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APPLICATION OF NEGATIVE REFRACTION TO ULTRASONIC NONDESTRUCTIVE EVALUATION

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Abstract

Negative refraction has been verified for sound waves using phononic crystals as focussing lens. It has been demonstrated for producing an image of a point object using the phononic crystals as lens. It shows improvement in the resolution limit. in this paper, we apply negative refraction to ultrasonic nondestructive evaluation. An experimental system for nondestructive evaluation with negative refraction is given. The transmission mode is used. This is the transmission of sound wave from the source through the object and incident onto the negative refraction lens (NRL). The object has to be immersed in water which acts as a couplant. Since the solid is immersed in water, we have to consider the process via a fluid-solid interface. We consider the theoretical approach to a homogeneous, isotropic solid bordered by a plane interface in a fluid and we discuss how the different modes of wave propagation are involved in the process. The expression for the transmitted wave through the object is obtained. This will be the incident wave impinging onto the negative focussing lens (NFL), which is made of phononic crystals. The multiple scattering theory (MST) is used to calculate the transmission coefficient through the NFL. We show how a dramatic variation in wave propagation with both frequency and propagation direction leads to novel focussing phenomena associated with large negative refraction. This is chaotic behavior. These effects are then used using ultrasonic techniques to image the transmitted wave field. The fields scattered are calculated theoretically using a Fourier imaging technique in which wave propagation through the crystal is accurately described by the 3D equi-frequency surfaces predicted from the MST. We then analyze the role played by the evanescent waves by including them in the image together with the propagating waves. This shows an improvment in the resolution.

INTRODUCTION

Negative refraction has been verified for sound waves using phononic crystals as focussing lens. It has been demonstrated for producing an image of a point object using the phononic crystals as lens [1]. it shows improvement in the resolution limit. The key issue of this paper is the design of the ultrasonic nondestructive testing system based on negative refraction. We calculate the various wave scattering processes involved in this NDT system.

DESIGN OF A NONDESTRUCTIVE TESTING SYSTEM

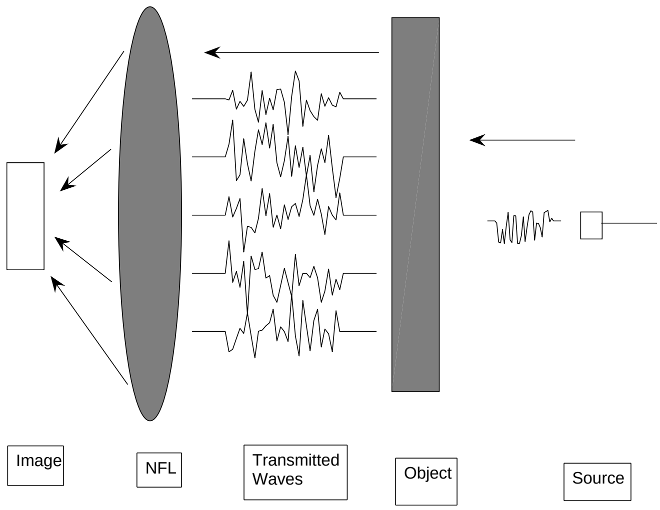

Figure 1: An ultrasonic nondestructive testing system with negative refraction

We propose the ultrasonic nondestructive testing system with negative refraction as follows.

In this paper, we consider a homogeneous isotropic solid bordered by a plane interface to a fluid. The fluid acts as a couplant. The ultrasound source is in the air. Upon arrival at the fluid/solid interface, the wave emits both the longitudinal (P) and transverse (S) waves in the solid. We describe the displacement vector $\mathfrak{u}$ of the incident elastic field as a function of the potentials ϕ nd ψ of the P and S waves propagating in the positive x direction:

$$\mathfrak{u}(x, y, z, t) = \nabla\phi(x, y, z, t) + \nabla_{\Lambda}\nabla_{\Lambda}\left[0, 0, \psi(x, y, z, t)\right] \tag{1}$$

There is in existence the 2-D Fourier transform $FT_{2-D}\left[\ldots\right]$ over x and y in the frequency domain of the potentials $\tilde{\Phi}(k_x, k_y, z, \omega)$ and $\tilde{\Psi}(k_x, k_y, z, \omega)$:

$$\phi(x,y,z,t) = \frac{1}{2\pi}\int d\omega \tilde{\phi}(x,y,z,\omega) exp(-j\omega t)$$
$$\tilde{\phi}(x,y,z,\omega) = \left(\frac{1}{2\pi}\right)^2 \int\int dk_x dk_y \tilde{\Phi}(k_x,k_y,z,\omega)$$
$$exp(j(k_x x + k_y y)) = FT^{-1}_{2-D}\left[\tilde{\Phi}\right]$$

Due to the wave equation, the dependence on z can be written explicitly

$$\tilde{\Phi}(k_x,k_y,z,\omega) = \tilde{\Phi}(k_x,k_y,x=0,\omega) exp\left[j\nu_a(k_r)x\right]$$

where ν_a is a function of $k_r = \sqrt{k_y^2 + k_z^2}$ and the longitudinal wave speed α and is defined by

$$\nu_a(\omega) = \begin{cases} \sqrt{w^2/\alpha^2 - k_r^2} & \text{if } k_r \leq w/\alpha \\ j\sqrt{k_r^2 - w^2/\alpha^2} & \text{if } k_r > w/\alpha \end{cases} \tag{2}$$

If ν_α is real, i.e. if $k_r \leq w/\alpha$, it can be considered as the x component k_x of the wave vector $\mathbf{k} = (k_x, k_y, k_z)$ of a propagative wave. If ν_α is imaginary, the wave is called evanescent, its amplitude decreases exponentially with the propagation in the x direction.

With similar definitions, we obtain for the shear-wave field:

$$\tilde{\Phi}(k_x,k_y,z,\omega) = \tilde{\Phi}(k_x,k_y,z=0,\omega) exp\left[j\nu_\beta(k_r)x\right]$$

Each wave type in the solid generates at the interface a wavefront. Describing the transmitted sound wave by its pressure field P, we can split it formally into the part created by the P wave P_P and the part played by the S wave P_S:

$$P(x,y,z,t) = P_P(x,y,z,t) + P_S(x,y,z,t)$$

giving

$$\tilde{P}_P(k_x,k_y,z,\omega) = \tilde{\Phi}(k_x,k_y,z=0,\omega)T_{Pf}\cdot exp(j\nu_\alpha h)exp[j\nu_c(z-h)]$$
$$\tilde{P}_S(k_x,k_y,z,\omega) = \tilde{\Psi}(k_x,k_y,z=0,\omega)T_{Sf}\cdot exp(j\nu_\beta h)exp[j\nu_c(z-h)] \tag{3}$$

The transmission coefficients T_{Pf} and T_{Sf} are functions of k_r and ω.

MULTIPLE SCATTERING THEORY

Here we extend the multiple-scattering theory (MST) for elastic waves by taking into account the full vector character [2]. Multiple scattering theory [3], through its success in electronic structure calculations for both ordered and disordered systems, show great promise for the study of elastic wave scattering and propagation in both ordered and disordered media. MST, usually known as the KKR (Korringa, Kohn and Rostoker) approach was developed mainly for the calculation of electronic band structures, although it originated from the study of classical waves (including acoustic waves). MST, in the spirit of the KKR approach, has been developed for the electromagnetic wave and was successfully applied to the band structure calculation of photonic crystals. At the same time, a layer MST theory [4] of electromagnetic waves was also successfully implemented, thus enabling rigorous calculation of the transmission and reflection coefficients for a slab of periodically arranged scatterers, and providing a direct way to compare theory with experiment.

In a homogeneous medium, the elastic wave equation may be written as

$$(\lambda + 2\mu)\underline{\nabla}(\underline{\nabla} \cdot \mathrm{u}) - \mu\underline{\nabla}_\Lambda\underline{\nabla}_\Lambda \mathrm{u} + \rho\omega^2\mathrm{u} = 0 \tag{4}$$

where ρ is the density and λ, μ are the Lam$\acute{e}$ constants of the medium. Zhengyou Liu *et. al.* [2] has derived using MST and vector theory an expression for the transmission coefficient for sound wave through the solid medium as:

$$T(R) = \frac{\sum_g \left[(\lambda+2\mu)U_{\alpha g}^{trn(ref)}U_{\alpha g}^{trn(ref)} \cdot U_{\alpha g}^{trn(ref)*}K_{\alpha gz}^{+} + \mu U_{\beta g}^{trn(ref)} \cdot U_{\beta g}^{trn(ref)*}K_{\rho gz}^{+}\right]}{\sum_g \left[(\lambda+2\mu)U_{\alpha g}^{inc} \cdot U_{\alpha g}^{inc*}K_{\alpha gz}^{+} + \mu U_{\beta g}^{inc} \cdot U_{\beta g}^{inc*}K_{\beta gz}^{+}\right]} \tag{5}$$

where Us are the amplitudes of the sound wave pressure.

Hence the expressions for the transmitted waves through the phononic crystals are given by from (3) and (5)

$$\begin{aligned} P_t(k_x, k_y, z, \omega) = {} & \tilde{\Phi}(k_x, k_y, z=0, \omega)T_{Pf}exp(j\nu_\alpha h)exp(j\nu_c(z-h)) \\ & + \tilde{\Psi}(k_x, k_y, z=0, \omega)T_{Sf}exp(j\nu_\beta h)exp(j\nu_c(z-h)) \cdot T(R) \end{aligned} \tag{6}$$

CALCULATION OF WAVE FIELD PATTERN USING A FOURIER TRANSFORM TECHNIQUE

Here the transmission coefficient is equivalent to the transfer function. Let $F_o(k_x, k_y)$ represent the Fourier spectrum of the sound wave transmitted by the lens and $F_i(k_x, k_y)$ the Fourier spectrum of the sound wave from the object incident onto the lens. Then the spatial wave field pattern is given by the inverse Fourier transform of $P_t(k_x, k_y, z, \omega)$:

$$P_t(k_x, k_y, z, t) = \left(\frac{1}{2\pi}\right)^2 \int_0^{k_x}\int_0^{k_y} P_t(k_x, k_y, z, \omega)exp(j2\pi(xk_x + yk_y))dk_xdk_y \tag{7}$$

RESOLUTION LIMIT

X. Zhang and Z. Liu [1] have shown that by using phononic crystals they are able to achieve negative refraction. Using the effect of negative refraction, they can fabricate a superlens which can break through the traditional limitation of Rayleigh resolution limit and achieve a transverse size (full size at half-maximum) of the image spot as 0.14λ. John Pendry [5] using a 'perfect lens' also based on negative refraction achieved even better resolution. In this paper we follow John Pendry [6]'s method of constructing the 'perfect lens' by incorporating the evanescent waves and we hope to defeat the Rayleigh resolution limit.

CONCLUSION

By incorporating negative refraction and including the evanescent wave, it is possible to design an ultrasonic nondestructive testing system with higher resolution. The next step will be to study the details of the experimental system.

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