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Control Method of an Underactuated Space Manipulator Using High-Frequency Excitation

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Abstract

An underactuated manipulator is defined as a manipulator with fewer actuators than degree of freedom of the system, which, for example, corresponds to the situation that one of actuators of a manipulator breaks down. Because it is especially hard to fix the broken actuator in space, control methods of the underactuated manipulator are generally considered to be employed as emergency control methods in space. However, the most past studies on the underactuated manipulator have targeted ground-fixed manipulators although no fixed plane exists in space. In this paper, a control method of a two-link underactuated manipulator considering the application of the manipulator in space is proposed. The control method utilizes bifurcation phenomena occurring in the free link by the effect of high-frequency excitation, which also makes state feedback of the free joint not necessary. We finally confirm the validity of the theoretical analysis by simulations.

INTRODUCTION

Many studies of nonholonomic systems represented by underactuated manipulators have been made in recent years [1] [2]. Arai and Tachi [3] theoretically and experimentally proposed a control method for a two-link underactuated manipulator with a brake at the passive joint by using the coupling characteristics of the system. Yu *et al.* [4] presented a control method using friction at the free joint for an underactuated manipulator without such devices as brakes.

However, there have not been any studies on underactuated manipulators considering the environment where the manipulator is neither fixed on the ground nor affected by gravity although the major application considered for underactuated manipulators is utilization in space. In this paper, we propose a control method of a two-link space underactuated manipulator. We particularly utilize bifurcation phenomena occurring in the free link by high-frequency excitation [5] and control position of the free link without state feedback of the link [6][7]. The validity of the control method is theoretically clarified.

ANALYTICAL MODEL AND EQUATION OF MOTION

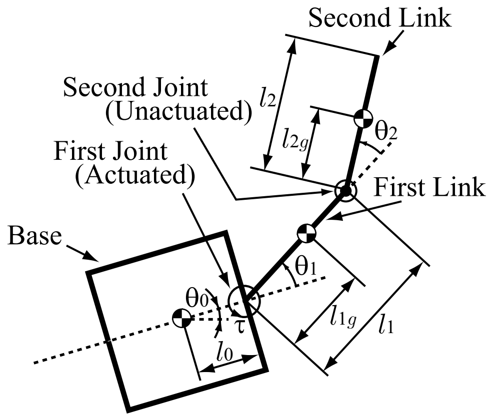


Figure 1: Analytical model of the manipulator

The analytical model of a two-link space underactuated manipulator mounted on a base is shown in Fig. 1. No external force is applied to the links, and the links and the base rotate on a two-dimensional plane. The first joint of the manipulator has an actuator giving torque τ for the first link. The second joint has neither actuator nor sensor and is called the free joint. The parameters are as follows:

m_0: mass of the base (15.2 kg)
m_1: mass of the first link (1.234×10^{-1} kg)
m_2: mass of the second link (3.92×10^{-2} kg)
l_0: distance between the center of mass of the base and the first joint(1.55×10^{-1} m)
l_1: length of the first link (1.50×10^{-1} m)
l_2: length of the second link (1.025×10^{-1} m)
l_{1g}: distance between the first joint and the center of mass of the first link (7.60×10^{-2} m)
l_{2g}: distance between the second joint and the center of mass of the second link (2.285×10^{-2} m)
I_0: moment of inertia of the base about the center of mass (1.8468×10^{-1} kgm^2)
I_1: moment of inertia of the first link about the center of mass (2.965×10^{-4} kgm^2)
I_2: moment of inertia of the second link about the center of mass (4.4094×10^{-4} kgm^2)
θ_0: relative angle of the base to the initial condition[rad]

θ_1: relative angle of the first link to the base[rad]
θ_2: relative angle of the second link to the first link[rad]

where the values in the parentheses correspond to those of the experimental apparatus mentioned later, and they are also used in the after-mentioned numerical simulations.

In this paper, we set that angular velocity of the base Ω_0 is kept constant and relative angle of the first link to the base θ_1 is excited as

$$\theta_1 = a_{\theta_1} \cos \omega t + \theta_{1off}, \tag{1}$$

where a_{θ_1} is excitation amplitude, ω is excitation frequency and θ_{1off} is the center of excitation. We then obtain the following dimensionless equation of motion about the second (free) link:

$$\begin{aligned} &\frac{d^2\theta_2}{dt^{*2}} + \mu \frac{d\theta_2}{dt^*} - (1 + c_2 \cos \theta_2) a_{\theta_1} \cos t^* \\ &\quad + c_1 \Omega_0^{*2} \sin(a_{\theta_1} \cos t^* + \theta_{1off} + \theta_2) + c_2 (\Omega_0^* - a_{\theta_1} \sin t^*)^2 \sin \theta_2 = 0, \end{aligned} \tag{2}$$

where the dimensionless time t^* is expressed as $t^* = \omega t$, and the dimensionless parameters are shown as follows:

$$\begin{aligned} &c_1 = \frac{m_0 m_2 l_0 l_{2g}}{(m_0 + m_1 + m_2)(I_2 + m_2 l_{2g}^2)}, \quad c_2 = \frac{m_2 l_1 l_{2g}(m_0 l_1 + m_1 l_1 - m_1 l_{1g})}{(m_0 + m_1 + m_2)(I_2 + m_2 l_{2g}^2)}, \\ &\mu = \frac{c}{(I_2 + m_2 l_{2g}^2)\omega}, \quad \Omega_0^* = \frac{\Omega_0}{\omega}. \end{aligned} \tag{3}$$

THEORETICAL ANALYSIS

Averaged Equation

We set that the excitation frequency ω is sufficiently larger than the angular velocity of the base Ω_0, which is what "high-frequency" means in this paper, namely, $\Omega_0^* (= \Omega_0/\omega) = O(\epsilon)$. We evaluate the magnitudes of the dimensionless parameters as

$$c_1 = O(1), \quad c_2 = O(1), \quad \mu = O(\epsilon), \quad a_{\theta_1} = O(\epsilon). \tag{4}$$

Using the method of multiple scales [8], Eq. (2) can be simplified to the following averaged equation:

$$\frac{d^2\theta_2}{dt^{*2}} + \mu \frac{d\theta_2}{dt^*} + \sigma\{c_1 \sin(\theta_{1off} + \theta_2) + c_2 \sin \theta_2\} - \frac{a_{\theta_1}^2 c_2^2}{2} \sin \theta_2 \cos \theta_2 = 0, \tag{5}$$

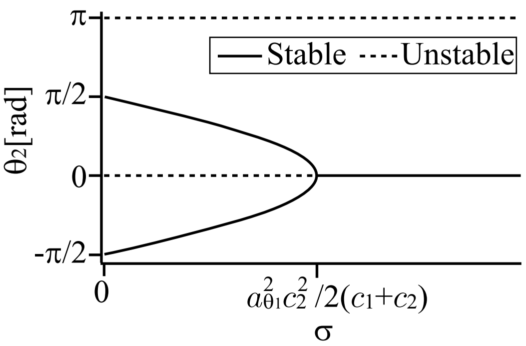
where $\sigma \equiv \Omega_0^{*2} (= \Omega_0^2/\omega^2)$. The first term is inertial force, the second is damping force, the third is centrifugal force caused by the rotation of the base, and the last is the effect of excitation.

Letting the time derivative be zero yields the equilibrium equation

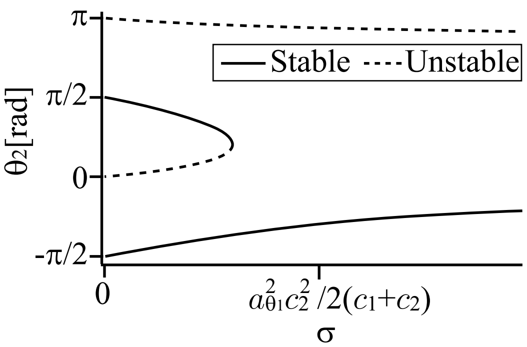
$$\sigma\{c_1 \sin(\theta_{1off} + \theta_{2eq}) + c_2 \sin \theta_{2eq}\} - \frac{a_{\theta_1}^2 c_2^2}{2} \sin \theta_{2eq} \cos \theta_{2eq} = 0, \tag{6}$$

where θ_{2eq} is equilibrium points of θ_2, which are determined by the values of σ and θ_{1off}.

Bifurcation Phenomena



(a) $\theta_{1off} = 0$



(b) $\theta_{1off} = \pi/4$

Figure 2: Bifurcation diagrams of the position of the second link

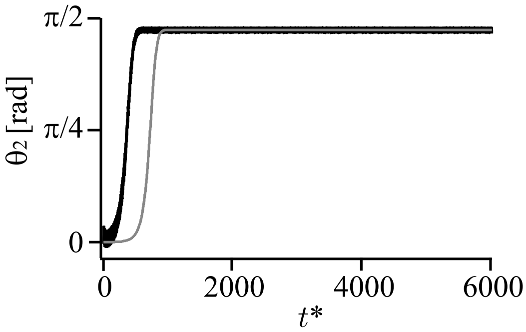
By solving Eq. (6) about θ_{2eq}, we obtain Figs. 2 (a) and (b) which show the relationships between σ (proportional to $1/\omega^2$) and equilibrium points of θ_2 when $\theta_{1off} = 0$ and $\theta_{1off} = \pi/4$, respectively. The solid and dashed lines respectively denote the stable and unstable equilibrium states in the figures.

When $\theta_{1off} = 0$, the second link undergoes a supercritical pitchfork bifurcation. The stable equilibrium states change from $\theta_{2eq} = \pm\pi/2$ at $\sigma = 0$ to $\theta_{2eq} = 0$ beyond $\sigma = a_{\theta_1}^2 c_2^2/2(c_1+c_2)$. Since σ is proportional to $1/\omega^2$, θ_{2eq} becomes nontrivial when the excitation frequency ω exceeds a certain value. That shows, by changing the excitation frequency ω, it is possible to control the stable equilibrium point where the position of the second link θ_2 converges. On the other hand, perturbated supercritical pitchfork bifurcations occur when $\theta_{1off} \neq 0$.

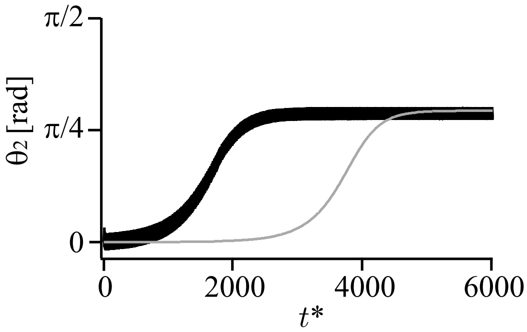
NUMERICAL SIMULATIONS

To confirm the validity of the theoretical analysis, we numerically solve Eq. (2), which is the equation before the averaging, and the averaged equation (5), using the Runge-Kutta method. Figures 3 (a), (b) and (c) show the results of the simulations when $\omega/2\pi = 20$ Hz ($\sigma = 0.1736\times10^{-4}$), $\omega/2\pi = 7.5$ Hz ($\sigma = 1.235\times10^{-4}$) and $\omega/2\pi = 4.5$ Hz ($\sigma = 3.429\times10^{-4}$), respectively. The black lines and the grey lines denote the solutions of the original equation 2 and the averaged equation 5, respectively. Angular velocity of the base is kept $60\Omega_0/2\pi = 5$ rpm, and the center of excitation $\theta_{1off} = 0$. The critical value of σ, where bifurcation occurs and nontrivial equilibrium points begin to exist, is 2.038×10^{-4} in the considered condition.

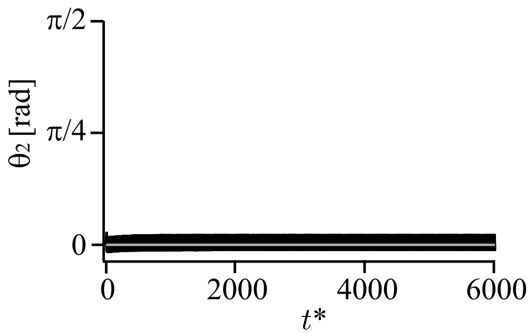
In Fig. 3 (a), the value of σ is quite smaller than the critical value, so the stable equilibrium point of θ_2 is near $\pi/2$ according to Fig. 2 (a). Corresponding to the analytical result,



(a)Time histories of θ_2 ($\omega/2\pi = 20$ Hz; $\sigma = 0.1736 \times 10^{-4}$)



(b)Time histories of θ_2 ($\omega/2\pi = 7.5$ Hz; $\sigma = 1.235 \times 10^{-4}$)



(c)Time histories of θ_2 ($\omega/2\pi = 4.5$ Hz; $\sigma = 3.429 \times 10^{-4}$)

Figure 3: Numerical simulations of the motion of the second link (comparison between the original and the averaged equation; black lines: the original, grey lines: the averaged)

angle of the second link θ_2 converges to the value about $\pi/2$.

When the excitation frequency $\omega/2\pi = 7.5$ Hz (Fig. 3 (b)), σ is closer to the critical point, and θ_2 becomes stable at the value about $\pi/4$ as the theoretical analysis shows.

Fig. 3 (c) shows that stable equilibrium point of θ_2 is 0 for the values of σ beyond the critical point.

For every condtion of σ above, the solution of the averaged equation lies on the solution of the original equation in the steady state, while the transient states of them are not exactly corresponding. That tells the theoretical analysis is valid enough in the steady state.

CONCLUSIONS

This paper proposed a position control method of an underactuated manipulator mounted on a rotating base. We theoretically clarified dynamics of the manipulator and the control method utilizing bifurcation phenomena which occur in motion of the free link under high-frequency excitation. We then confirmed the control method by simulations.

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