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QUANTUM ACOUSTIC IMAGING

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Abstract
In a previous paper we propose quantum acoustical imaging based on the quantum effect of phonons entanglement. It reduces the resolution limit to λ/N where λ is the sound wavelength and N is the number of phonons involved in the entanglement. In this paper we use a different approach by harnessing on the property that entanglement is highly sensitive to a slight change in the hamiltonian of the system which is attenuation in our case. We derive the perturbation Hamiltonian density by taking account of the two phonons entanglement. The intensity of the propagated ultrasonic wave in the solid is derived in terms of attenuation. Its sensitivity dependence on the attenuation increases at an order of amplitude in proportion to the number of phonons involved in entanglement. A quantum acoustical imaging system is proposed.

INTRODUCTION

In a previous paper [1] we propose quantum acoustical imaging based on the quantum effect of phonons entanglement. It reduces the resolution limit to λ/N where λ is the sound wavelength and N is the number of phonons involved in the entanglement. In this paper we use a different approach by harnessing on the extremely high sensitivity of entanglement to a slight change in the hamiltonian of the system which is attenuation in our case.

SENSITIVITIES OF QUANTUM EFFECTS

The followings are some examples of the extremely high sensitivities of quantum effects:

1. Tunnelling Current

The quantum mechanically induced tunnelling current in solid state physics has extremely high sensitive dependence on the distance travelled. This sensitivity has been harnessed in the design of the scanning tunnelling microscope (STM) which produces images of sample surface up to atomic resolution.

2. Quantum Gyroscope

   Entanglement between the quanta employed in the quantum gyroscope enhances the accuracy. Here the atoms are entangled with each other. In this case, we insert into the two input ports of the interferometer, the Fock state of N particles in the $j = 1, 2$ input mode, corresponding to two atoms entanglement. Such a quantum gyroscope ought to be about 10^8 times more sensitive to rotation than the standard one. Entanglement has already led to an improvement in quantum clock synchronization.

3. Entanglement in Quantum Computer

   The pairwise entanglement present in a quantum computer can be simulated as a dynamically localized system. Here the entanglement is exponentially sensitive to changes in the Hamiltonian of the simulated system. Moreover, the entanglement is exponentially sensitive to the logic position of the qubits chosen.

HARNESSING THE SENSITIVITY OF THE RESPONSE OF ATTENUATION IN A PHONONS ENTANGLEMENT SYSTEM

We will begin with a quantum mechanical treatment of ultrasonic propagation in solids. Many authors [2]-[5] have considered the absorption of ultrasonic waves in an ideal crystal as a result of the sound quanta interacting via the anharmonic terms of the Hamiltonian with the lattice vibration quanta. In this paper, we will start with the consideration of three-phonon interactions and extend it to the case of two phonons in entanglement. With the advent of experimental techniques, it is now possible to study three-phonon interactions in detail. This is done by experimentally generating two noncollinear beams of ultrasonic phonons and by standard experimental procedures, detecting the phonon beam created by the interaction of the initial phonon beams [6].

Following Slonimskii [2]'s approach, the deformation of a solid under stress is described by the components $\omega_{\alpha,\beta}$ of the deformation tensor:

$$\omega_{\alpha,\beta} = \frac{1}{2}\left(U_{\alpha,\beta} + U_{\beta,\alpha} + U_{\gamma,\alpha}U_{\gamma,\beta}\right) \tag{1}$$

$$U_{\alpha,\beta} = \frac{\partial U_\alpha}{\partial x_\beta} \tag{2}$$

where U_α is the displacement of a point in the x_α direction, and the Einsteinian notation is used, i.e. repeated indices denote a summation over those indices. In terms of the deformation tensor, the elastic energy density $\mathcal{H}$ of an isotropic solid is written as [7]

$$\mathcal{H} = \mu\omega_{\alpha,\beta}^2 + (\frac{1}{2}K - \frac{1}{3}\mu)\omega_{\alpha,\alpha}^2 + \frac{1}{3}A\omega_{\alpha,\beta}\omega_{\beta,\gamma}\omega_{\gamma,\alpha} + B\omega_{\alpha,\beta}^2\omega_{\gamma,\gamma} + \frac{1}{2}CW_{\alpha,\alpha}^3 \tag{3}$$

where μ is the modulus of rigidity, K is the modulus of compression and A, B, C are the third-order elastic constants.

Define two new tensors:

$$\begin{aligned} U_{\alpha,\beta} &= \frac{1}{2}(U_{\alpha,\beta} + U_{\beta,\alpha}) \\ V_{\alpha,\beta} &= \frac{1}{2}(U_{\alpha,\beta} - U_{\beta,\alpha}) \end{aligned} \tag{4}$$

the Hamiltonian density can be written as a sum of two Hamiltonian densities:

$$\mathcal{H}_0 = \mu U_{\alpha,\beta}^2 + (\frac{1}{2}K - \frac{1}{3}\mu)U_{\alpha,\alpha}^2 \tag{5}$$

$$\begin{aligned} \mathcal{H}' = &\frac{1}{3}CU_{\alpha,\alpha}^3 + (B + \frac{1}{2}K - \frac{1}{3}\mu)U_{\alpha,\beta}^2 U_{\gamma,\gamma} + (\frac{1}{3}A + \mu)U_{\alpha,\beta}U_{\beta,\gamma}U_{\gamma,\alpha} \\ &- (\frac{1}{2}K - \frac{1}{3}\mu)U_{\gamma,\gamma}V_{\alpha,\beta}V_{\beta,\alpha} - \mu U_{\alpha,\beta}V_{\beta,\gamma}V_{\gamma,\alpha} \end{aligned} \tag{6}$$

where the symmetry properties of $U_{\alpha,\beta}$ and $V_{\alpha,\beta}$ have been used. Using time-dependent perturbation theory, $\mathcal{H}_0$ is the unperturbed Hamiltonian density, and $\mathcal{H}'$ is the perturbation Hamiltonian density which produces nonzero probabilities for transitions between available phonon states.

In general, the displacement vector $\underline{\mathrm{U}}(\underline{\mathrm{r}})$ is the sum of the displacement vectors associated with each harmonic wave:

$$\underline{\mathrm{U}}(\underline{\mathrm{r}}) = \sum_{n=1}^{3} e_n(a_n e^{iK_n \cdot r} + a_n^* e^{-iK_n \cdot r}) \tag{7}$$

where e_n is a unit vector in the direction of polarization, a_n is the amplitude of the nth phonon, and $\underline{\mathrm{K}}_n$ is the polarization vector. In quantum theory, the amplitudes of Eq. (7) and the annihilation and creation operators of the linear harmonic oscillator whose only nonzero matrix elements are

$$\left\langle N \pm 1 \middle| \begin{pmatrix} a^* \\ a \end{pmatrix} \middle| N \right\rangle = \left[\frac{\hbar}{2m\omega} \begin{pmatrix} N+1 \\ N \end{pmatrix} \right] \frac{1}{2} e^{\pm i\omega t} \tag{8}$$

where N is the initial number of phonons, $\hbar\omega$ is the phonon energy, t is the time, and m is the mass of the volume of interaction V.

The matrix elements of the components $U_{\alpha,\beta}$ and $V_{\alpha,\beta}$ are obtained by differentiating the displacement components:

$$\langle N \pm 1 \left| U_{\alpha,\beta} \right| N \rangle = \mp \frac{i}{2} e^{\mp iK \cdot r}(e_\alpha K_\beta + e_\beta K_\alpha) \left\langle N \pm 1 \middle| \begin{pmatrix} a^* \\ a \end{pmatrix} \middle| N \right\rangle \tag{9}$$

$$\langle N \pm 1 \left| V_{\alpha,\beta} \right| N \rangle = \mp \frac{i}{2} e^{\mp iK \cdot r}(e_\alpha K_\beta - e_\beta K_\alpha) \left\langle N \pm 1 \middle| \begin{pmatrix} a^* \\ a \end{pmatrix} \middle| N \right\rangle \tag{10}$$

Assuming that the initial phonons interact for a sufficiently long time, the transition probability (the rate of occurrence of a process per unit time) P between the initial i and final f states is given by [8]

$$P = \frac{2\pi}{\hbar}\mathcal{H}'^2_{if}\mathcal{D}_f(\epsilon_i) \tag{11}$$

where ϵ_i is the energy of the initial state and $\mathcal{D}_f(\epsilon_i)$ is the density of final states about ϵ_i. The perturbing Hamiltonian $\mathcal{H}'$ is obtained by integrating the perturbing Hamiltonian density over the volume of interaction.

Envisioning a classical elastic wave as being an ideally dense homogeneous beam of phonons, the beam intensity is given by

$$I_n = \frac{1}{2}10^3\rho C_n\omega_n^2 X_n^2 = 10^3\hbar C_n\omega_n\eta_n \tag{12}$$

where C_n is the phonon speed, η_n is the phonon density, and the 10^3 factor is the conversion factor from the mks to the cgs units used in this paper. Eq. (12) is the classical expression where X_n is the displacement amplitude.

Of all the interactions occurring between the η_1 phonons and the η_2 phonons, only a small number, given by the transition probability, will generate η_3 phonons. Each newly created η_3 phonon may be visualized as the centre of a Huygens' spherical wave which interacts with adjacent Huygens waves to produce a diffraction pattern. Integrating the Huygens' spherical waves over all angles, we obtain

$$X_3^2 = \frac{\hbar P}{2\pi r^2\rho\omega_3 C_3} \tag{13}$$

TWO PHONONS ENTANGLEMENT

The two phonons state is given by

$$|\phi\rangle = |\phi_1\rangle|\phi_2\rangle \tag{14}$$

So for two phonons entanglement, the matrix element of $\mathcal{H}'_{if}$ will be given by

$$\langle N \pm 1\left|\mathcal{H}'_{if}\right| N\rangle \cdot \langle N \pm 1\left|\mathcal{H}'_{if}\right| N\rangle = \left[\langle N \pm 1\left|\mathcal{H}'_{if}\right| N\rangle\right]^2 \tag{15}$$

Hence for two phonons entanglement, the beam intensity I_3 will be given by

$$\begin{aligned} I_3 &= \frac{1}{2}10^3\rho C_3\omega_3^2 X_3^2 \\ &= \frac{1}{2}10^3\rho C_3\omega_3^2\frac{\hbar P}{2\pi r^2\omega_3\rho C_3} \\ &= \frac{1}{2}10^3\rho C_3\omega_3^2\frac{\hbar}{2\pi r^2\omega_3\rho C_3}\left(\frac{2\pi}{\hbar}\right)\mathcal{D}_f(\epsilon_i)\cdot\left[\langle N \pm 1\left|\mathcal{H}'_{if}\right| N\rangle\right]^4 \\ &= \frac{1}{2}10^3\omega_3^2\frac{\mathcal{D}_f(\epsilon_i)}{r^2}\cdot\left[\langle N \pm 1\left|\mathcal{H}'_{if}\right| N\rangle\right]^4 \end{aligned} \tag{16}$$

The attenuation is given by the Hamiltanian density which is $\mathcal{H}'_{if}$. So from Eqn. (16) we realize that the intensity's sensitivity dependence on the attenuation is two orders of magnitude higherfor the case of two phonons entanglement and will be three orders of magnitude more sensitive for the case of three phonons entanglement and so on.

DESIGN OF A QUANTUM ACOUSTICAL IMAGING SYSTEM BASED ON THE SENSITIVITY DEPENDENCE ON ATTENUATION

The design will be based on the interference of two nonlinear beams of two entangled phonons. Quantum-entangled phonons have been produced in KT_aO_3 [9]. The experimental setup will follow that of Ref 5 with modification of the ultrasound source to entangled phonons source. Also the experimental system of Ref. 5 has to be extended to meet the requirement of a 2D image scanning system, so that the image will be an attenuation image. The diffraction tomography imaging system will be considered.

CONCLUSION

It is feasible to construct an imaging system which can harness on the high sensitivity of the two-phonons entanglement to the attenuation of ultrasound propagation. A detailed calculation of the perturbed Hamiltonian which yields attenuation with the presence of two phonons entanglement will be our next work.

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