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NUMERICAL SIMULATION OF SOUND WAVE PROPAGATION WITH SOUND ABSORPTION IN TIME DOMAIN

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Abstract

Numerical simulation of the sound wave propagation with the sound absorption in the time domain is discussed in this paper. To implement the sound absorption effect with complex frequency dependency in the time domain such as the relaxation absorption in air or water, a convolver using the digital filter is embedded in the element of the digital Huygens' modeling (DHM) or the transmission line matrix modeling (TLM). Some numerical demonstrations are made for the one-dimensional sound propagation in seawater.

INTRODUCTION

For the accurate numerical analysis for the sound propagation, the introduction of the sound absorption is required in the numerical scheme. There are two main reasons of the sound absorption; one is the viscosity of the medium, and another is the relaxation process of the translational and the rotational molecular motion in the fluid [1]. The absorption coefficient due to the viscosity is in proportion to square of frequency. This type of classical absorption can be easily expressed in the wave equation. The expression of the sound absorption due to the relaxation in the wave equation is however not so easy because the relaxation phenomena show the complex frequency dependency.

There are two methods to implement the frequency dependent characteristics into the numerical scheme in time domain such as FDTD method; one is the method using FFT and inverse FFT in which the waveform is transformed into the frequency domain by FFT then it is transformed into time domain by inverse FFT after the filtering process is performed. Another is the method using a covolution technique in time domain. The method using FFT and inverse FFT is however unsuitable to the time domain scheme because it expends the computation time.

In this paper, the convolution technique using the digital filter is applied to the digital Huygens' modeling (DHM) [2]-[4] or transmission line matrix modeling (TLM) [5], [6] in order to implement the sound absorption with the frequency dependency. DHM is a physical model in which the propagation and the scattering of waves are simulated as the sequences of impulses scattering as Huygens' principle states. The formulation of DHM is very simple because time and space are not explicitly appeared in the formulation due to the synchronization between time and space. The digital equivalent circuit can be simply developed based on the DHM. It is easy to implement the digital filter into DHM [3]. Some numerical demonstrations are made for the one-dimensional sound propagation in seawater.

WAVE EQUATION WITH VELOCITY DISPERSION

The governing equations for the acoustic field with the velocity dispersion are given as follows [7]

$$\frac{\partial p}{\partial t} = -\kappa \nabla \cdot \boldsymbol{u} - \delta_1 \frac{\partial}{\partial t} \nabla \cdot \boldsymbol{u} \tag{1}$$

$$\rho_0 \frac{\partial \boldsymbol{u}}{\partial t} = -\nabla p + \delta_2 \nabla^2 \boldsymbol{u} \tag{2}$$

where p is sound pressure, $\boldsymbol{u}$ is particle velocity vector, ρ_0 is density and κ is bulk modulus, respectively. Equation (1) is the continuity equation and (2) is the equation of motion. δ_1 and δ_2 are respectively given as

$$\delta_1 = \kappa \left(\frac{1}{c_v} - \frac{1}{c_p} \right) \tag{3}$$

$$\delta_2 = \zeta + \frac{4}{3}\eta \tag{4}$$

where c_v and c_p are specific heat at constant volume and one at constant pressure, respectively, ζ is bulk viscosity and η is shear viscosity, respectively. Eliminating the particle velocity from equations (1) and (2), the wave equation for the sound pressure is derived as

$$\left(1 + \frac{\delta_1 + \delta_2}{\rho_0 c_0^2} \frac{\partial}{\partial t} \right) \nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = 0 \tag{5}$$

where $c_0 = \sqrt{\kappa/\rho}$ is sound velocity.

Under the assumption of the steady state sound propagation, the wave equation (5) is transformed as

$$\left(1 + j\omega \frac{\delta}{\rho_0 c_0^2} \right) \nabla^2 P + k^2 P = 0 \tag{6}$$

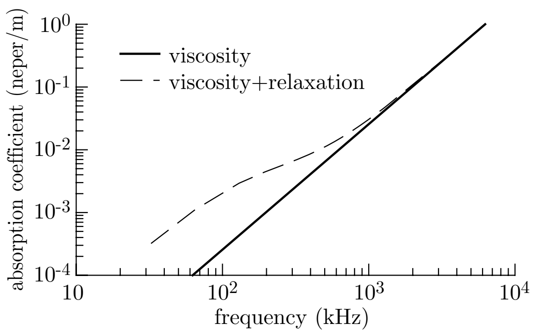

Figure 1: Frequency characteristics of the absorption coefficient of seawater.

where P is amplitude, ω is angular frequency, $k = \omega/c_0$ is wave number, and $\delta = \delta_1 + \delta_2$. Introducing the propagation constant β, the Helmholtz equation (6) is transformed into

$$\nabla^2 P + \beta^2 P = 0 \tag{7}$$

where

$$\beta^2 = \frac{k^2}{1 + j\omega \dfrac{\delta}{\rho_0 c_0^2}} \tag{8}$$

Under the assumption of $\delta\omega/\rho_0 c_0^2 \ll 1$, the propagation constant for the wave propagating to $+x$ direction is approximately expressed as

$$\beta = k + j\alpha = k + j\frac{\delta\omega^2}{2\rho_0 c_0^3} \tag{9}$$

where α is classical absorption coefficient. For an example, figure 1 shows the frequency characteristics of the absorption coefficient of the seawater at 20°C. The solid line indicates the classical absorption characteristics due to the viscosity predicted by equation (9), which is in proportion to the square of frequency. The dashed line indicates the characteristics including the effect of relaxation, which are not so simple compared with the classical one. In the case of time domain analysis such as FDTD, the introduction of the relaxation effect is not so easy.

DIGITAL HUYGENS' MODEL (DHM) FOR ACOUSTIC FIELD

In the digital Huygens' model, a three-dimensional minute acoustic field can be described by a cubic element consisting of six transmission lines and a stub with length $\Delta\ell$ which are connected at the center node of the element as shown in figure 2. Each transmission line or branch has the characteristic impedance of $Z_0 = \rho_0 c_0$. The stub has the characteristic impedance of Z_0/ξ where ξ is the normalized impedance and is non-reflectively terminated at the other end. The non-reflective termination at the stub end means that the stub is terminated with the characteristic impedance of the stub to match.

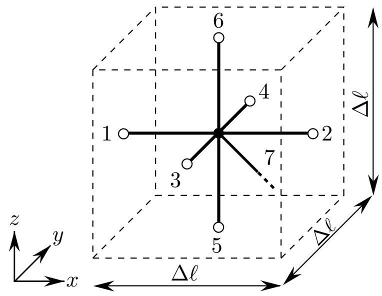

Figure 2: Three dimensional DHM element with stub.

The particular feature of DHM is that the sequences of the impulses are traced in time domain. When an input pulse P is applied to the branch 1 at the time $t = n\Delta t$ where $n = 1, 2, \cdots$ and $\Delta t = \Delta\ell/c_0$, the pulse is scattered at the node because of the impedance discontinuity at the connecting node. So the impulse of amplitude $-(4+\xi)/(6+\xi)P$ is reflected back to the incident branch 1 and the impulses of amplitude $2/(6+\xi)P$ are scattered to the other five branches and the stub. The scattering matrix is thus given as

$$\begin{bmatrix} S_1(n+1) \\ S_2(n+1) \\ S_3(n+1) \\ S_4(n+1) \\ S_5(n+1) \\ S_6(n+1) \\ S_7(n+1) \end{bmatrix} = a \begin{bmatrix} b & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & b & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & b & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & b & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & b & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & b & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_1(n) \\ P_2(n) \\ P_3(n) \\ P_4(n) \\ P_5(n) \\ P_6(n) \\ P_7(n) \end{bmatrix} \tag{10}$$

where P and S are incident and scattered pulses, subscripts indicate the number of the branch or stub, and $a = 2/(6+\xi)$ and $b = -(2+\xi/2)$. The pulse scattered to the stub is not reflected back into the node because of the non-reflective termination, which expresses the propagation loss. The scattered pulses then become the input pulses to the adjacent elements. The sequence of this process creates the propagation of the waves that corresponds to the Huygens' principle, as the field consists of the connection of the elements forming a network. This process is easily implemented on a computer.

The amplitude $P_a(n)$ at the node is evaluated as

$$P_a(n) = \frac{2}{6+\xi}\sum_{i=1}^{6} P_i(n) \tag{11}$$

The absorption coefficient α' for an element is estimated from the equation (11) as

$$\alpha' = \ln\frac{6}{6+\xi} \tag{12}$$

This equation shows that the absorption coefficient introduced by the stub is independent of the frequency and can not be applied to the simulation of the sound propagation.

EXPRESSION OF SOUND ABSORPION WITH DIGITAL FILTER

Equation (10) shows the impulse responses which can be expressed as a multi-port digital filter as shown in figure 3. The stub is not appeared in the equivalent circuit because the pulse scattered into the stub is not reflected back into the node. To express the sound absorption with frequency dependency in DHM, a FIR digital filter is inserted after the multiplier a in the digital equivalent circuit. The filter coefficients h corresponding to the frequency characteristics of the sound absorption coefficient shown in figure 1 are calculated as shown in table 1. They are calculated by the least P-norm optimal design method provided by MATLAB filter design toolbox [8]. The filter order of 20 is required to express the characteristics due to the relaxation, while the order of 10 is only required in the case of the classical absorption.

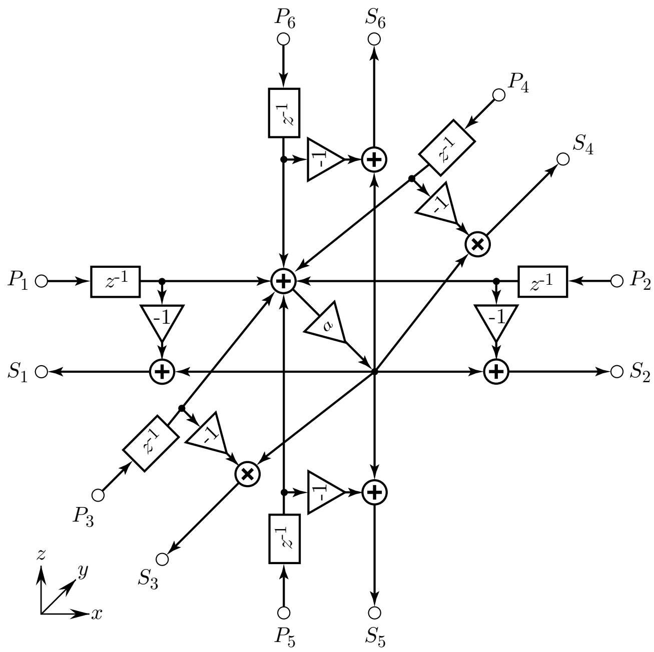

Figure 3: Digital equivalent circuit for DHM.

NUMERICAL EXPERIMENTS

To verify the validity of the present scheme, the numerical examinations are made for the sound propagation under the sea. A thin acoustic pipe of 1m in length is considered for the one-dimensional model. The pipe is driven at one end by the velocity with the waveform of the Gaussian shape and another is terminated by the sound absorber with the surface acoustic impedance of $\rho_0 c_0$. The pipe is divided into 5000 DHM elements whose element length is $\Delta\ell = 0.2$mm. The time step Δt is chosen to be 76.98nS and the sound speed c_0 is 1500m/s.

Figure 4 shows the calculated frequency characteristics of the absorption coefficient with the stub for $\xi = 10^{-6}$. The flat characteristic is obtained which is predicted in equation

Table 1: Filter coefficients of the FIR filter.

viscosity				viscosity + relaxation	
h_0	0.999926989383839	h_{11}	0.000001743005226	h_0	0.999918199250317
h_1	0.000087125979812	h_{12}	0.000004625332505	h_1	0.000092386365086
h_2	-0.000021323612414	h_{13}	-0.000002372229596	h_2	-0.000010755621423
h_3	0.000010894708465	h_{14}	-0.000002393075925	h_3	0.000001801494141
h_4	-0.000004558063368	h_{15}	0.000002660267627	h_4	-0.000009651685625
h_5	0.000003624315413	h_{16}	0.000000182498428	h_5	0.000013627816579
h_6	-0.000005587720839	h_{17}	-0.000001494547394	h_6	-0.000003897009810
h_7	0.000002499293207	h_{18}	0.000000494887689	h_7	-0.000008086409101
h_8	0.000001693911703	h_{19}	0.000000435189536	h_8	0.000005163473544
h_9	-0.000002933975550	h_{20}	-0.000000057588599	h_9	0.000006170243507
h_{10}	0.000001599822954			h_{10}	-0.000008892522503

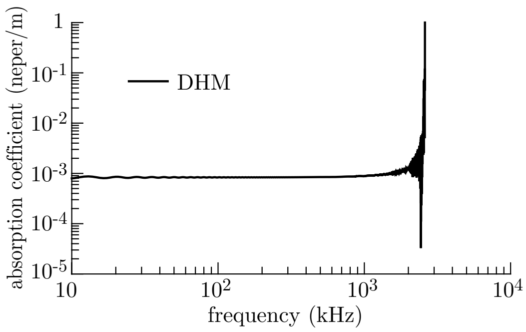

Figure 4: Frequency characteristics of the absorption coefficient with the stub.

(12), however it can not be applied to the simulation of the sound wave propagation. The divergent error in the higher frequency region is observed due to the cut-off frequency of the DHM network.

Figure 5 shows the calculated frequency characteristics of the classical absorption coefficient due to the viscosity. The bold line indicates the characteristics calculated by DHM with

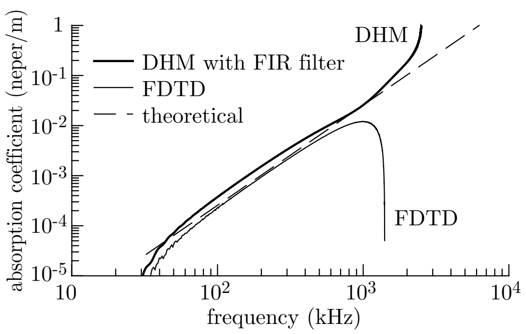

Figure 5: Frequency characteristics of the classical absorption coefficient.

the FIR filter and the dashed line the theoretical one predicted in equation (9). The characteristics in proportion to square of frequency is realized in the frequency range of 50kHz$\sim$1.5MHz. The large error above 1.5MHz is again due to the cut-off frequency of the DHM network. The thin line indicates the characteristic calculated by FDTD with $\delta = 2 \times 10^{-6}$. Both numerical results well agree.

Figure 6 shows the propagation characteristics at 800 kHz. The solid line indicates the characteristics calculated by DHM and the dashed line indicates the theoretical one at 800kHz where $\alpha = 0.0164$ neper/m. The result calculated by DHM well agrees with the theoretical in which the amplitude decreases exponentially.

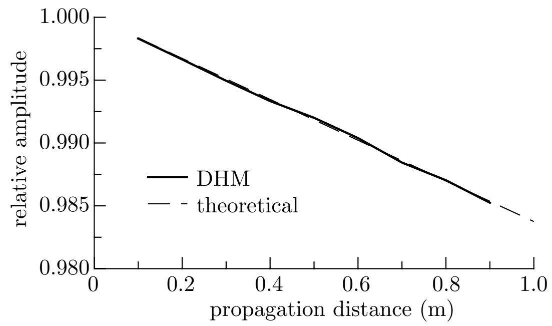

Figure 6: Propagation characteristics at 800kHz.

Figure 7 shows the frequency characteristics of the absorption coefficient including the relaxation effect. The desired characteristics are realized in the frequency range of 50kHz$\sim$1.5MHz. It is found that the arbitrary frequency characteristics can be included by the use of the digital filter.

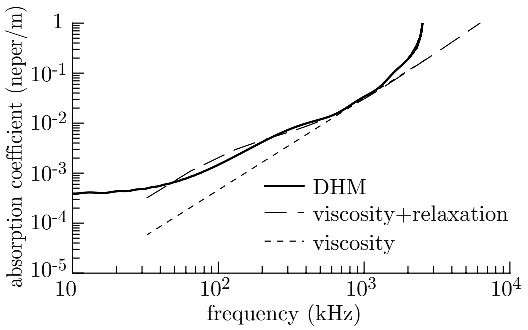

Figure 7: Frequency characteristics of the absorption coefficient including the relaxation effect with FIR filter.

CONCLUSIONS

To implement the frequency characteristics of the sound absorption coefficient in the sound propagation, the digital filter is applied to the digital Huygens' model (DHM). Some numerical demonstrations are made for the one-dimensional sound propagation in seawater. It is found that the complex frequency characteristics such as the absorption coefficient including the relaxation can be easily realized using the FIR filter.

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