ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

LATERAL VIBRATION OF ROTOR-BEARING SYSTEM WITH FLEXIBLE COUPLING AND PARALLEL OFFSET

Chao-Yang Tsai and Shyh-Chin Huang*

Department of Mechanical Engineering, National Taiwan Univ. of Sci. and Tech.,
43, Kee-Lung Rd, Sec.4, Taipei, Taiwan 106.
schuang@mail.ntust.edu.tw

Abstract

A rotor system composed of a flexible shaft, unbalanced disks, elastic supports and a coupling of parallel offset/misalignment was investigated. The authors first derived the transfer matrix method (TMM) for a rotating shaft and discovered that its boundary shears were mutual coupling and time dependent in two perpendicular directions due to rotation. The coupling shears affected the shaft's critical speeds up to 50%. The transfer matrix of a flexible coupling with parallel offset was then derived and its effects on critical speeds and whirling response were particularly focused on. Numerical results showed that the flexibility of a coupling significantly influenced the rotor-bearing critical speeds. The parallel offset, yet, acted as an excitation force similar to an unbalance except it affected through the whole driven part rather than a single point excitation. The coexistence of disk unbalance and coupling offset revealed that the offset caused more significant effects at rational speeds but the unbalance increased its weight with the rotation speed due to the centrifugal force. Response amplitude and whirl orbits across the offset were illustrated and it was discovered that in certain rotation range the shaft whirled asynchronously across the offset.

INTRODUCTION

Approaches to dynamic analysis of rotor systems can be basically divided into two main streams. The first one is the FEM [1-2], and the second one is the TMM [3].As to the existing literature related to TMM, Prohl [4] employed it for the dynamic analysis of rotor systems. Lund and Orcutt [5] established the transfer matrix of a shaft in a continuous concept but neglected both rotary inertia and gyroscopic effect. Chao and Huang [6] introduced a modified transfer matrix extended from Myklestad's transfer matrix but employed the Euler beam and rigid disk as fundamental elements and obtained better natural frequencies and shapes than those of discrete model. Many researchers [7-8] continuously added efforts into TMM such as developing oil-film bearing matrix, including rotary inertia, gyroscopic effects of disks and many others.

Though TMM has been extensively applied for rotor analysis, to the authors' knowledge, none of them included the commonly seen case of coupling offset. Dewell and Mitchell [9] experimentally studied parallel and angular offset of a metallic-disk-type coupling. They used real time analyzer and verified that frequencies of n× speed appeared due to offset. They suggested the 2× and 4× components be used for misalignment diagnosis. Xu and Marangon adopted a universal joint for misalignment and employed the component mode synthesis to analytically [10] studied and experimentally [11] validated the calculations. They concluded that the unbalance and misalignment could be characterized by 1× and 2× components, respectively. Lee and Lee [12] employed FEM for misaligned rotor-bearing system. In their studies, angular, parallel, and combined effects but no coupling were discussed with extensively shown whirling orbits. Al-Hussain and Redmond [13] analytically derived the equations of two Jeffcott rotors with rigid coupling of parallel offset. In their conclusions, they did not obtain the 2× component as predicted by the others.

The authors here derive the lateral transfer matrix for rotor-bearing system with flexible coupling and parallel offset. Through the derivation of a rotating shaft, the authors discover that the shaft's boundary shears are time-dependent and coupled in two perpendicular directions. That was, to the authors' knowledge, neither described nor noticed in the existing literature. Numerical results enhanced that these coupling shears could drastically reduce the shaft's critical speeds up to 50% at high rotations. The coupling stiffness is found to affect the rotor's critical speeds and the offset acts as an excitation similar to an unbalance but influences through the whole driven shaft. The whirling orbits are investigated as well. The results showed that the two ends of a misalignment may whirl asynchronously as rotation falls into some regions.

EFFECT OF BOUNDARY COUPLING SHEARS OF ROTATING SHAFTS

Figure 1 shows the fundamental elements in TMM, in which there are shafts, disks, and bearings. The new one is the parallel offset. First, the equations of motion and the boundary equations of a rotating shaft is derived to be

$$\begin{cases} EI_{zz}v'''' + \rho[A\ddot{v} - I_{zz}\ddot{v}'' - \Omega(I_{yy}+I_{zz})\dot{w}'' + \Omega^2 I_{yy}v''] = 0 \\ EI_{yy}w'''' + \rho[A\ddot{w} - I_{yy}\ddot{w}'' + \Omega(I_{yy}+I_{zz})\dot{v}'' + \Omega^2 I_{zz}w''] = 0 \end{cases} \tag{1}$$

$$\begin{cases} \left\{V_Y + [EI_{zz}v''' \underline{- \rho I_{zz}\ddot{v}' - \rho\Omega(I_{yy}+I_{zz})\dot{w}' + \rho\Omega^2 I_{yy}v'}]\right\}\delta v = 0\,,\ \left\{M_Y + EI_{yy}v''\right\}\delta v' = 0 \\ \left\{V_Z + [EI_{yy}w''' \underline{- \rho I_{yy}\ddot{w}' + \rho\Omega(I_{yy}+I_{zz})\dot{v}' + \rho\Omega^2 I_{ZZ}w'}]\right\}\delta w = 0\,,\ \left\{M_Z - EI_{zz}w''\right\}\delta w' = 0 \end{cases} \tag{2}$$

Note that Eq. (2) reveals a very important phenomenon that boundary shears couple with the time derivatives of displacements in Y-Z directions, as underlined, due to rotation. Unlike a non-rotating shaft, in which the boundary shears, said V_y and V_z, are uncoupled and time independent. The coupling terms could be significant at very high rotational speed as to be shown. Solve the boundary value problem of eqs.(1,2) and draw its frequency loci on Figure 2, which illustrates the coupling shears effects on the shaft's natural frequencies with the rotational speed, where $\omega_n^* = \omega_n / \omega_0$, $\Omega^* = \Omega / \omega_0$ and ω_n is the rotating shaft's nth natural frequency and ω_0 is shaft's first flexural natural frequency at $\Omega = 0$. It is noticed that the exact (coupled) solutions have lower natural frequencies. That means the shaft's critical speeds are overestimated if the coupling effects were ignored, e.g., around 50% higher for free-free case.

TRANSFER MATRIX OF AN OFFSET MISALIGNMENT AND TOTAL TRANSFER MATRIX

There are two types of coupling missalignment, one is of parallel offset, and the other is of angular offset. In the present studies a flexible coupling is modeled as a translational spring combined with a bending spring. In between two ends of the coupling, there is a parallel offset as shown in Figure 3. According to the equilibrium relations, the authors derive the misalignment transfer matrix in the following

$$v^R = v^L + \frac{V_y}{K_L} + e \cdot \sin(\Omega t + \gamma_e) \quad , \quad \phi^R = \phi^L - \frac{1}{K_B} M_z^L \tag{3}$$

$$w^R = w^L - \frac{V_z}{K_L} - e \cdot \cos(\Omega t + \gamma_e) \quad , \quad \theta^R = \theta^L - \frac{1}{K_B} \cdot M_Y^L \tag{4}$$

Neglect coupling inertia so that, $M^R = M^L$, $V^R = V^L$. K_L and K_B are linear and bending stiffness of the coupling $\phi_e = \Omega t + \gamma_e$, γ_e is the phase relative to rotor's reference. Note that the torsional vibration is not considered so that $\phi_e = \Omega t + \gamma_e$ retains all the times. $r = e + \delta_e$ is dynamic offset, δ_e is linear displacement of the spring. The transfer matrix of a parallel offset coupling yields to be

$$\{S\}^R = [M]\{S\}^L + \{C\} \tag{5}$$

$$[M] = \begin{bmatrix} [M_1]_{8\times 8} & [M_2]_{8\times 8} & \{0\}_{8\times 1} \\ [M_3]_{8\times 8} & [M_4]_{8\times 8} & \{0\}_{8\times 1} \\ \{0\}_{1\times 8} & \{0\}_{1\times 8} & 1 \end{bmatrix} \tag{6}$$

$$\{C\}_{17\times 1} = \{\{C_1\}\ \{C_2\}\ 1\}^T \tag{7}$$

$$\begin{cases} \{C_1\} = \{\, e\sin\gamma_e \;\; 0 \;\; 0 \;\; 0 \;\; e\sin\gamma_e \;\; 0 \;\; 0 \;\; 0 \,\} \\ \{C_2\} = \{\, e\cos\gamma_e \;\; 0 \;\; 0 \;\; 0 \;\; -e\cos\gamma_e \;\; 0 \;\; 0 \;\; 0 \,\} \end{cases} \tag{8}$$

Equation (5) is, for the first time derived, the transfer matrix of a coupling with parallel misalignment. $[M]$ [14], similar to the others [15], is a coupling matrix links the left and right states, and $\{C\}$ is the misalignment vector. It will be seen that after multiplication to its right matrices, all components right to the misalignment contribute to the excitation. If there is no misalignment (e=0), $\{C\}$ vanishes and Eq. (5) simply represents a transfer matrix of a coupling. Now, assume a typical misaligned rotor system of a misaligned coupling between the k^{th} and $k+1^{th}$ elements, as Figure 1.The overall transfer matrix containing unbalanced disk and misaligned shaft is derived to be

$$\{S\}_n^R = [T^u]\{S\}_1^L + [T^m]\{C\} \tag{9}$$

where $\{S\}_1^L$ represents the left state of unit 1, $\{S\}_n^R$ is the right state of unit n, and

$$\begin{cases} [T^u] = [T]_n\,[T]_{n-1}\cdots\cdots[T]_{k+1}[M][T]_k[T]_{k-1}\cdots\cdots[T]_2[T]_1 \\ [T^m] = [T]_n\,[T]_{n-1}[T]_{n-2}\cdots\cdots[T]_{k+1} \end{cases} \tag{10}$$

Note that $[T]_i$ denotes the i^{th} element's transfer matrix, a shaft, a bearing, or a disk. $[T^u]$ is the overall transfer matrix yielded by the multiplication of all transfer matrices and $[T^m]$ is the multiplication of the transfer matrices to the right of misalignment, *i.e.*, from $k+1^{th}$ to the n^{th}. Substituting the boundary conditions, a 9 × 9 matrix yields

$$\begin{Bmatrix} \{0\}_{8\times1} \\ 1 \end{Bmatrix} = \begin{bmatrix} [T^u]'_{8\times8} & \{u\}_{8\times1} \\ \{0\}_{1\times8} & 1 \end{bmatrix} \begin{Bmatrix} \{S'\}_{8\times1}^L \\ 1 \end{Bmatrix} + \begin{Bmatrix} [m]_{8\times4} \\ \{0\}_{1\times4} \end{Bmatrix} \cdot \{C'\}_{4\times1} \tag{11}$$

with

$$\{C'\}_{4\times1} = \{-e\sin\gamma_e, -e\sin\gamma_e, -e\cos\gamma_e, e\cos\gamma_e\}^T \tag{12}$$

Note that elements of $[T^u]'$, $\{u\}$ and $[m]$ are the degeneration of $[T^u]$ and $[T^m]$ matrices [15]. Simplify Eq. (11) and rearrange it , one can write it as

$$\begin{bmatrix} [T^u]'_{8\times8} & \{u\}_{8\times1} \\ \{0\}_{1\times8} & 1 \end{bmatrix} \begin{Bmatrix} \{S'\}_{8\times1}^L \\ 1 \end{Bmatrix} = \begin{Bmatrix} -[m]_{8\times4}\cdot\{C'\}_{4\times1} \\ 1 \end{Bmatrix} \tag{13}$$

or

$$[T^u]'_{8\times8} \times \{S'\}_{8\times1}^L = -\{u\}_{8\times1} - [m]_{8\times4}\cdot\{C'\}_{4\times1} \tag{14}$$

There are two effects in the Eq. (14). On the right side, the first term is the unbalanced excitation and the second term is the misalignment excitation. Provided the misalignment is zero (e = 0), Eq. (14) yields an unbalanced response analysis. If the coupling stiffness K_B and K_L approach infinite, $[M]$ matrix becomes an identity matrix, representing a rigid coupling.

RESULTS, DISCUSSION AND CONCLUSIONS

The rotor system consists of three bearings, four rigid disks, and seven-section flexible shaft as Figure 4 shows. The three bearings are assumed of the same constants in Y and Z direction. Figure 5 shows the FRF of the rotor with three different offsets. Refer to Figure 5 and Eqs. (8,9) it is realized that the coupling stiffness affects the rotor's critical speeds but the offset acts just like an excitation. As seen, resonance occurs at the same critical speeds with no offset and FRF amplitude is proportional to the offset.

The mutual effects of shaft offset and disk unbalance are calculated and as shown in Figure 6. From the curves, it is seen the offset predominates rather than unbalance. Figure 7 shows the whirling orbits of the system with merely shaft offset (solid) and combine effects of unbalance and offset (dashed). Three plots correspond to (a) $\Omega < \Omega_{cr1}$ (b) $\Omega_{cr1} < \Omega < \Omega_{cr2}$ (c) $\Omega_{cr2} < \Omega < \Omega_{cr3}$. It is seen that at lower rotation speed, shaft offset predominates. The unbalance only slightly changes the orbit orientation. With the increase of rotational speeds, due to centrifugal force generated by disk unbalance, the orbit change significantly in both orientation and magnitude. It is further noted that in Figure 7(c), the two ends of the coupling whirl in opposite directions.

The present research derived the transfer matrix for flexible coupling with parallel offset. The investigation reveals that coupling stiffness affects the critical speeds and the offset plays as an excitation. During the derivation of a rotating shaft transfer matrix, the authors found that the shaft's boundary shears in Y-Z directions coupled together due to rotation. The coupling shears affected the most as the shaft was free at both ends. It could reduce the shaft's first critical speed up to 50%.The derivation of TMM and numerical results revealed that misalignment induced lateral response of the same frequency as rotational speed (1×) and that was unlike most of the researches where multiple integers (n×**)** components were found. We believe that the reason of n× components disappearing in our derivation is due to the coupling's torsional vibration was not considered. The coupling will transmit torque as well and if the torsional flexibility of the coupling is taken into account the driven shaft will fluctuate and it causes non-constant rotation. The non-constant speed in conjunction with the misalignment and unbalance will consequently generate n× frequencies of cyclic forces and moments on lateral vibration.

The combined effects of disk unbalance and shaft misalignment showed that shaft misalignment imposed much greater effect than the disk unbalance at most of rotational speeds. That means the shaft misalignment usually plays a dominating role. The effect of disk unbalance will be of the same significance as the rotor is at very high rotational speeds.

ACKNOWLEDGMENT

The authors are grateful to the NSC of Taiwan for its support under the grant NSC94-2218-E-011-014.

REFERENCES

[1] Nelson H.D. and McVaugh J.M., "The dynamics of rotor bearing systems using finite elements", ASME J. Eng. for Industry, 593-600(1976)

[2] Özgüven H.N., "On the critical speed of continuous shaft-disk systems", ASME J. Vibr. Acoust., Stress, and Reliability in Design,106, 59-61(1984)

[3] Gu J., "An improved transfer matrix-direct integration method for rotor dynamics", ASME J. Vibr., Acoust., Stress, and Reliability in Design, 108, 182-188(1986)

[4] Prohl M.A., "A general method for critical speeds of flexible rotors", J. Applied Mechanics, 67, 142-148(1945)

[5] Lund J.W. and Orcutt F.K., "Calculations and experiments on the unbalance response of a flexible rotor", ASME J. Eng. for Industry, 89, 785-796(1967)

[6] Chao S.W. and Huang S.C., "On the flexural vibrations of shaft-disk systems using a modified transfer matrix method", Proceedings of the 6th National Conference of the CSME, 1607-1618 (1989)

[7] Rieger N.F. and Zhou S., "Development and verification of transfer matrix unbalance response procedure for three-level rotor-foundation systems", ASME J. Vibr. Acoust.,120(1), 240-251 (1998)

[8] Zu J.W. and Ji Z., "An improved transfer matrix method for steady-state analysis of nonlinear rotor bearing systems", ASME J. Eng. Gas Turbines Power, 124(1), 303-310(2002)

[9] Dewell D.L. and Mitchell L.D., "Detection of a misaligned disk coupling using spectrum analysis", ASME J. Vibr., Acoustics, Stress, and Reliability in Design, 106, 9-16(1984)

[10] Xu M. and Marangoni R.D., "Vibration analysis of a motor-flexible coupling-rotor system subject to misalignment and unbalance, Part Ⅰ: Theoretical model and Analysis", J. Sound Vib., 185(3), 663-679(1994)

[11] Xu M. and Marangoni R.D., "Vibration analysis of a motor-flexible coupling-rotor system subject to misalignment and unbalance, Part II: Experimental Validation", J. Sound Vib.,176(8), 681-691(1994)

[12] Lee Y.S. and Lee C.W., "Modeling and vibration analysis of misaligned rotor-bearing system", J. Sound Vib.,224(1) , 17-32(1999)

[13] Al-Hussain K.M.and Redmond I., "Dynamic response of two rotors connected by rigid mechanical coupling with parallel misalignment", J. Sound Vib., 249(2) , 483-498(2002)

[14] Tsai C.Y., "Transfer matrix method to misalign vibration analysis in rotor systems", Master Thesis, Department of Mechanical Engineering, National Taiwan University of Science and Technology (1997)

[15] Rao J.S., *Rotor Dynamics*. Wiley Eastern Limited (1983)

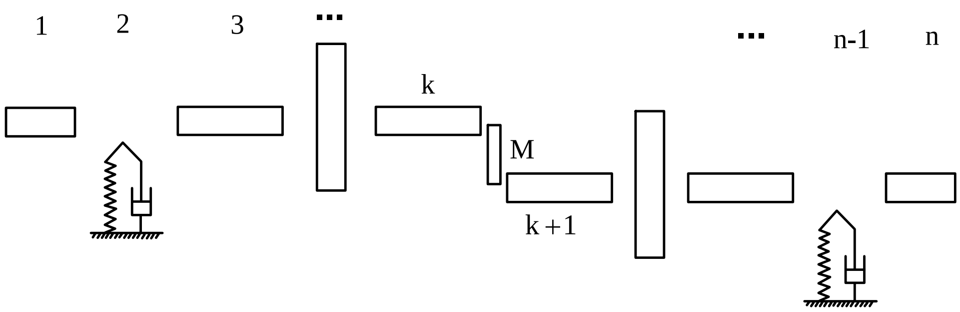


Figure 1 - Schematic diagram of a misaligned rotor in transfer matrix method

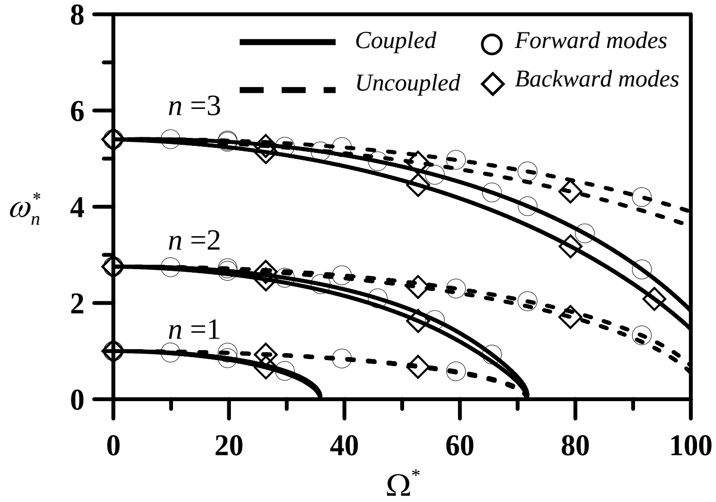


Figure 2 – Natural frequencies of a rotating shaft with free-free boundary

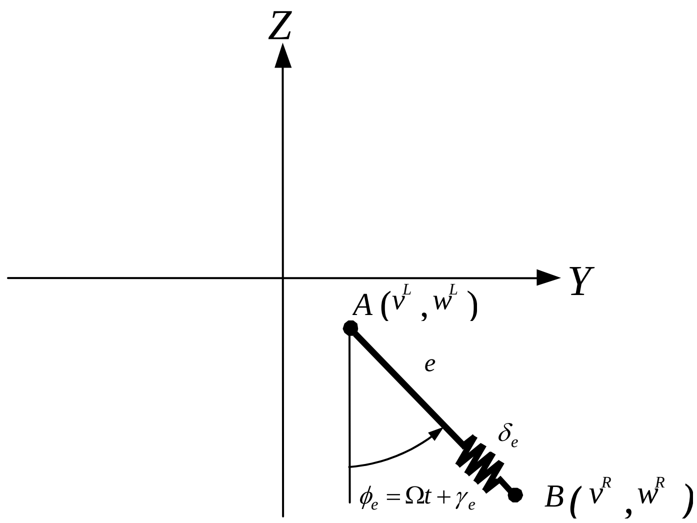


Figure 3 - Displacement of misalignment

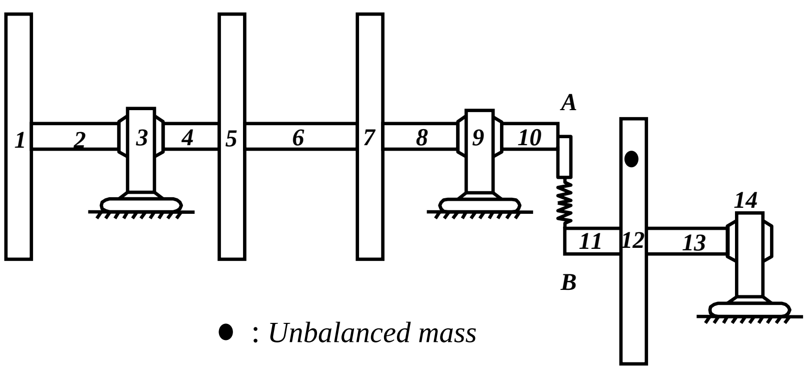


Figure 4 - Example of a misaligned

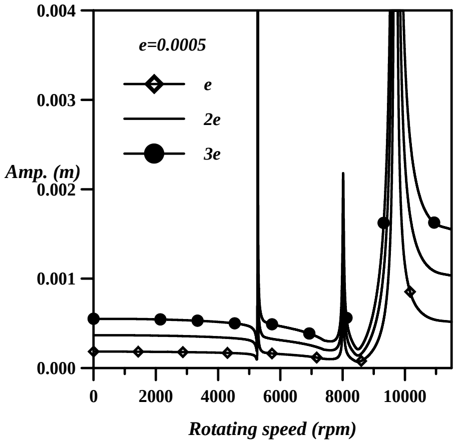


Figure 5 - FRF due to misalignment

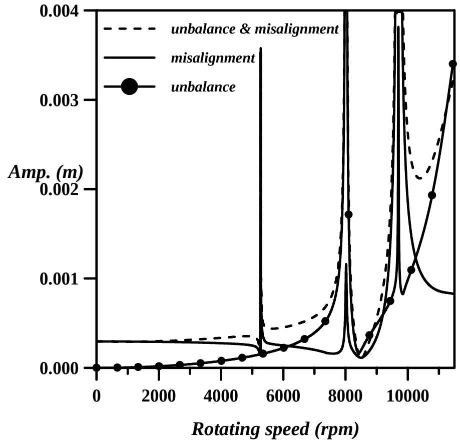
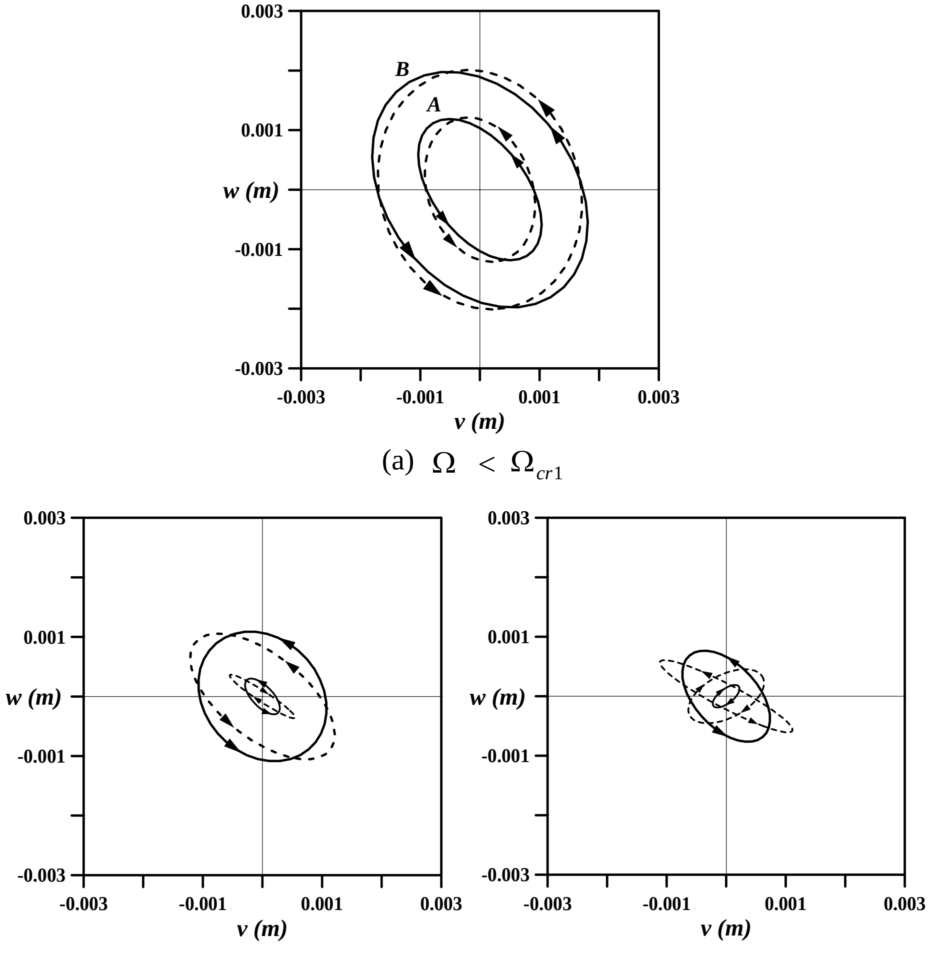


Figure 6 - FRF due to misalignment and unbalance



(a) $\Omega < \Omega_{cr1}$

(b) $\Omega_{cr1} < \Omega < \Omega_{cr2}$ (c) $\Omega_{cr2} < \Omega < \Omega_{cr3}$

Figure 7 - Whirling orbits at different rotational speed due to mutual effect of shaft misalignment and disk unbalance

Misalignment only *Misalignment & Unbalance*