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ALLOCATION OF THREE CONTROL FORCES TO FOUR ACTUATORS FOR 3-DOF HYBRID VIBRATION ISOLATION SYSTEM

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Abstract
This paper treats a hybrid type vibration isolation system for 3-DOF vibrations, that is, bouncing, pitching and rolling vibrations. The hybrid type system is composed of passive spring-damper system and active system with electromagnetic actuators. The controller for the vibration isolation system derives one control force for bounce and two control moments for pitch and roll in regard to the center of gravity. Usually, an actuator to supply vibration control force is set in each supporting part at the four corners of the loading platform. The allocation of three control outputs to four actuators is not unique because of redundancy. In this paper, the allocation of the control output is discussed in detail.

INTRODUCTION

This paper treats a hybrid type vibration isolation system composed of a passive spring-damper system and an active system with electromagnetic actuators. The passive system reduces vibrations of high frequency range and an active one reduces those of low frequency range. The hybrid system isolates 3-DOF vibrations, that is, bouncing, pitching and rolling vibrations in this paper. Some control theory, e.g. PID, LQG, H_∞, etc., is applied to design the active system (Mizutani et al, Doyle et al), and the optical control gains to give the effective vibration isolation performance are provided. The controller of the vibration isolation system derives one control force for bouncing vibration and two control moments for pitching and rolling ones in regard to the centre of gravity. Usually, an actuator to supply vibration control force is set in each

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supporting part at four corners of the loading platform. The allocation of three control outputs to four actuators is not unique because of redundancy, so that we discuss the allocation of the control force and moments about the centre of gravity to each actuator in detail.

EQUATIONS OF MOTION AND STATE EQUATION

Equations of Motion

Figure 1 shows the analytical model of the hybrid type vibration isolation system. The loading platform is assumed a rigid body and assumed to have three DOF: bounce, pitch, and roll for the centre of gravity (Shannan). The loading platform is supported by four sets of vibration isolation parts, each part of which is consisted of a passive spring-damper part, and an active part with electromagnetic actuators. Using symbols in Figure 1, the equations of motion for 3-DOF, that is, the bouncing displacement x_G, the pitching angle θ_P and the rolling angle θ_R, are derived as follows:

$$\begin{aligned}
m\ddot{x}_G &= -k_G x_G - k_{GP}\theta_P - k_{GR}\theta_R - c_G\dot{x}_G - c_{GP}\dot{\theta}_P - c_{GR}\dot{\theta}_R + u_G + k_{FL}d_{FL} \\
&\quad + k_{FR}d_{FR} + k_{RL}d_{RL} + k_{RR}d_{RR} + c_{FL}\dot{d}_{FL} + c_{FR}\dot{d}_{FR} + c_{RL}\dot{d}_{RL} + c_{RR}\dot{d}_{RR} \\
J_p\ddot{\theta}_P &= -k_{GP}x_G - k_P\theta_P - k_{PR}\theta_R - c_{GP}\dot{x}_G - c_P\dot{\theta}_P - c_{PR}\dot{\theta}_R + u_P \\
&\quad - k_{FL}L_{bF}d_{FL} - k_{FR}L_{bF}d_{FR} + k_{RL}L_{bR}d_{RL} + k_{RR}L_{bR}d_{RR} \\
&\quad - c_{FL}L_{bF}\dot{d}_{FL} - c_{FR}L_{bF}\dot{d}_{FR} + c_{RL}L_{bR}\dot{d}_{RL} + c_{RR}L_{bR}\dot{d}_{RR} \\
J_R\ddot{\theta}_R &= -k_{GR}x_G - k_{PR}\theta_P - k_R\theta_R - c_{GR}\dot{x}_G - c_{PR}\dot{\theta}_P - c_R\dot{\theta}_R + u_R \\
&\quad - k_{FL}L_{tL}d_{FL} + k_{FR}L_{tR}d_{FR} - k_{RL}L_{bR}d_{RL} + k_{RR}L_{tR}d_{RR} \\
&\quad - c_{FL}L_{tL}\dot{d}_{FL} + c_{FR}L_{tR}\dot{d}_{FR} - c_{RL}L_{tL}\dot{d}_{RL} + c_{RR}L_{tR}\dot{d}_{RR}
\end{aligned} \tag{1}$$

where k_{XY} and c_{XY} are the coupled spring constant and the coupled viscous damping coefficient in the *XY* direction, respectively. The symbol u_G is the control force in the x_G direction, and symbols u_P and u_R are control moments in θ_P and θ_R directions.

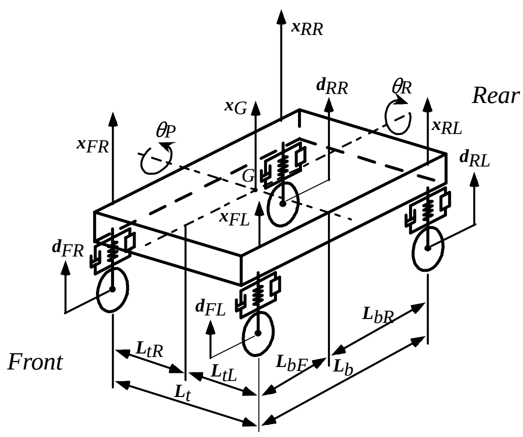

Fig.1 Analytical model

State Equation and Output Equation

The control subjects of this system are the bouncing acceleration $\ddot{x}_G$, and the pitching and rolling angular accelerations $\ddot{\theta}_P$ and $\ddot{\theta}_R$, since acceleration of the loading platform should be decreased to avoid the damage of carried fragile products. The angular accelerations $\ddot{\theta}_P$ and $\ddot{\theta}_R$ are hardly measured for the practically in-plant vehicle, so that accelerations of four supported points $\ddot{x}_{FL}$, $\ddot{x}_{FR}$, $\ddot{x}_{RL}$ and $\ddot{x}_{RR}$ are measured instead of $\ddot{x}_G$, $\ddot{\theta}_P$ and $\ddot{\theta}_R$, and fed back to the controller. The equation for transformation from $\ddot{x}_{FL}$, $\ddot{x}_{FR}$, $\ddot{x}_{RL}$ and $\ddot{x}_{RR}$ to $\ddot{x}_G$, $\ddot{\theta}_P$ and $\ddot{\theta}_R$ are geometrically derived as follows:

$$\begin{pmatrix} \ddot{x}_G \\ \ddot{\theta}_P \\ \ddot{\theta}_R \end{pmatrix} = \frac{1}{4} \begin{pmatrix} (1-A-B) & (1-A+B) & (1+A-B) & (1+A+B) \\ -\frac{2}{L_{bF}+L_{bR}} & -\frac{2}{L_{bF}+L_{bR}} & \frac{2}{L_{bF}+L_{bR}} & \frac{2}{L_{bF}+L_{bR}} \\ -\frac{2}{L_{tL}+L_{tR}} & \frac{2}{L_{tL}+L_{tR}} & -\frac{2}{L_{tL}+L_{tR}} & \frac{2}{L_{tL}+L_{tR}} \end{pmatrix} \begin{pmatrix} \ddot{x}_{FL} \\ \ddot{x}_{FR} \\ \ddot{x}_{RL} \\ \ddot{x}_{RR} \end{pmatrix} \tag{2}$$

where A and B are

$$A = \frac{L_{bF} - L_{bR}}{L_{bF} + L_{bR}},\ B = \frac{L_{tL} - L_{tR}}{L_{tL} + L_{tR}}$$

The state vector and the control force vector are defined as $\boldsymbol{x} = [\dot{x}_G \quad \dot{\theta}_P \quad \dot{\theta}_R \quad x_G \quad \theta_P \quad \theta_R]^T$ and $\boldsymbol{u} = [u_G \quad u_P \quad u_R]^T$, respectively, and the system disturbance vector as $\boldsymbol{d} = [\dot{d}_{FL} \quad \dot{d}_{FR} \quad \dot{d}_{RL} \quad \dot{d}_{RR} \quad d_{FL} \quad d_{FR} \quad d_{RL} \quad d_{RR}]^T$ where symbols d_{FL}, d_{FR}, d_{RL} and d_{RR} are absolute displacements of the supports as shown in Figure 1. When the out put vector is defined as $\boldsymbol{y} = [\ddot{x}_G \quad \ddot{\theta}_P \quad \ddot{\theta}_R]^T$, the state equation and the output equation are expressed as follows:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{Ax} + \boldsymbol{Bu} + \boldsymbol{Ed} \\ \boldsymbol{y} &= \boldsymbol{Cx} \end{aligned} \tag{3}$$

COTROLLER DESIGHN

Since the subject of this paper is to discuss how to allocate 3-DOF control forces to four actuators, an appropriate control theory can be used to design the active vibration isolation system. As an example, we use the H_∞ control theory in this paper.

Table 1 Parameters for simulation

Weight [kg]	m=3.9
Length [m]	L_{bF}= L_{bR}=0.182 L_{tL}= L_{tR}=0.072
Spring Constant [N/m]	k_{FL}=k_{FR}=3.7×10^3 k_{RL}=k_{RR}=3.7×10^3
Damping Coefficient [Ns/m]	c_{FL}=c_{FR}=5.0 c_{RL}=c_{RR}=5.0
Moment of Inertia [kgm^2]	J_P=0.11 J_R=0.02

Table 1 indicates parameters correspond to the experimental apparatus. For simulations, the harmonic disturbance that the amplitude is 0.25mm and differences in the phase are 45° for d_{FR}, 90° for d_{RL} and 135° for d_{RR} against d_{FL} is applied to each support.

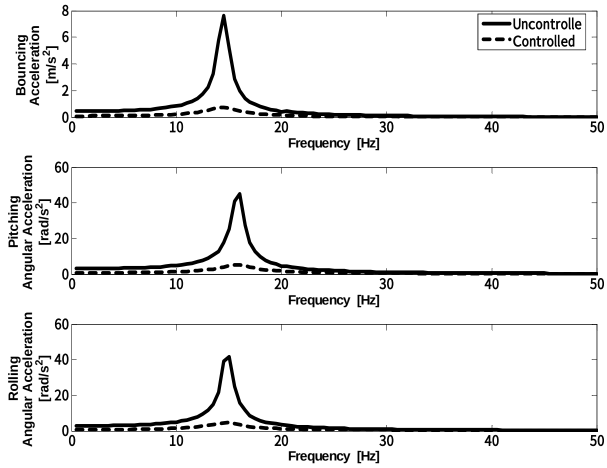

Fig.2 Acceleration responses of acceleration and angular acceleration

Figure 2 shows the bouncing acceleration, and the pitching and rolling angular accelerations with and without H_∞ control for parameters in Table 1.

ALLOCATION OF CONTROL FORCE AND MOMENTS TO ACTUATORS

The 3-DOF control force and moments are derived as the bouncing force and the pitching and rolling moments for the centre of gravity of the loading platform by applying H_∞ control theory to the state equation and the output one in Equation (3). An actuator to supply vibration control force is set in each supporting part at four corners of the loading platform as shown in Figure 1.

The allocation of 3-DOF control force and moments to four actuators is not unique because of redundancy of actuators to the control force and moments.

In this chapter, as an example, we investigate the possibility whether 3-DOF control force and moments can be uniformly allocated to the output of each actuator or not.

Application of Pseudo-Inverse Matrix

From the geometrical condition of four actuators set at four corners of the loading platform, the output vector of the actuator $\boldsymbol{u_{ac}}$ is related to the control force vector $\boldsymbol{u}$ by the following equation.

$$\boldsymbol{u} = \begin{pmatrix} u_G \\ u_P \\ u_R \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -L_{bF} & -L_{bF} & L_{bR} & L_{bR} \\ -L_{tL} & L_{tR} & -L_{tL} & L_{tR} \end{pmatrix} \begin{pmatrix} u_{FL} \\ u_{FR} \\ u_{RL} \\ u_{RR} \end{pmatrix} = \boldsymbol{G_{eq} u_{ac}} \tag{4}$$

The output vector $\boldsymbol{u}_{ac}$ seems to be derived from the control force vector $\boldsymbol{u}$ when both sides of Equation (4) are premultiplied by the inverse matrix $\boldsymbol{G}_{eq}^{-1}$ of the translation matrix $\boldsymbol{G}_{eq}$, but the inverse matrix $\boldsymbol{G}_{eq}^{-1}$ cannot be defined because the matrix $\boldsymbol{G}_{eq}$ is a non-square matrix of (3,4)-type. Defining a square translation matrix instead of the inverse matrix $\boldsymbol{G}_{eq}^{-1}$ as the pseudo-inverse matrix $\boldsymbol{G}_{eq}^{*}$, the vector $\boldsymbol{u}_{ac}$ can be transformed from the vector $\boldsymbol{u}$ as follows:

$$\boldsymbol{u}_{ac} = \boldsymbol{G}_{eq}{}^{*}\boldsymbol{u} = \left(\boldsymbol{G}_{eq}{}^{T}\boldsymbol{G}_{eq}\right)^{-1}\boldsymbol{G}_{eq}\boldsymbol{u} \qquad (5)$$

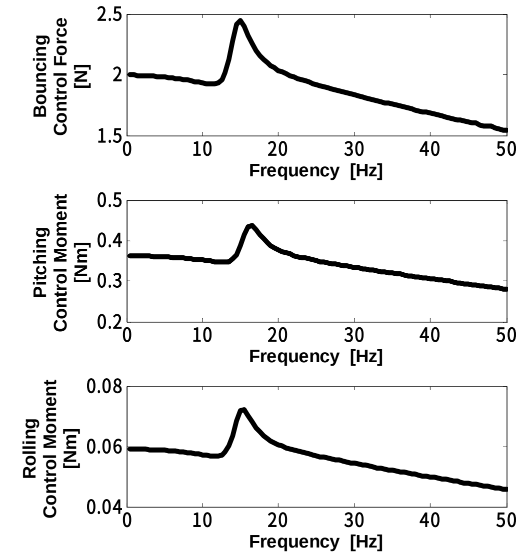

Fig.3 Control force and moments for centre of gravity

The control force and moments for the centre of gravity obtained by numerical simulation is as shown in Figure 3. Figure 4 shows the output force of each actuator allocated from the control force and moments as shown in Figure 3 by Equation (5). From Figure 4, both output forces $\boldsymbol{u}_{FL}$ and $\boldsymbol{u}_{RR}$ are larger than the others, and then output forces transformed by the pseudo-inverse matrix $\boldsymbol{G}_{eq}^{*}$ do not become the similar values.

A set of output forces in Figure 4 is made to a standard set hereafter and termed by $\boldsymbol{P}_{inv}$.

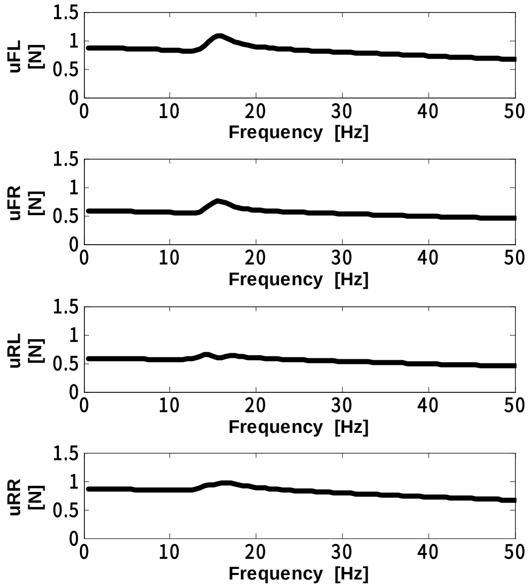

Fig.4 Actuator outputs calculated by pseudo inverse matrix

Application of Generalized Inverse Matrix

There are many equivalent allocations from 3-DOF control force and moments to four actuators because of redundancy. In this section, the transformation is represented by the generalized inverse matrix.

We consider the case that the control force vector $\boldsymbol{u}_{ac}$ is translated by the generalized inverse matrix $\boldsymbol{G}_{eq}{}^{*} + \boldsymbol{z}_{m} - \boldsymbol{G}_{eq}{}^{*}\boldsymbol{G}_{eq}\boldsymbol{z}_{m}\boldsymbol{G}_{eq}\boldsymbol{G}_{eq}{}^{*}$, which a zero space matrix is added to the pseudo-inverse matrix to give generality to the transformation matrix (Yanai), as follows:

$$\boldsymbol{u}_{ac} = \left(\boldsymbol{G}_{eq}{}^{*} + \boldsymbol{z}_{m} - \boldsymbol{G}_{eq}{}^{*}\boldsymbol{G}_{eq}\boldsymbol{z}_{m}\boldsymbol{G}_{eq}\boldsymbol{G}_{eq}{}^{*}\right)\boldsymbol{u} \tag{6}$$

where $\boldsymbol{z}_m$ is called the zero space matrix that is a non-square matrix of (4,3) type. The properties of the generalized inverse matrix (6) depend on the zero space matrix that can be given arbitrarily. Since the pseudo-inverse matrix $\boldsymbol{G}_{eq}^{*}$ has the relation $\boldsymbol{G}_{eq}{}^{*}\boldsymbol{G}_{eq} = \boldsymbol{I}$, Equation (6) is rewritten as

$$\boldsymbol{u}_{ac} = \left\{\boldsymbol{G}_{eq}{}^{*} + \left(\boldsymbol{I} - \boldsymbol{G}_{eq}{}^{*}\boldsymbol{G}_{eq}\right)\boldsymbol{z}_{m}\right\}\boldsymbol{u} \tag{7}$$

It is obvious from Equation (7) that the zero space matrix $\boldsymbol{z}_m$ acts multiplicatively on the vector $\boldsymbol{u}$. For the same parameters as Figure 3, Equation (7) is expressed as follows:

$$\boldsymbol{u}_{ac} = \left\{\boldsymbol{G}_{eq}{}^{*} + 0.25\begin{pmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \\ z_{41} & z_{42} & z_{43} \end{pmatrix}\right\}\boldsymbol{u}$$

$$= \left\{\boldsymbol{G}_{eq}{}^{*} + 0.25\begin{pmatrix} Z_1 & Z_2 & Z_3 \\ -Z_1 & -Z_2 & -Z_3 \\ -Z_1 & -Z_2 & -Z_3 \\ Z_1 & Z_2 & Z_3 \end{pmatrix}\right\}\boldsymbol{u} = \boldsymbol{G}_{eq}{}^{*}\boldsymbol{u} + 0.25\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}(\mathrm{Z}_1 \quad \mathrm{Z}_2 \quad \mathrm{Z}_3)\boldsymbol{u} \tag{8}$$

where

$$Z_1 = z_{11} - z_{21} - z_{31} + z_{41}$$
$$Z_2 = z_{12} - z_{22} - z_{32} + z_{42}$$
$$Z_3 = z_{13} - z_{23} - z_{33} + z_{43}$$

The translation of Equation (8) maintains the same mean even if elements of $\boldsymbol{z}_m$ except one row elements are put zero, because elements Z_1, Z_2 and Z_3 depend on elements of first, second and third rank of $\boldsymbol{z}_m$, respectively.

Figure 5 shows frequency responses of the maximum output for each actuator obtained by Equation (8) for parameters as $Z_1 = \pm 0.4$, $Z_2 = 0$ and $Z_3 = 0$. For $Z_1 = -0.4$,

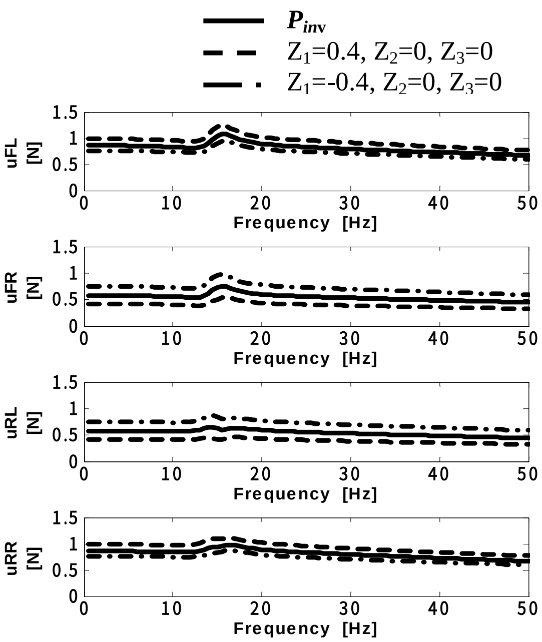

Fig.5 Actuator outputs calculated by general inverse matrix

the frequency response curve of each actuator shows the similar output in Figure 5. The output of each actuator can be adjusted to a certain value by using the generalized inverse matrix in Equation (6), but elements of one row of $\boldsymbol{z}_m$ should be obtained by trials and errors in order to make the output of each actuator the similar value.

Application of Dynamic Equilibrium

From dynamic equilibrium among the output of each actuator, and the control force and moments for the centre of gravity, the following relation is derived.

$$\begin{aligned} u_G &= u_{FL} + u_{FR} + u_{RL} + u_{RR} \\ u_P &= -(u_{FL} + u_{FR})L_{bF} + (u_{RL} + u_{RR})L_{bR} \\ u_R &= -(u_{FL} + u_{RL})L_{tL} + (u_{FR} + u_{RR})L_{tR} \end{aligned} \qquad (9)$$

where the control force u_G and control moments u_P and u_R are known, and the displacements between the centre of gravity and each actuator are also known. Equation (9) cannot be solved, because there are three relations but unknown parameters are four. If the output of one actuator, for example u_{FL}, can be obtained by some means, the following simultaneous equations are set up.

$$\boldsymbol{u}_{ac} = \begin{pmatrix} u_{FL} \\ u_{FR} \\ u_{RL} \\ u_{RR} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & \dfrac{L_{bR}}{L_{bF} + L_{bR}} & -\dfrac{1}{L_{bF} + L_{bR}} & 0 \\ -1 & \dfrac{L_{tR}}{L_{tL} + L_{tR}} & 0 & -\dfrac{1}{L_{tL} + L_{tR}} \\ 1 & 1 - \dfrac{L_{bR}}{L_{bF} + L_{bR}} - \dfrac{L_{tR}}{L_{tL} + L_{tR}} & \dfrac{1}{L_{bF} + L_{bR}} & \dfrac{1}{L_{tL} + L_{tR}} \end{pmatrix} \begin{pmatrix} u_{FL} \\ u_G \\ u_P \\ u_R \end{pmatrix} \qquad (10)$$

Let us consider an example that the magnitude of the actuator output u_{FL}, which is transformed from the control force vector $\boldsymbol{u}$ by using the pseudo-inverse matrix, is arranged in a desired magnitude, and substituted instead of u_{FL} in the right hand side of Equation (10). Figure 6 shows frequency responses of the maximum output for each actuator obtained by Equation (10) for parameters 0, $0.8u_{FL}$ and $1.2u_{FL}$ instead of u_{FL}. It is obvious from Figure 6 that the frequency response curve of each actuator but u_{RR} shows the similar degree of output. As a result, the output of each actuator can be adjusted to the given output of each actuator by the allocation used the dynamic equilibrium of the system.

SUMMARY

Concerning to the allocation of 3-DOF control forces to four actuators of the hybrid vibration isolation system proposed in this paper, the major conclusions obtained can be summarized as follows:

The control force and moments for the centre of gravity, that is, the bouncing force and the pitching and rolling moments can be allocated to the output force of each actuator by the pseudo-inverse matrix. The output of each actuator is given unconditionally and cannot be adjusted to the similar values for this allocation.

By using the generalized inverse matrix, the output of each actuator is adjustable to a certain value, but trials and errors have to be performed to make the output of each actuator the given value.

If the output of one actuator can be obtained by some means, the simultaneous equations are obtained from the dynamic equilibrium of the system. The control force and moments can be allocated to the output force of each actuator by solving the simultaneous equations of the dynamic equilibrium, and the output of each actuator can be adjusted to the given output of each actuator.

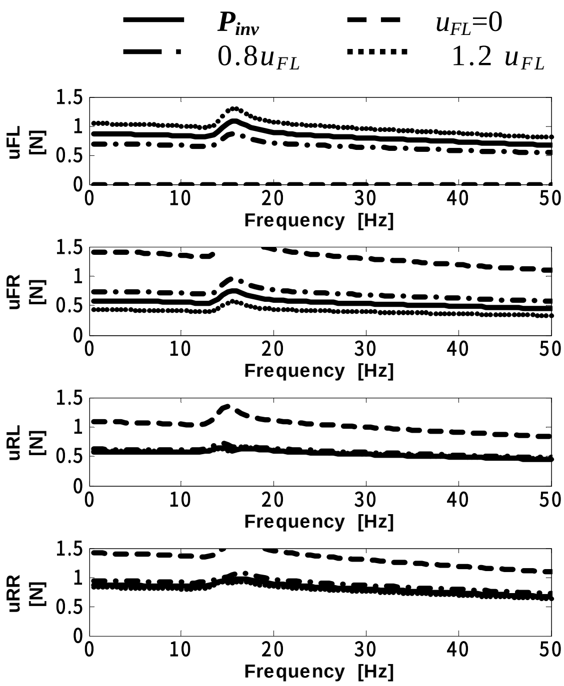

Fig.6 Actuator outputs calculated by dynamic equilibrium

REFERENCES

K. Mizutani, Y. Fujita and R. Ikeura, "On an Improvement of Hybrid Vibration Isolation System", Proc. of DETC'97: 1997 ASME Design Engineering Technical Conferences, CD-ROM #VIB-3805, pp.1-6, (1997)

J. C. Doyle, K. Glover, P. P. Khargonekar and B. A. Francis, "State-Space Solutions to Standard H_2 and H_∞ Control Problems", IEEE Trans., **AC-32**, 8, pp.831-847 (1989)

J. E. Shannan and M. J. Vanderploeg, "A Vehicle Handling Model With Active Suspensions", J. of Mechanisms, Transmissions, and Automation in Design, Trans. ASME, **111**, pp.375-381 (1989)

H. Yanai and K. Takeuchi, *Projection Matrix, Generalized Inverse Matrix, and Singular Value Decomposition* (in Japanese). (Tokyo University Press, Tokyo, 1997)