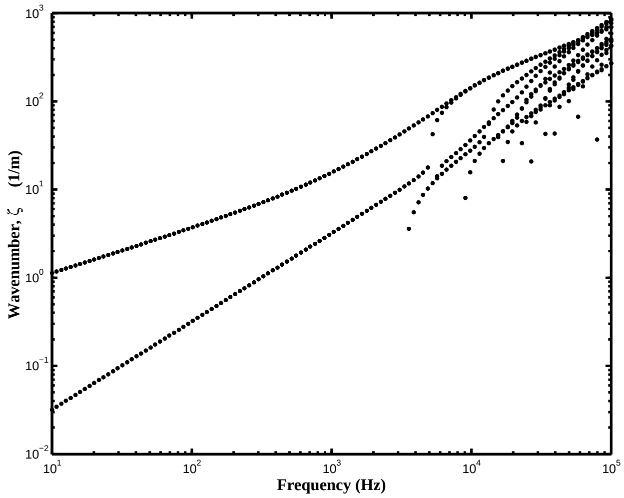

Figure 2: Propagating wavetypes for a 50.0 *mm thick white ceramic sandwich beam: dots, SPECFE computed results.*

FORCED RESPONSE

To provide a comparison with the numerical results here comparisons were made, in addition with the measurements, with FEMLAB 3.1 results [4]. The geometric properties and the total mass density of the structure are assumed to be known. For the spectral finite element method and standard finite element method the aluminium foam sandwich sample was modelled using 33 degrees of freedom and 14978 degrees of freedom respectively. The standard finite element method used cubic polynomials over a triangulated domain; the mesh of 1593 elements is shown in Fig.1. Comparable numbers of degrees of freedom were used for the steel beam and white ceramic sandwich sample and unit magnitude vertical point forces were included in each numerical model.

Finally measurements for different positions of accelerometers and hammer were carried out. The signals from the impulse hammer force gauge and the accelerometers were conditioned by Bruel & Keir type 2635 charge amplifiers before being sent to a digital spectrum analyser. For each measurement, 10 signals were acquired and the average response determined. The ordinary coherence function was checked to ensure that the data were of acceptable quality. The frequency range of the measurements was 5 Hz to 500 Hz for the simple steel beam and 5 Hz to 1.5 kHz for the sandwich beam with a resolution of 2.5 Hz for each. The lower frequency limit was determined by the low accelerometer sensitivity. At 1.5 kHz, the flexural wavelength would be at least 10 times the depth of the beam so there was no risk of the motion tending toward that of a Timoshenko beam [6].

Comparison for amplitude of the acceleration between experimental results, SFEM and FEMLAB [4] between SFEM and measurements are reported in Figs.3 to 4.

STEEL BEAM

Figure compares the predicted transfer accelerance for the steel beam structure from Table 1, using a standard finite element scheme and the spectral element method and the measured response. The following observations are made.

(a) Both numerical methods agree with the measured response for the homogeneous steel beam in the frequency range given; almost indistinguishable transfer accelerance in Fig.3 except at anti–resonances. Differences may be seen in the coherence results

(b) The transfer accelerance is over–predicted by the numerical models except in narrow band resonant regions. This may be due to small mass sensitivity effecting vibrations in these regions, which are evident in a phase diagram.

CERAMIC PANEL

Figure 4 compares the predicted transfer accelerance for the white ceramic sandwich structure from Table 1, using a standard finite element scheme, the spectral element method and the measured response. Generally the response is different to the two previous examples, the core is much thicker and lighter than the aluminium foam panel and the following observations were made for the given source and receiver position.

(a) The two numerical methods agree in the frequency range up to 1500 Hz but discrepancies with the measured data appear as low as 800 Hz. The difference is evident in the phase behaviour and the coherence.

(b) Both numerical methods slightly overpredict the transfer accelerance below 600 Hz. Above this frequency the numerical methods significantly underpredict the accelerance. The phase showing curious behaviour between 400 Hz and 800 Hz not observed in the measurement data.

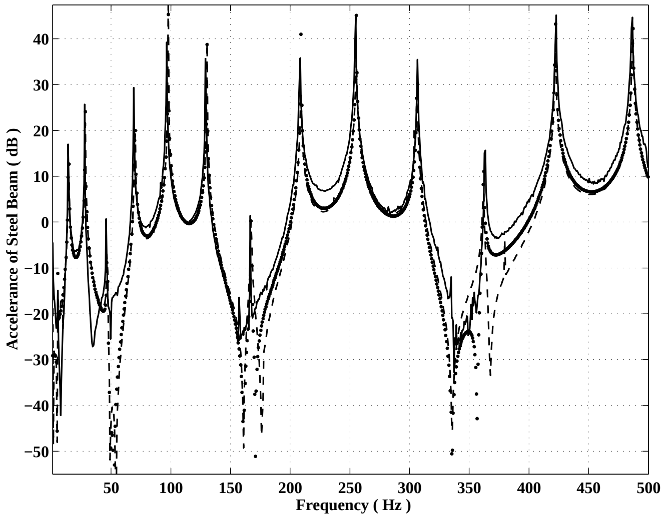

Figure 3: Acceleration levels at 0.27 *m and hammer at* 0.74 *m from edge. Solid line (-) measurements; dashed line SFEM (–); dots FEMLAB (.) for steel beam.*

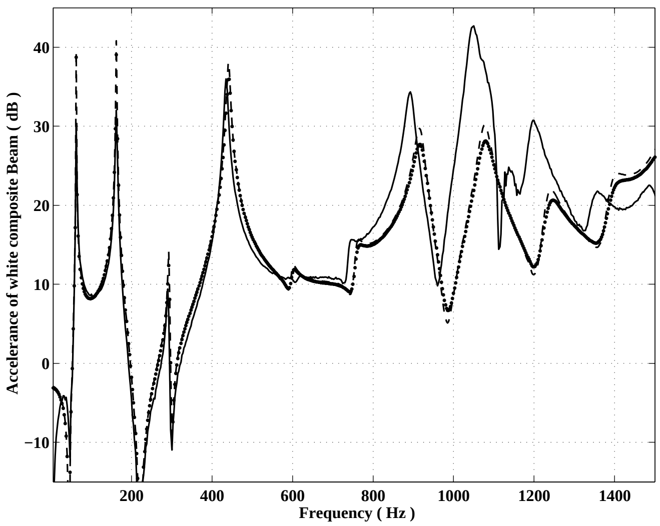

Figure 4: Acceleration levels at 1.07 *m and hammer at* 0.68 *m from edge. Solid line (-) measurements; dashed line SFEM (–); dots FEMLAB (.) for white ceramic sandwich beam.*

(c) Resonant peaks are over–predicted by the numerical methods above 800 Hz, clearly seen in Fig.4. This is possibly due to the difficulty in estimating the material parameters for the sandwich core, which may not display elastic behaviour.

CONCLUSION

A spectral finite element developed for the analysis of structural vibration at high frequencies for sandwich beams has been presented. It describes the motion as a combination of travelling and decaying waves along the structure using a low number of degrees of freedom. The developed element is useful for structures that are uniform along a single coordinate axis but otherwise arbitrary in material composition and geometry. Furthermore, since the cross–section displacement functions are formulated in terms of the nodal displacements using high order polynomials it can easily be coupled to a conventional finite element method.

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