# PERFORMANCE ANALYSIS OF VIBRATION ISOLATION FOR VEHICLE ER SUSPENSION SYSTEMS USING $H_2$ and $H_\infty$ CONTROL METHODS

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## Abstract

This paper investigates the performance of vibration isolation for single-wheel vehicle suspension model using the Electrorheological(ER) mount system. The main contribution of this paper is that the optimal damping coefficients of the ER mount are obtained by using $H_2$ and $H_\infty$ control methods and some initial experiment is also conducted to verify the control method. The control technique of optimal electric field associated with the optimal damping coefficients for ER fluids in squeeze mode with sinusoidal and broadband vibration excitation is also presented in this work. Both of the theoretical derivation and experimental investigation are conducted in present study. In the theoretical derivation, the optimal damping coefficients of the fundamental ER squeeze mode and numerical simulations from model are presented. In the experimental work, the relation of damping coefficients and electric fields is obtained from preliminary experiments. The results indicate that the proposed control methods are effective in suppressing the sinusoidal and broadband vibration. The work presented in the paper is the first part of this study.

KEYWORDS: Electrorheological(ER), optimal damping coefficients, $H_2$ and $H_\infty$, sinusoidal and broadband vibration excitation.

## INTRODUCTION

Vibration transmission control for vehicles can be basically classified into three configurations; passive, active and semi-active. The passive method uses rubber mount to suppress vibration. This method is only effective for the large-size flexible structures at high frequencies [1]. The active method makes use of smart material actuators to attenuate vibration. This method is effective for the small-sized flexible structures and its performance is fairly good in a wide frequency range. Vibration control with semi-active actions has been of popular interest in recent years. In such system, the damping characteristics of adaptable energy dissipation devices are varied on-line according to feedback signals and control commands. Since energy is always being dissipated, this method is more stable than a fully-active approach and is insensitive to the spillover problem [2]. In general, the semi-active method is featured by electrorheological (ER) or magnetorheological (MR) fluids. ER fluids belong to the general class of smart materials whose rheological properties can be modified by applying an electric field. These fluids have been known since the late 1940s when Winslow [3] suggested their first potential engineering application. Recently, many investigations have been conducted in the field of vibration isolation and control due to the rapid phase change of an ER fluid and hence the fast conversion of its physical properties [4]. The aims of the work presented in this paper are to investigate the performance of electrorheological fluid-based mount for vibration isolation using optimal controllers. The performance of ER mount for vibration isolation using $H_2$ and $H_\infty$ control methods has been analyzed through simulations. Preliminary Experiments have also been carried out to validate the feasibility of using optimal controllers on ER mount. The results show that good performance can be obtained by using $H_2$ and $H_\infty$ control methods for sinusoidal and broadband vibration. The paper is organized as follows. We first describe the configuration of the ER mount and the control method. Then the performance of ER mount for vibration isolation using $H_2$ and $H_\infty$ control methods is analyzed and experimental set-up is shown. Finally conclusions are made.

## SYSTEM CONFIGURATION AND CONTROL METHODS

### System Configuration

Figure 1(a) shows an SDOF isolation system with base excitation for single-wheel vehicle suspension model. The governing equation of the motion is

$$m_s\ddot{x}_s + c(\dot{x}_s - \dot{x}_b) + k(x_s - x_b) = 0 \,, \tag{1}$$

where $m_s$ is the system mass, $c$ is the damping coefficient, $k$ is the stiffness coefficient, $\ddot{x}_s$ is the vertical acceleration of the mass, $\dot{x}_s$, $x_s$ are the vertical velocity and displacement of the mass, $\dot{x}_b$ , $x_b$ are the velocity and displacement of the base. If we replace the viscous damper with an ER damper, the SDOF isolation system with the ER mount can be shown in Fig. 1(b). The governing equation of the motion can now be expressed as

$$m_s \ddot{x}_s + f_{ER} + k(x_s - x_b) = 0 \,, \tag{2}$$

where $f_{ER}$ is the controllable damping force, which can also be expressed as

$$f_{ER} = \tilde{c}(\dot{x}_s - \dot{x}_b) \,, \tag{3}$$

where $\tilde{c}$ is the controllable damping coefficient. Substituting Eq. (3) into Eq. (2) we obtain

$$m_s \ddot{x}_s + \tilde{c}(\dot{x}_s - \dot{x}_b) + k(x_s - x_b) = 0 \,. \tag{4}$$

Laplace transforming both sides of the Eq. (4) and assuming zero initial conditions ( $x_s(0), \dot{x}_s(0), x_b(0), \dot{x}_b(0) = 0$ ) show that the transfer function of the displacement between the mass and the base is given by

$$H(s) = \frac{\mathrm{X}_s(s)}{\mathrm{X}_b(s)} = \frac{\tilde{c}s + k}{ms^2 + \tilde{c}s + k} \,. \tag{5}$$

If we substitute $s = j\omega$ into Eq. (5), then the transfer function becomes

$$H(j\omega) = \frac{\mathrm{X}_s(j\omega)}{\mathrm{X}_b(j\omega)} = \frac{j\tilde{c}\omega + k}{-m\omega^2 + j\tilde{c}\omega + k} \,, \tag{6}$$

where $H(j\omega)$ is the frequency response function.

## Control Methods

In this work $H_2$ and $H_\infty$ control were used for attenuating vibration. The squared $H_2$ norm [5] of the frequency response function is now defined as

$$\|H\|_2^2 = \sum_{\omega} \left| \frac{j\tilde{c}\omega + k}{-m\omega^2 + j\tilde{c}\omega + k} \right|^2 . \tag{7}$$

The optimal damping coefficient, which minimizes the $H_2$ norm of the frequency response function in some given frequency range can then be calculated by minimizing the sum of the square magnitude of the frequency response function as follows

$$
\begin{aligned}
\min_{\tilde{c}} \|H\|_2^2 &= \min_{\tilde{c}} \left\| \frac{j\tilde{c}\omega + k}{-m\omega^2 + j\tilde{c}\omega + k} \right\|_2^2 \\
&= \min_{\tilde{c}} \sum_{\omega} \left| \frac{j\tilde{c}\omega + k}{-m\omega^2 + j\tilde{c}\omega + k} \right|^2
\end{aligned}
\tag{8}
$$

The optimal damping coefficient which will minimize the sum of square magnitude of the frequency response function can be calculated by using the function fmins() in MATLAB[6]. For $H_\infty$ control the optimal damping coefficient, which minimizes the $H_\infty$ norm of the frequency response function in some given frequency range can then be calculated by minimizing the maximum value of the frequency response function as follows

$$
\min_{\tilde{c}} \|H\|_\infty = \min_{\tilde{c}} \max_{\omega} \left| \frac{j\tilde{c}\omega + k}{-m\omega^2 + j\tilde{c}\omega + k} \right| \tag{9}
$$

The optimal damping coefficient which will minimize the maximum value of the frequency response function can be calculated by using the function constr() in MATLAB[6]. The $H_\infty$ control will result in minimal peak of the magnitude in the frequency response function. This approach ensures that the magnitude in the frequency response function has the lowest possible maximum value.

## SIMULATIONS AND EXPERIMENTAL SETUP

In this section performance of vibration isolation for single-wheel vehicle ER suspension model using $H_2$ and $H_\infty$ control methods will be evaluated through simulations and some initial experiment will be carried out to validate the simulation results. Figure 2 shows simulation results of the displacement transmissibility using $H_2$ control for original, fixed and varied optimal damping coefficients. The original damping coefficient is determined from the measured data. From the Figure it can be seen that the resonance frequency occurs at around 20 Hz. The magnitude of displacement transmissibility using $H_2$ control method decreases at the frequencies below 30 Hz for fixed optimal damping coefficient. However some amplification appears at the frequencies above 30 Hz. Figure 2 also shows the results for varied optimal damping coefficient which varies with frequency and is calculated by using

$H_2$ control method. As can be seen the magnitude of displacement transmissibility for varied damping coefficient decreases significantly at the frequencies below 30 Hz. This is because the damping coefficient varies with frequency. In optimization process the damping coefficient at every frequency point can be adjusted in order to achieve the best performance. It suggests that the performance of the proposed vibration isolation system would be good at the frequencies below 30 Hz for fixed damping coefficient and better performance could be obtained over the whole frequency range for varied damping coefficient by using $H_2$ control method.

Figure 3 shows simulation results of the displacement transmissibility using $H_\infty$ control method for original, fixed and varied optimal damping coefficients. Comparing Fig. 3 and Fig. 2 we can see that the performance is similar for fixed damping coefficient using $H_2$ or $H_\infty$ control method. Whilst the performance for varied damping coefficient using $H_\infty$ control is better than that using $H_2$ control. The reason for this is that $H_\infty$ control is to minimize the maximum value of the frequency response function of the displacement, however, $H_2$ control is to minimize the sum of the squared magnitude of the frequency response function. Therefore the frequency response function of the displacement in $H_\infty$ control is flatter than that in $H_2$ control.

In this work an ER mount has been used and installed in the experimental suspension system. Some initial experiments have also been done to validate the simulation results. Only $H_2$ control method for the fixed damping coefficient has been used for simplicity. Figure 4 shows the experimental ER suspension system. The relationship between optimal electric field and damping force can be determined from Fig. 5. Fig. 6(a) shows the base excitation signal used in the experiments and Fig. 6(b) presents the performance of the vibration isolation system using $H_2$ control with the fixed optimal damping coefficient. We can see that good attenuation is obtained around the resonance frequency as expected in simulations. From those results presented in this section we may expect that ride comfort can be improved by employing the proposed ER suspension system using $H_2$ and $H_\infty$ control methods.

## CONCLUSIONS

In this paper the formulation of $H_2$ and $H_\infty$ control which calculated the optimal damping coefficients in the squeeze mode ER mount for the single-wheel vehicle suspension model has been presented. The results showed that good attenuation was achieved at around the resonance frequency for $H_2$ and $H_\infty$ control with the fixed optimal damping coefficient, and better attenuation was obtained over the whole frequency range for $H_2$ and $H_\infty$ control with the varied optimal damping coefficient. Some initial experiments have also been carried out to evaluate the performance of the proposed vibration isolation system. The experimental results showed that the proposed control approach was effective in suppressing the broadband vibration and the ride comfort could be improved by using the proposed ER suspension system.

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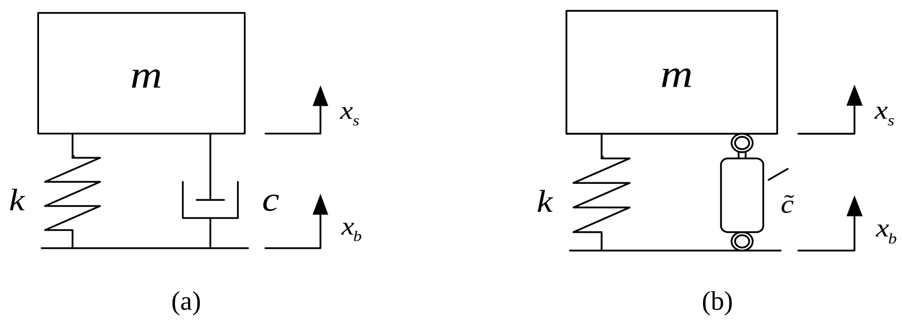



Figure 1 - The SDOF isolation system with the base excitation. (a) With viscous damper. (b) With the ER mount.

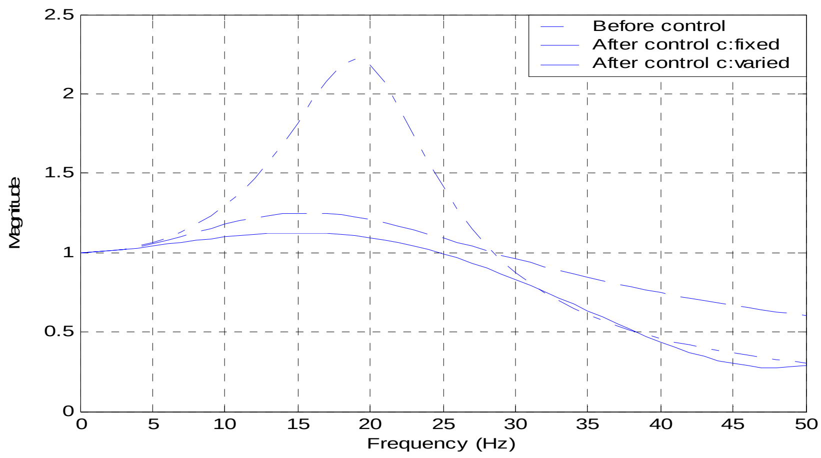



Figure 2 - Displacement transmissibility of the ER mount system using $H_2$ control for fixed and varied damping coefficients.

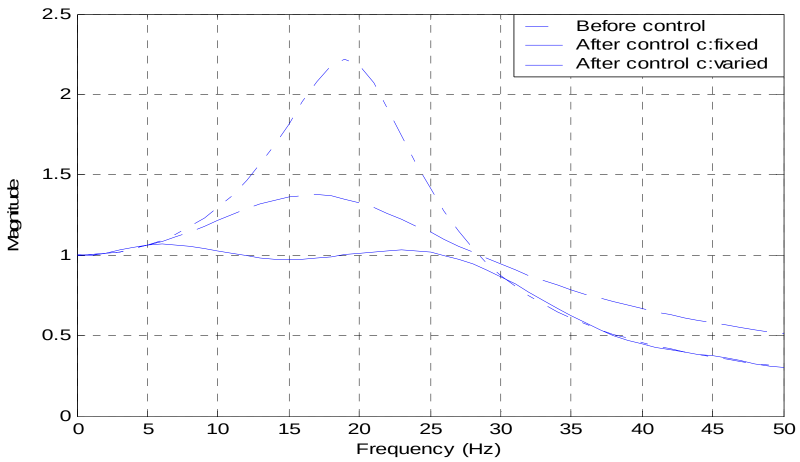



Figure 3 - Displacement transmissibility of the ER mount system using $H_\infty$ control for fixed and varied damping coefficients.

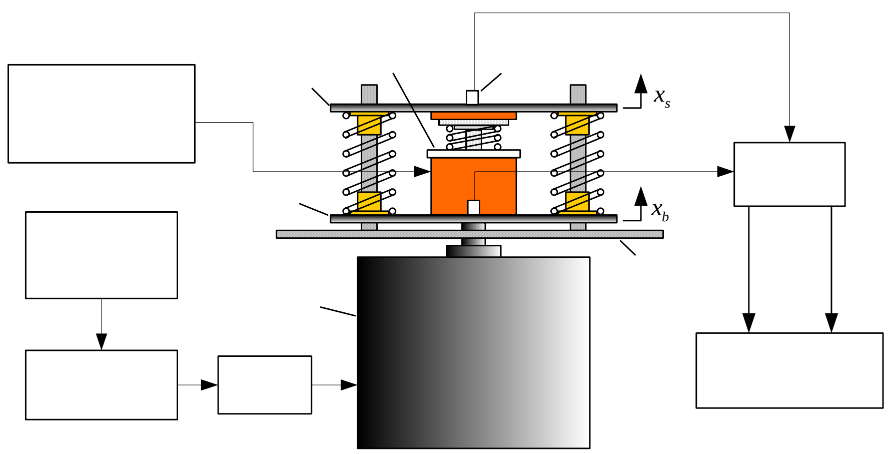



Figure 4 - Experimental set-up.

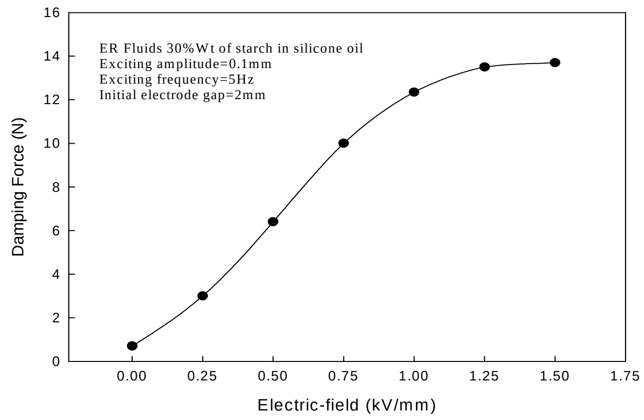
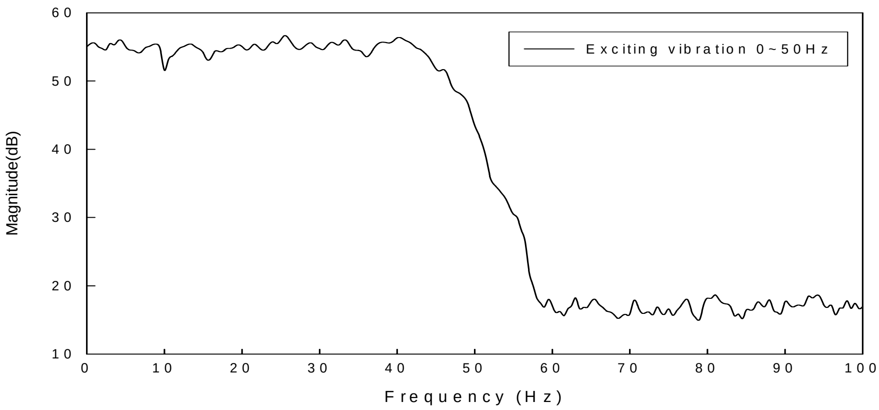
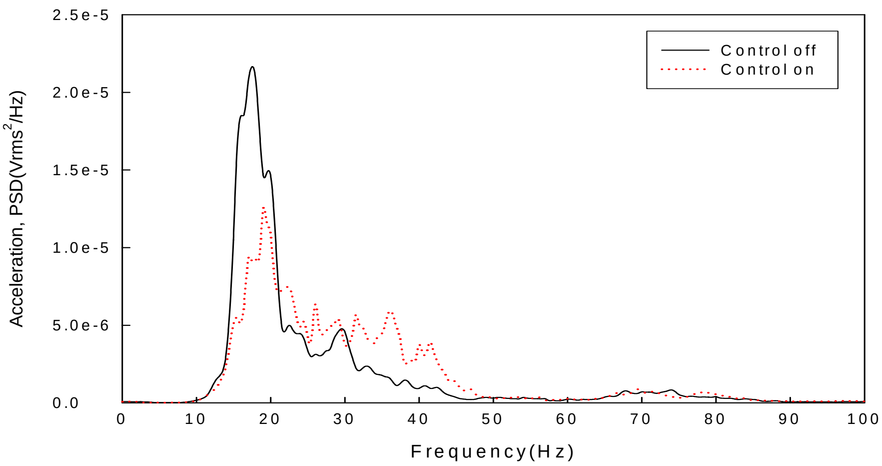



Figure 5 - Relationship between the damping force and the electric field.




(a)




(b)

Figure. 6 - Control performance. (a) Random vibration excitation signal. (b) Performance of the proposed vibration isolation system.