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A SYSTEMS APPROACH TO POWERTRAIN DYNAMICS

Andrew Guzzomi* and Brian Stone

School of Mechanical Engineering
The University of Western Australia
35 Stirling Hwy, Crawley, Western Australia 6009.
aguzzomi@mech.uwa.edu.au

Abstract

Complex components may be modelled in various ways. However when a system is made up of many complex components some methods of modelling are very inefficient and their accuracy also becomes questionable. A systems approach has the advantage that each component may be modelled in the most appropriate way and then the models of the components may be assembled to produce a model of the complex system. As an illustration of this approach this paper presents a discussion of the various components involved in powertrain dynamics. Powertrain systems are commonly modelled in the frequency domain and these techniques are well known and accepted. This paper however uses a Time Domain Receptance approach to model torsional vibration. The assumptions currently made in modelling such systems are discussed. Some recent experimental work is described that shows that some major assumptions that are commonly made are not valid. The most significant components are identified and the ways of modelling them described. Components included in the time domain model are the: engine, valvetrain, camshaft drive belt, clutch, continuous shafts, a gear pair and the tyres. Simulations demonstrate the potential of the modelling technique and the ease with which modifications to the model can be achieved. Finally the areas that require further investigation are identified.

INTRODUCTION

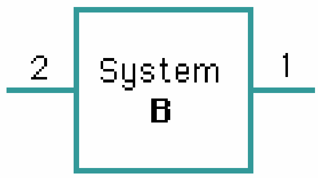
The systems approach based on the concept of receptances is well documented [1], and has been used to model dynamic systems. A receptance is the transfer function in the frequency domain relating the ratio of the resulting displacement of a system and the applied excitation on the system. When a systems approach is applied, the system being modelled is first broken up into a number of sub-systems which can be modelled easily. Then the receptances of the sub-systems are derived. Because of some useful properties of the receptance, the receptance of the original system can be obtained by adding the receptances of the sub-systems in the appropriate manner. Thus when a systems approach is applied, the problem of modelling a system becomes one of modelling the interaction of the sub-systems into which the original system may be broken.

During the process of adding the sub-systems only two sub-systems are added at a time. After one addition the resultant system is considered to be one sub-system, and can be added with another sub-system. The receptance of a system containing n sub-systems can be developed by repeating the process of adding two sub-systems n-1 times. The original system may be very complex, but the addition of two of its sub-systems is always simple, and the number of sub-systems contained in the complex system only affects the times the simple addition is repeated. Thus another advantage of the systems approach is that it is possible to write generalized computer programs modelling a large variety of systems.

The time domain receptance was developed by Li [2], [3] on the basis of a numerical solution of the differential equations governing the motion of the systems. It is the purpose of this paper to outline the concepts of time domain receptances and to apply them to powertrain dynamics.

THE TIME DOMAIN RECEPTANCE

Consider the system B, shown at some time t, we wish to determine its state at time t+Δt. If during the time interval Δt there are mean forces $F_1(\Delta t)$ and $F_2(\Delta t)$ acting at co-ordinates 1 and 2 respectively, then if the system may be considered linear over the time interval Δt the following equations are applicable,



$$x_1(t+\Delta t) = BI_{11}P_1(\Delta t) + BI_{12}P_2(\Delta t) + BII_1 \tag{1}$$

$$x_2(t+\Delta t) = BI_{21}P_1(\Delta t) + BI_{22}P_2(\Delta t) + BII_2 \tag{2}$$

Where, BII_1 and BII_2 represent the values that $x_1(t+\Delta t)$ and $x_2(t+\Delta t)$ would have attained if there had been no external forces. They correspond to complimentary functions and depend on the state at time t. BI_{11}, BI_{12}, BI_{21} and BI_{22} relate the responses to the impulse magnitudes alone with no initial displacements or velocities and correspond to particular integrals. Note that for a linear system $BI_{12} = BI_{21}$.

Excitation at a Remote Coordinate

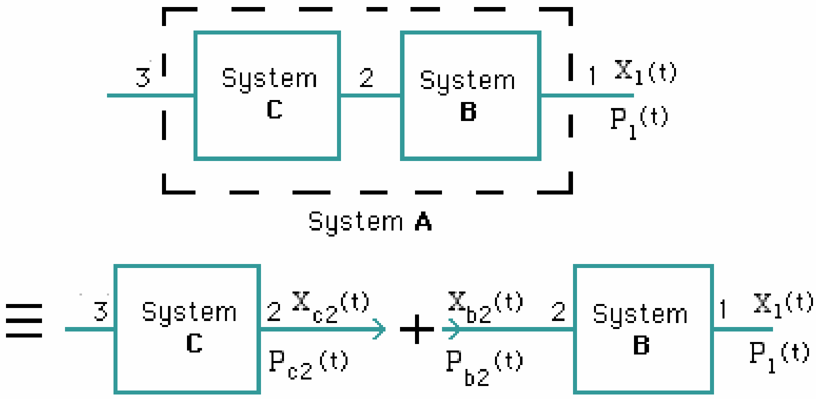


Figure 1 System addition

Excitation at a remote coordinate is when the excitation is applied at a co-ordinate other than that at which two systems are joined. The excitation may be a force or torque and the response a displacement or rotation. Consider system A as shown in figure 1 that has two sub-systems connected at coordinate 2, and is subject to an excitation $P_1(\Delta t)$ at coordinate 1 while the response of A is measured at coordinate 3. Thus the addition, excitation and response are at different coordinates. By dividing system A at coordinate 1 as shown in figure 1 yields the equation of compatibility of displacements

$$x_{b2}(t+\Delta t) = x_{c2}(t+\Delta t) = x_2(t+\Delta t) \tag{3}$$

Equilibrium of the forces at the join requires

$$P_{b2}(\Delta t) + P_{c2}(\Delta t) = 0 \tag{4}$$

and the equations for the displacements of sub-system B are

$$x_1(t+\Delta t) = P_1(\Delta t)BI_{11} + P_{b2}(\Delta t)BI_{12} + BII_1 \tag{5}$$
$$x_{b2}(t+\Delta t) = P_1(\Delta t)BI_{21} + P_{b2}(\Delta t)BI_{22} + BII_2 \tag{6}$$

and for sub-system B

$$x_3(t+\Delta t) = P_{c2}(\Delta t)CI_{32} + CII_3 \tag{7}$$
$$x_{c2}(t+\Delta t) = P_{c2}(\Delta t)CI_{22} + CII_2 \tag{8}$$

Substituting (6) and (8) in (3) and from (4) $P_{c2}(\Delta t) = -P_{b2}(\Delta t)$

$$P_1(\Delta t)BI_{21} + P_{b2}(\Delta t)BI_{22} + BII_2 = -P_{b2}(\Delta t)CI_{22} + CII_2$$

$$\therefore P_{b2}(\Delta t) = \frac{-P_1(\Delta t)BI_{21} - BII_2 + CII_2}{BI_{22} + CI_{22}} \tag{9}$$

Substituting in (8)

$$x_1(t+\Delta t)=\frac{P_1(\Delta t)\left(BI_{11}BI_{22}+BI_{11}CI_{22}-BI_{12}^2\right)-BII_2BI_{12}+CII_2BI_{12}}{BI_{22}+CI_{22}}+BII_1$$

$$x_1(t+\Delta t)=P_1(\Delta t)AI_{11}+AII_1$$

where

$$AI_{11}=BI_{11}-\frac{BI_{12}^2}{BI_{22}+CI_{22}} \quad \text{and} \quad AII_1=BII_1+\frac{BI_{12}\left(CII_2-BII_2\right)}{BI_{22}+CI_{22}}$$

similarly since

$$x_2(t+\Delta t)=P_1(\Delta t)\left(BI_{21}-\frac{BI_{21}BI_{22}}{BI_{22}+CI_{22}}\right)+\frac{CII_2BI_{22}-BII_2BI_{22}}{BI_{22}+CI_{22}}+BII_2$$

$$x_2(t+\Delta t)=P_1(\Delta t)AI_{21}+AII_2$$

where

$$AI_{21}=BI_{21}-\frac{BI_{21}BI_{22}}{BI_{22}+CI_{22}} \quad \text{and} \quad AII_2=\frac{BI_{22}\left(CII_2-BII_2\right)}{BI_{22}+CI_{22}}+BII_2$$

similarly

$$x_3(t+\Delta t)=\frac{P_1(\Delta t)BI_{21}CI_{32}}{BI_{22}+CI_{22}}+\frac{BII_2CI_{32}-CII_2CI_{32}}{BI_{22}+CI_{22}}+CII_3$$

$$x_3(t+\Delta t)=P_1(\Delta t)AI_{31}+AII_3$$

where

$$AI_{31}=\frac{BI_{21}CI_{32}}{BI_{22}+CI_{22}} \quad \text{and} \quad AII_3=\frac{CI_{32}\left(BII_2-CII_2\right)}{BI_{22}+CI_{22}}+CII_3$$

We now have the time domain receptance of system A. Using the same approach we could now add another system and so on until the complete complex system is reached. At this stage a time increment is made and then the system is subdivided again while determining all the forces/torques at each join using equation (9). The forces/torques on each side of each sub-system are then known and a time increment is made on each sub-system. In this way using successive time increments

The modelling of torsional vibration of power train dynamics in the time domain requires that the characteristics of the various components are known. We have so far investigated, reciprocating engines, valve trains, drive belts, flexible couplings, gear boxes and tyres, (some examples include [4],[5],[6] and [7]). The major areas of interest have been those that are significant but have previously been ignored. Thus we have investigated friction effects in engines, are working on friction effects in valve trains and have begun investigations into the tyre/road stiffness and damping. In a short paper such as this a simple example will suffice to illustrate the method and its possible applications.

AN EXAMPLE

Consider a simple model of a drivetrain that includes a single cylinder engine, prop shaft, differential, lay shafts, wheels and tyres plus the vehicle body.

Engine

The engine is modelled as shown in figure 2.

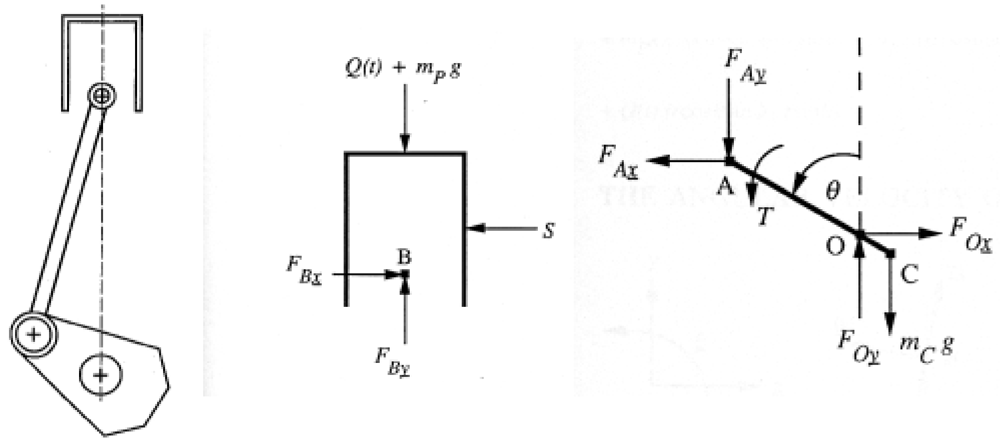


Figure 2. Free body diagrams of the piston and crank as identified by Hesterman [3].

Hesterman [3] determined the torque required to be applied to the crank so that it would have a given angular velocity and acceleration at any particular angular position. The equation obtained [3] was,

$$\ddot{\theta}_C = \frac{T - \frac{1}{2} I'(\theta).\dot{\theta}_C^2 - g(\theta) - Q(t,\theta)}{I(\theta)} \tag{10}$$

where $I(\theta)$, $I'(\theta)$, $g(\theta)$ and $Q(t,\theta)$ may be found in [3].

We can determine the time domain receptance by using equation (10) for the engine as follows. Let the piston loading term $Q(t,\theta)$ represent the gas force on the piston applied at coordinate 1 and the angular rotation be coordinate 2. Then we need to find BI_{12}, BI_{22} and BII_2. For BI_{12} we need the situation where the response to the excitation alone is found. In this case $x_2(t+\Delta t) = P_1(\Delta t)BI_{12}$. Hence we put T=0 and $Q(t,\theta) = P_1(\Delta t) = 1.0$ and on substituting in (10) obtain,

$$\ddot{\theta}_C = \frac{-\frac{1}{2} I'(\theta).\dot{\theta}_C^2 - g(\theta) - 1.0}{I(\theta)}$$

Any numerical method may now be used to obtain θ_C which is equal to $x_2(t+\Delta t)$ and hence,

$$BI_{12} = \frac{x_2(t+\Delta t)}{P_1(\Delta t)} = \frac{x_2(t+\Delta t)}{1.0} = \theta_C(t+\Delta t)$$

It should be noted that BI_{12} will not be constant.

BI_{22} is found in a similar manner. We use $x_2(t+\Delta t) = P_2(\Delta t)BI_{22}$ and put $T_2(\Delta t) = P_2(\Delta t) = 1.0$ and $Q(t,\theta) = P_1(\Delta t) = 0$. Substituting in (10) gives,

$$\ddot{\theta}_C = \frac{1.0 - \frac{1}{2} I'(\theta).\dot{\theta}_C^2 - g(\theta)}{I(\theta)}$$

Any numerical method may now be used to obtain θ_C which is equal to $x_2(t+\Delta t)$ and hence,

$$BI_{22} = \frac{x_2(t+\Delta t)}{P_2(\Delta t)} = \frac{x_2(t+\Delta t)}{1.0} = \theta_C(t+\Delta t)$$

Again it should be noted that BI_{12} will not be constant.

Finally to find BII_2 we need the case where there is no excitation. In this case $x_2(t+\Delta t) = BII_2$. From (10)

$$\ddot{\theta}_C = \frac{-\frac{1}{2} I'(\theta).\dot{\theta}_C^2 - g(\theta)}{I(\theta)}$$

Any numerical method may now be used to obtain θ_C which is equal to $x_2(t+\Delta t)$ and hence we have BII_2.

Drivetrain

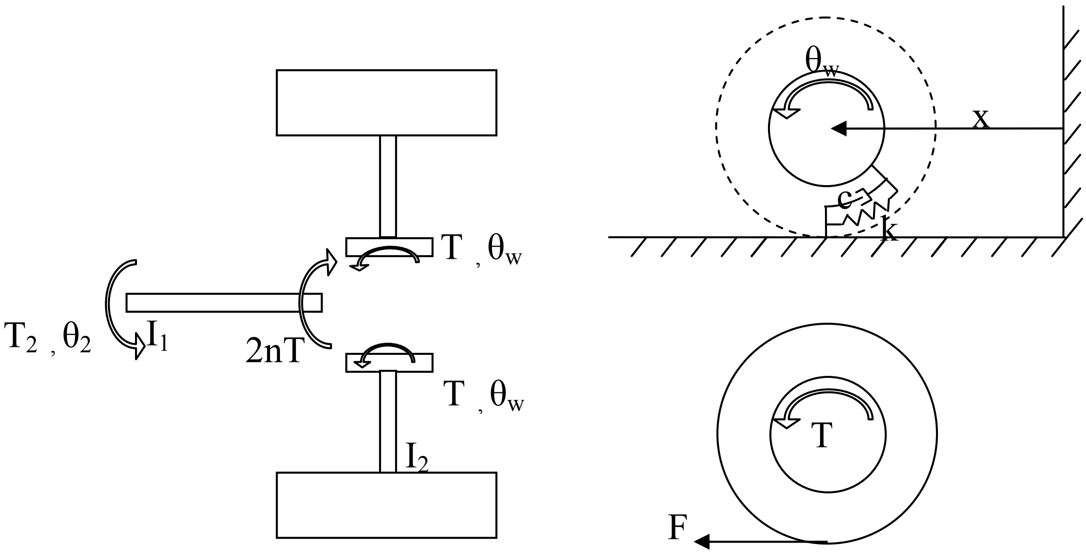


Figure 3 Drivetrain attached to the engine.

The engine is coupled to a prop shaft, differential, lay shafts, wheels and tyres. These may be considered as a single system (figure 3) as it is relatively easy to model this system if the shafts are all considered rigid.

The equation of motion for the prop-shaft is, $I_1\ddot{\theta}_2 = T_2 - 2nT$ (11)

where n is the gear ratio in the differential and T is the torque on each lay shaft.

For each lay shaft and wheel the equation of motion is, $I_2 n\ddot{\theta}_2 = T - FR$ (12)

Where F is the force on the tyre and R is the tyre radius.

For the vehicle the equation of motion is, $m_{total}\,\ddot{x} = 2F - c_D\,\dot{x}$ (13)

Where m_{total} is the total mass of the vehicle, and c_D is the drag coefficient.

It remains to determine the force F. The contact stiffness and damping act as the wheel rotation is greater or less than that expected from the linear motion of the vehicle. Thus

$$F = k(nR\theta - x) + c(nR\dot{\theta} - \dot{x}) \tag{14}$$

Rearranging and manipulating these equations to eliminate T gives

$$\ddot{\theta}_2 = \frac{T_2 - 2nFR}{I_1 + 2n^2 I_2} \tag{15}$$

$$\ddot{x} = \frac{2F - c_D\,\dot{x}}{m_{total}} \tag{16}$$

Using an identical approach to that used for the engine the time domain receptance may be found. The two systems (the engine and drivetrain) may then be added. For a small single cylinder engine mounted in a 'go-cart' and starting from rest the results obtained are shown in figures 4 and 5.

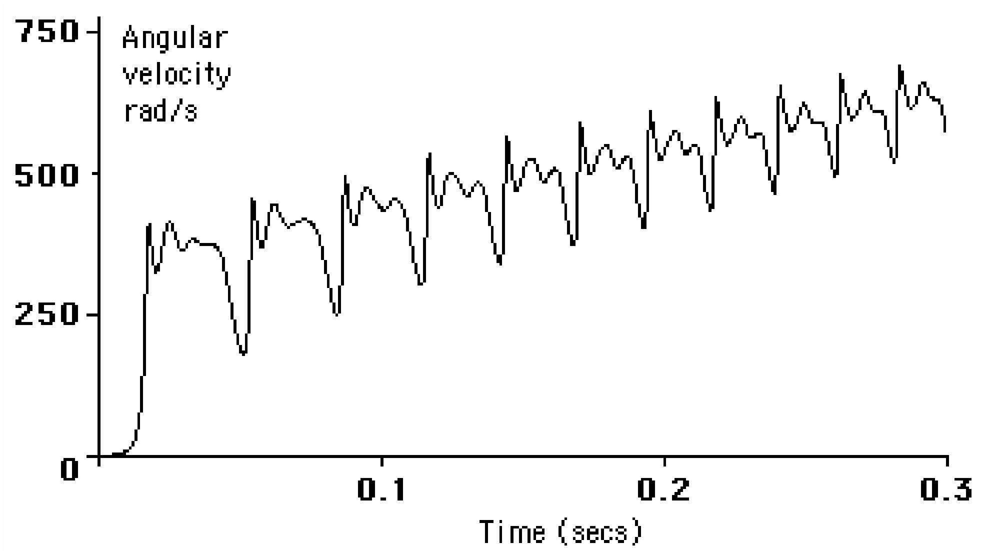


Figure 4 Engine speed as a function off time

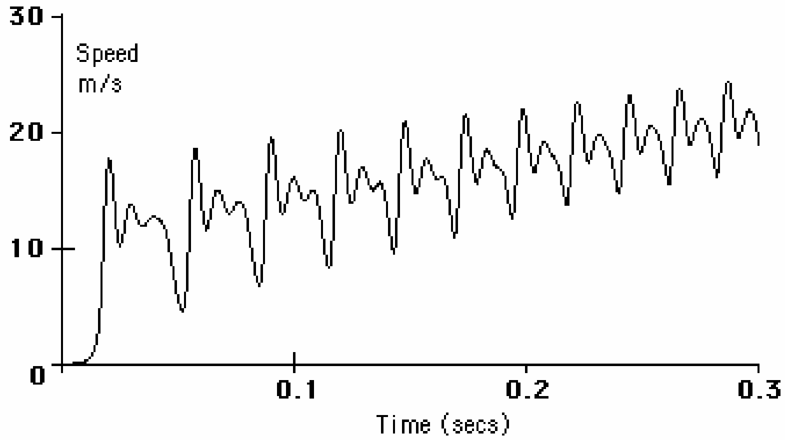


Figure 5 Vehicle speed as a function of time

CONCLUSIONS

The application of the systems approach to time domain modelling has been used utilizing the time domain receptance. An example including an engine, drivetrain and including tyre contact deflections has been presented. In this simple model it was seen that the varying speed of the engine, which results from both the gas force and the varying inertia of the engine mechanism, couples to the speed of the vehicle. For a single cylinder engine this variation is quite large. The tyre stiffness and damping cause the form of the oscillation seen by the vehicle to change. Although the system was represented by addition of 2 sub-systems the model could easily be extended to include other powertrain components by further sub-system addition without complete re-derivation of the equations. It is this property that makes time domain receptance techniques so attractive for modelling complex systems.

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