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# FREE VIBRATION ANALYSIS FOR STIFFENED PLATES OF SHIP TANK SIDE IN CONTACT WITH FLUID USING ASSUMED MODE METHOD

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## Abstract

In this study, the assumed mode method using characteristic polynomials of Timoshenko beam is applied to the free vibration analysis for the stiffened plate of the ship tank side in contact with fluid. The hydro-elastic effect of the fluid-structure interaction is considered by fluid velocity potential, derived from boundary conditions for fluid and structure, and utilized for the calculation of added mass using assumed modes. To verify the validity and effectiveness of the present method, free vibration analysis for the stiffened plates in contact with finite and infinite fluid have been carried out and its results were compared with those obtained by a general purpose FEA software based on the coupled FE-BEM.

## INTRODUCTION

Stiffened plate structures have been extensively used for ship local structures and their free vibration analyses are required to avoid resonance with relevant excitation sources in the ship design stage. Especially, being in contact with fluid, the plate structure is more likely to resonant with one of the excitation sources because the added mass effect of fluid decreases significantly its natural frequencies. Therefore it is very important to predict accurate vibration characteristics of stiffened plates in contact with fluid for anti-resonance design of ship structure.

There have been many research works on the dynamic interaction between structure and fluid using numerical or semi-analytical method. Numerical methods such as finite element method (FEM) [5] and coupled finite element-boundary element method (FE-BEM) [3] can be generally applied to such problems, but it may require considerable time and efforts in modelling and analysis. As semi-analytical approaches, Amabili [6] and Kwak [8] presented the vibration of plates placed in a circular or annular aperture in contact with fluid using the Rayleigh-Ritz method. In addition, Kwak [7] obtained the added mass incremental factor for rectangular plates using the Rayleigh-Ritz method combined with the boundary element method. Recently, Zhou and Cheung [1] presented solutions for vertical rectangular plates placed in a rectangular aperture of a vertical rigid wall in contact with fluid of finite width and depth using analytical-Ritz method. Up to now, however, most of the studies are about the bare plates and it is hardly to find studies for stiffened plates.

In this paper, the assumed mode method using characteristic orthogonal polynomials of Timoshenko beam [4] is applied to the free vibration analysis for the stiffened plates of ship tank side in contact with fluid. We have utilized velocity potential for the calculation of added mass to solve the fluid-structure interaction problem. The plate considered in this study is the Mindlin plate including the shear deformation and rotary inertia effects with rotational constraints along the edges. To verify the validity and effectiveness of the present method, free vibration analysis for the various stiffened plates in contact with fluid have been carried out and compared its results with those obtained by a general purpose FEA software based on the FE-BEM.

## KIENTIC ENERGY OF THE FLUID

For the free vibration analysis of stiffened plate in contact with fluid, it is essential to estimate the kinetic energy attributed to the fluid. In this study, we consider a rectangular plate with width $a$ and height $b$ in contact with fluid on one side which lies in the $\tilde{x}-\tilde{y}$ plane and the ideal and irrotational fluid in a rectangular domain with finite width $d$ and depth $e$, but with infinite length along the $\tilde{z}$ axis as shown in Figure 1. In the figure, $x, y$ and $\tilde{x}, \tilde{y}, \tilde{z}$ are coordinate notations to describe the motions of the plate and fluid, respectively. The other walls and the bottom of rectangular container are assumed as rigid except the rectangular plate part in contact with fluid. In addition, the surface waves and hydrostatic pressure effects are neglected in the present study. As a consequence of these hypotheses, the velocity potential $\phi(\tilde{x},\tilde{y},\tilde{z},t)$ exists satisfying the following Laplace equation and boundary conditions in fluid domain $\Omega$:

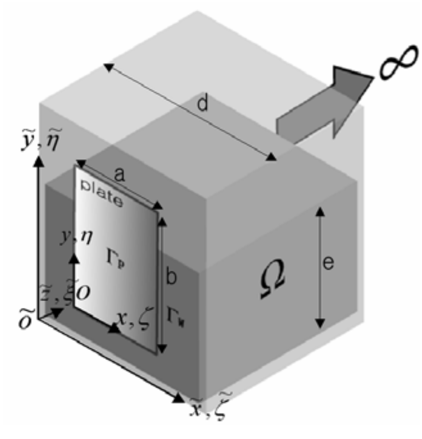



*Figure 1 – A wetted side plate of rectangular tank structure*

$$\Delta\phi = \frac{\partial^2\phi}{\partial\tilde{x}^2} + \frac{\partial^2\phi}{\partial\tilde{y}^2} + \frac{\partial^2\phi}{\partial\tilde{z}^2} = 0, \qquad \text{in } \Omega \tag{1}$$

$$\frac{\partial\phi}{\partial\tilde{x}} = 0, \qquad \text{on } \tilde{x} = 0,\ d \text{ (rigid wall)} \tag{2}$$

$$\frac{\partial\phi}{\partial\tilde{y}} = 0, \qquad \text{on } \tilde{y} = 0 \text{ (rigid bottom)} \tag{3}$$

$$\phi = 0, \qquad \text{on } \tilde{y} = e \text{ (no surface waves)} \tag{4}$$

$$\phi = 0, \qquad \text{at } \tilde{z} \to \infty \text{ (infinite fluid)} \tag{5}$$

$$-\left.\frac{\partial\phi}{\partial\tilde{z}}\right|_{\tilde{z}=0} = \begin{cases} \dfrac{\partial w}{\partial t} & \text{on wetted plate surface } (\Gamma_P) \\ 0 & \text{on the other part } (\Gamma_W) \end{cases} \tag{6}$$

where $w = W(x,y)\sin(\omega t)$ is the transverse deflection of the plate.

Applying the method of separation of variables and considering the boundary conditions (2)-(5), the solution of the Laplace equation (1) is given by [1]

$$\phi = \omega d \cos(\omega t) \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} E_{rs} \cos(r\pi\tilde{\zeta}) \cos(\tau_s \pi \tilde{\eta}) e^{-C_{rs}\tilde{\xi}} \tag{7}$$

where $\omega$ is the circular frequency in radian/sec. and

$$C_{rs} = \pi\sqrt{r^2 + \left(\lambda\gamma\tau_s/\beta\right)^2}, \quad \tau_s = s + 0.5 \tag{8}$$

In the above equations, the following non-dimensional parameters and coordinates are introduced:

$$\zeta = x/a,\ \eta = y/b,\ \tilde{\zeta} = \tilde{x}/d,\ \tilde{\eta} = \tilde{y}/e,\ \tilde{\xi} = \tilde{z}/d,\ \lambda = a/b,\ \gamma = b/e,\ \beta = a/d \tag{9}$$

Substituting equation (7) into (6) and applying the orthogonality of the trigonometric functions and the Fourier series expansion, the coefficient $E_{rs}$ can be exactly derived in the form of integral equations as follows:

$$E_{rs} = \frac{\varepsilon_r}{C_{rs}} \iint_{\Gamma_P} W(\zeta,\eta) \cos(r\pi\tilde{\zeta}) \cos(\tau_s \pi \tilde{\eta}) d\Gamma \tag{10}$$

where if $r = 0$, $\varepsilon_r$ =2 and otherwise, $\varepsilon_r$ =4.

Therefore the kinetic energy of the fluid, whose density is $\rho_w$, is expressed as follows:

$$T_w = \frac{1}{2}\rho_w \iiint_{\Omega} (\nabla\phi)^2 d\Omega \tag{11}$$

where $\nabla$ is the gradient operator. By using Green's theorem to above equation and substituting equation (7), the volume integral in the fluid domain can be transformed into a surface integral for the plate region $\Gamma_P$ as follows:

$$\begin{aligned} T_w &= \frac{1}{2}\rho_w \iint_{\Gamma_P} \left( \phi \frac{\partial \phi}{\partial \tilde{z}} \right)\Bigg|_{\tilde{z}=0} d\Gamma \\ &= \frac{1}{2}\rho_w a^2 e\omega \cos(\omega t) \sum_{r=0}^{\infty}\sum_{s=0}^{\infty} \frac{\varepsilon}{C_{rs}} \iint_{\Gamma_P} \dot{w}\cos(r\pi\tilde{\zeta})\cos(\tau_s \pi\tilde{\eta}) d\Gamma \\ &\quad \times \iint_{\Gamma_P} w\cos(r\pi\tilde{\zeta})\cos(\tau_s \pi\tilde{\eta}) d\Gamma \end{aligned} \tag{12}$$

in which, transverse deflection $w$ and velocity $\dot{w}$ express the fluid-structure interaction effect and can be approximated by shape functions of the considered plate.

## FREE VIBRATION ANALYSIS OF STIFFENED PLATE IN CONTACT WITH FLUID

The free vibration analysis of stiffened plate in contact with fluid is carried out by the assumed mode method. The transverse deflection, $w$, and the rotations, $\theta_\zeta$ and $\theta_\eta$, of the rectangular Mindlin plate are assumed to be a linear combination of the characteristic polynomials multiplied by time-dependent generalized coordinates, $A_{mn}(t), B_{mn}(t), C_{mn}(t)$, in the $\zeta$ and $\eta$ directions as

$$\begin{aligned} w(\zeta,\eta,t) &= \sum_{m=1}^{p}\sum_{n=1}^{q} A_{mn}(t) X_m(\varsigma) Y_n(\eta) \\ \theta_\varsigma(\zeta,\eta,t) &= \sum_{m=1}^{p}\sum_{n=1}^{q} B_{mn}(t) \Psi_m(\varsigma) Y_n(\eta) \\ \theta_\eta(\zeta,\eta,t) &= \sum_{m=1}^{p}\sum_{n=1}^{q} C_{mn}(t) X_m(\varsigma) \Phi_n(\eta) \end{aligned} \tag{13}$$

where $p, q$ are the numbers of polynomials and $X_m(\varsigma), Y_n(\eta)$ and $\Psi_m(\varsigma), \Phi_n(\eta)$ are characteristic polynomials for transverse displacement and rotational angle in $\varsigma$ - and $\eta$ - directions, respectively. The polynomials are derived from satisfying the orthonormal condition for a uniform Timoshenko beam with ends elastically restrained against rotation as shown in Figure 2:

$$\int_0^1 \left( \rho A W_m W_n + \rho I \Theta_m \Theta_n \right) d\xi = \begin{cases} 0 & for\ m \neq n \\ 1 & for\ m = n \end{cases} \tag{14}$$

where $\rho$ is the density, $A$ is the cross-sectional area, $I$ is the moment of inertia, $L$ is the length and $K_R$ is the rotational spring constant of the beam. And, $W$ and $\Theta$ represent the transverse displacement and the rotational angle, respectively.

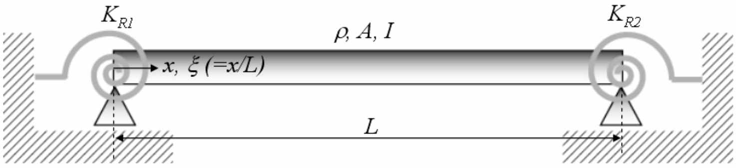



*Figure 2 – Uniform Timoshenko beam with ends elastically restrained against rotation*

The first and second sets of the characteristic polynomials are derived by applying the boundary conditions of the plate in each direction and the higher order polynomial sets are generated by applying the Gram-Schmidt orthogonalization procedure [4]. By the assumed mode method using the characteristic polynomials derived from the above described method, we can derive the total potential energy $V_{total}$ and the kinetic energy $T_{total}$ of the stiffened plate in contact with fluid as follows [2]:

$$V_{tatal} = V_p + V_b = \frac{1}{2}\sum_{i=1}^{3pq}\sum_{j=1}^{3pq} k_{ij} q_i(t) q_j(t) \tag{15}$$

$$T_{tatal} = T_p + T_b + T_w = \frac{1}{2}\sum_{i=1}^{3pq}\sum_{j=1}^{3pq} m_{ij} \dot{q}_i(t) \dot{q}_j(t) \tag{16}$$

where

$$\{q(t)\} = \{A_{11}, A_{12}, \cdots, A_{pq,} B_{11}, B_{12}, \cdots, B_{pq,} C_{11}, C_{12}, \cdots, C_{pq}\}^T \tag{17}$$

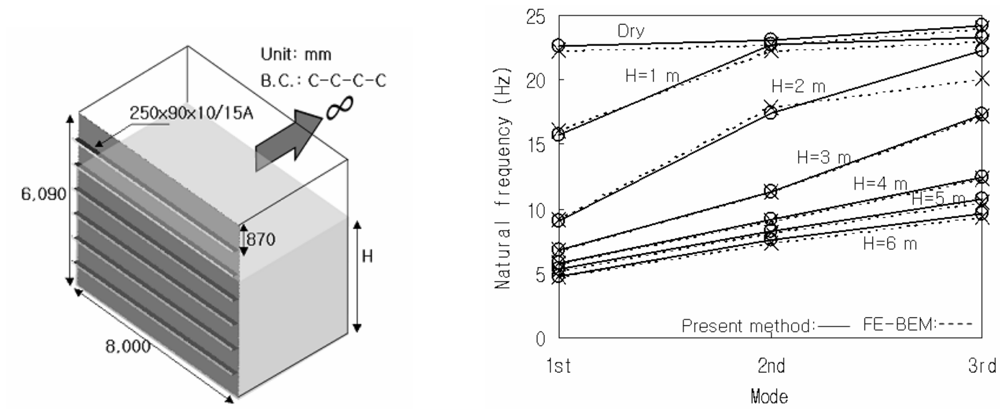
and $V_p$ is the potential energy of the bare plate, $V_b$ is that of the stiffener members, and $T_p$, $T_b$ and $T_w$ represent the kinetic energy terms contributed by the bare plate, the stiffener members and the contact fluid, respectively. In equations (15) and (16), $k_{ij}$ and $m_{ij}$ are the coefficients of stiffness and mass for the stiffened plate in contact with fluid, respectively. Inserting Equations (15) and (16) into Lagrange's equations of motion for conservative discrete systems gives

$$[M]\{\ddot{q}(t)\} + [K]\{q(t)\} = \{0\} \tag{18}$$

where $[M]$ and $[K]$ are mass and stiffness matrices, respectively. Assuming that the time dependence of $q(t)$ is harmonic, the natural frequency and its corresponding $\{q(t)\}$ can be calculated from the equation (18). Finally, we can determine the mode shape of the stiffened plate in contact with fluid can be determined by substituting $\{q(t)\}$ to equation (13).

## NUMERICAL EXAMPLES

To verify the validity of the present method, various numerical examples for stiffened




*(a)* *(b)*

*Figure 3 – Uni-directionally stiffened plate in partially contact with fluid and their natural frequencies: (a) the plate structure in a rectangular tank and (b) natural frequency.*

plate structures in contact with fluid are carried out and the results are compared with those obtained by a general purpose FEA software MSC/Nastran based on the coupled FE-BEM. In the numerical analysis, material characteristics of the stiffened plate are set to Young's modulus $E = 2.06E5\,MPa$, Poisson ratio $\nu = 0.3$, density $\rho_s = 7{,}850\,kg/m^3$. The contact fluid is assumed as sea water and its density is set to $\rho_w = 1{,}025\,kg/m^3$. In the numerical analysis, 11 polynomials for each direction are used as assumed modes.

**Stiffened plate in partially contact with fluid**

Using the present method and the MSC/Nastran, free vibration analysis has been carried out for a clamped uni-directionally stiffened plate in partially contact with fluid, corresponding to ship's tank structure, as shown in Figure 3(a). In the FE-BEM analysis, the infinite fluid condition at opposite side of the stiffened plate is truncated by modeling a rigid wall located 10 m apart. In Figure 3(b), its natural frequencies are shown as the water depth is varied from 0 to 6 m. From the results, natural frequencies up to 3rd mode obtained by the present method and the FE-BEM show good agreement.

**Stiffened plate in fully contact with semi-infinite fluid on the plate plane**

To verify the accuracy of the present method for the stiffened plate in fully contact with semi-infinite fluid on the plate plane, free vibration analysis is carried out for the total

*Table 1 – Model parameters of the uni-directionally stiffened plate (unit: mm)*

| | |
|---|---|
| Plate dimension | 2400×2400, 4000×2400, 3200×3200, 4000×4000, 2400×4800, 4000×4800 |
| Plate thickness | 11, 13, 15 |
| Stiffener type | 150×90×9/9A, 250×90×10/15A, 400×100×11.5/16A |
| Stiffener spacing | 800 |

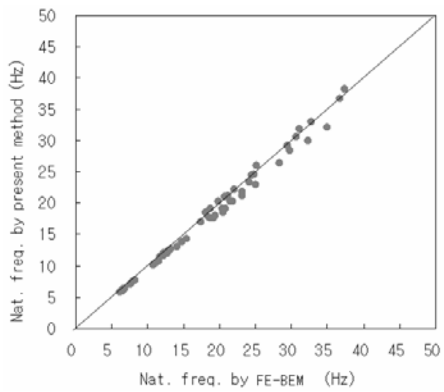



*Figure 4 – Fundamental natural frequencies of uni-directionally stiffened plate in contact with semi-infinite fluid*

54 uni-directionally stiffened plates, designed by varying three parameters of plate dimension, plate thickness and stiffener type. The parameters are represented in Table 1. In the analysis, the ratio of plate dimension and fluid domain on the plate plane is taken as $\gamma(=b/e)\le 0.1$, $\beta(=a/d)\le 0.1$ and the boundary conditions were regarded as simply supported on all edges. The fundamental natural frequencies for the 54 plates obtained by the present method and the FE-BEM are compared in Figure 4. From the results, the mean difference and its standard deviation of natural frequencies obtained by the two methods are -4.4% and 3.6%, respectively.

Finally, for the simply supported cross-stiffened plate in contact with semi-infinite water as shown in Figure 5, the natural frequencies and corresponding mode shapes are presented in Figure 6. As can be seen, the differences of natural frequencies up to 3$^{rd}$ mode obtained by the two methods are within 6.5% and their mode shapes are in good agreement.

## CONCLUSIONS

In this paper, the assumed mode method using characteristic polynomials of Timoshenko beam is applied for the free vibration analysis of the stiffened plate in contact with fluid of the ship tank side. The hydro-elastic effect of the fluid-structure interaction is considered by semi-analytical method using the fluid velocity potential, derived from boundary conditions for fluid and structure. From the numerical

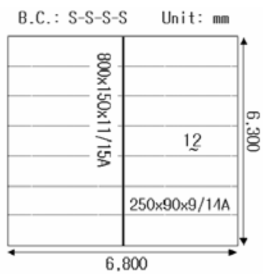



*Figure 5 – Cross-stiffened plate*

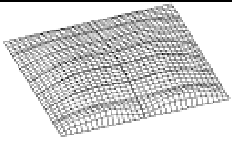
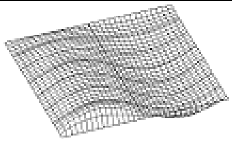
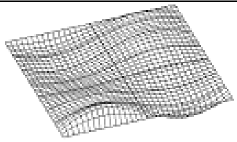
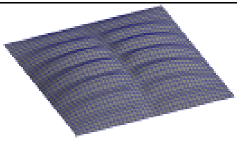
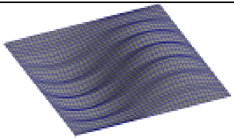
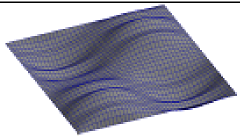
| | 1st mode | 2nd mode | 3rd mode |
|---|---|---|---|
| Present method | <br>9.14 Hz | <br>12.57 Hz | <br>14.54 Hz |
| FE-BEM | <br>9.27 Hz | <br>12.09 Hz | <br>13.66 Hz |

*Figure 6 – Natural frequencies and mode shapes of the cross-stiffened plate in contact with semi-infinite water*

investigation for the stiffened plate structures with different plate design, fluid-contact and boundary conditions, we can confirm the validity of the present method. Thus, the present method is usefully applicable to the free vibration analysis of stiffened plate structure of ship in contact with fluid and its anti-resonance design.

## ACKNOWLEDGMENT

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