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VIBRATION AND SOUND FIELD ANALYSIS OF A HOLED PLATE SUBJECTED TO A ROTATING SOUND SOURCE

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Abstract

This paper presents the vibration and the corresponding sound field analysis of a holed plate excited by a rotating sound source. The vibration characteristics of the holed plate are investigated by applying a layer of artificial "ectoplasm" on the entire plate including the holed area. Such an algorithm can avoid a non-unique problem with ill-conditioned mass and stiffness matrices. The external excitation on the plated is from a point sound source in a rotating motion. Rayleigh-Ritz method is performed for obtaining the equations of motion. Then, the dynamic responses can be determined. The radiated sound field, on the opposite side of the plate to the sound source, due to vibration of the baffled plate is found by using Rayleigh's integral. The diffraction field from sound wave propagating through the holes is also formulated. Total sound field is the combination of these two sound fields. Numerical results of the vibration responses of the plate and the associated sound field are showed and discussed.

INTRODUCTION

This manuscript investigates the dynamic characteristics of a holed plated subjected to a rotating simple sound source. Moreover, the corresponding sound pressure due to radiation of the vibrating plate and scattering through the holes is also studied. The holed plate is chosen as a simple model of a PC case while the rotating source represents the cooling fan. The results and the discussions of this research can give some design guidelines in the noise reduction aspect.

References related to this topic are two journal papers by Beslin and Guyader [1, 2]. In their research, they suggested the method of applying "ectoplasm" for the vibration analysis of a holed plate. Some associated noise analysis was also presented [2].

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FORMULATION OF THE HOLED PLATE

Consider a simply-supported, baffled plate, given in Fig. 1 with several holes on it. For solving dynamic behaviors of a holed plate, a traditional method, such as Galerkin's method, may yield non-positive definite mass and stiffness matrices [1]. In this research, a layer of ectoplasm of weak material is artificially covered on the entire plate including the holed area for the purpose of formulation [1,2]. The mechanical properties of the ectoplasm must satisfy

$$\tilde{e} << \frac{r_{ect}}{r} << 1 \quad \text{and} \quad \tilde{e} << \frac{D_{ect}}{D} << 1 \tag{1}$$

Here, $\tilde{e}$ is a small parameter, r and D are the density (mass per unit area) and bending stiffness of the plate material, r_{ect} and D_{ect} are those of the ectoplasm.

Hamilton's principle

The equations of motion are derived by using Hamilton' s principle and Rayleigh-Ritz method [3]. Hamilton's principle can be stated as

$$d H = d \int_{t_1}^{t_2} \int_{s} (T - U + FW) ds \ dt = 0 \tag{2}$$

Here, T represents the kinetic energy density of the composite plate, U represents the elastic potential energy density, F is the external loading, W is the transverse displacement of the plate. The formulation is integrated over the entire area s of the composite plate.

Equations of motion

In Rayleigh-Ritz method, the transverse displacement of the beam is expressed in a linear combination form of

$$W(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} w_{nm} f_{nm} e^{j \bar{w} t} \tag{3}$$

Here, w_{nm} is unknown and $\bar{w}$ is the vibrating frequency. The trial function is chosen as the mode shape of a solid plate written as

$$f_{nm}(x, y) = \sqrt{\frac{4}{b^2}} \sin\left(\frac{np\,x}{b}\right) \sin\left(\frac{mp\,y}{b}\right) \tag{4}$$

Substituting (4) into Hamilton's principle (2) yield the governing equations of the composite plate written as

$$K_{nmpq} w_{nm} - \bar{w}^2 M_{nmpq} w_{nm} = \int_{s_p} F f_{pq} ds \tag{5}$$

where K_{nmpq} and M_{nmpq} are constants [1].

A MOVING SIMPLE SOURCE

Moving simple source

The sound pressure on the plate due to a moving simple source of strength $Q(t) = qe^{jwt}$, located at $\vec{r}_s$ (t), can be expressed in a form of [4]

$$p(\vec{r},t) = \int \frac{Q(t)}{4p\, r} d\,(t - t - r/c)\, dt \tag{6}$$

Here, $r = |\vec{r} - \vec{r}_s|$ is the distance from the receiving point to the source, Q is defined as the mass rate per unit time, and c is the sound speed. From its definition, the function d must satisfy

$$\int_{-\infty}^{\infty} f(t)\, d\big(g(t)\big)\, dt = \left[\frac{f(t)}{|dg/dt|}\right]_{t=t^*} \tag{7}$$

In eq. (6), the retarded time t^* can be solved from $g(t) = t - t - r(t)/c = 0$ and $dg/dt = M_r - 1$. Here, Mach number is defined as $M_r = (\vec{n} \cdot \vec{v}_S)/c$ where $\vec{n}$ is the unit vector in the direction of $\vec{r}\,(t^*)$ and $\vec{v}_S$ is the source velocity. The sound pressure due to moving source therefore becomes [4]

$$p(\vec{r},t) = \frac{Q(t^*)}{4p\, r\, |1 - M_r|} \quad \text{with} \quad r = |\vec{r} - \vec{r}_s(t^*)| \tag{8}$$

where t actually depends on t^*.

Rotating simple source

The source considered in this manuscript is in a circular motion, in the xy plane, with radius a, frequency Ω, and about the center point (x_0, y_0, z_0). Then, the sound pressure at $\vec{r}$ can be written as

$$p(\vec{r},t) = \frac{Q(t^*)}{4p\, r \left|1 - \frac{a\Omega}{c}\left[\vec{n} \cdot \left(-\sin\Omega t^* \vec{i} + \cos\Omega t^* \vec{j}\right)\right]\right|} \tag{9}$$

where $\vec{r} = \left\{\left[x - \left(x_0 + a\cos\Omega t^*\right)\right]\vec{i} + \left[y - \left(y_0 + a\sin\Omega t^*\right)\right]\vec{j} + \left(-z_0\right)\vec{k}\right\}$. As discussed in the future section, $t^*(t)$ actually approximates to a periodic function of t with frequency Ω. Therefore, the sound pressure on the plate can be expressed as a Fourier series in the following form

$$p(x,y,z,t) = p(\vec{r},t) = \sum_{l=-\infty}^{\infty} \hat{F}_l e^{j(w + l\Omega)t} \tag{10}$$

The coefficient $\hat{F}_l$ are given in reference [5] and s_p is the area of the holed plate.

PLATE EXCITED BY THE ROTATING SOURCE

The loading, on a baffled plate from a sound source, is called blocked pressure and is proven twice the sound pressure on the plate [6]. The governing equations, an extension of eq. (5), therefore become

$$K_{nmpq} w_{lnm} - (w + l\Omega)^2 M_{nmpq} w_{lnm} = \int_{s_p} \frac{4}{\sqrt{b^2}} \hat{F}_l \sin\frac{pp\,x}{b} \sin\frac{qp\,y}{b} ds_p \tag{11}$$

with s_p the plate area not including the holes**.** The displacement of the holed plate is obtained by

$$W(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{l} w_{lnm} f_{nm}(x, y) e^{j(w+l\Omega)t} \tag{12}$$

SOUNG FIELD DUE TO THE VIBRATING HOLED PLATE

The output sound pressure is calculated at a point on the opposite side of the plate to the sound source. The total sound field is the combination of the radiated and the scattering sound fields.

Radiated sound field

The radiated sound field can be obtained by Kirchhoff-Helmholtz integral [7]

$$P(\vec{r}) = -\int_{s_p} \left[P(\vec{r}\,*) \frac{\partial G(\vec{r})}{\partial n} - G(\vec{r}) \frac{\partial P(\vec{r}\,*)}{\partial n} \right] ds_p(\vec{r}\,*) \tag{13}$$

where $\vec{r}$ is a point in the sound field, and $\vec{r}\,*$ is a point located on the holed plate. The Green's function in the integral should be chosen as $G(\vec{r}) = 2g(\vec{r}) = -2e^{-jkR^*}/(4pR^*)$ by image source method for a baffled problem where g is the free field Green's function [7] and $\vec{R}^* = \vec{r}\text{-}\vec{r}\,*$. By recognizing $\frac{\partial G(\vec{r})}{\partial n} = 0$ and substituting motion of the plate into (13), one can reduce Kirchhoff-Helmholtz integral to Rayleigh's integral given as

$$P(\vec{r}) = \frac{j\overline{w} r_0}{2p} \int_{s_p} \frac{V_p e^{-jkR^*}}{R^*} ds_p \tag{14}$$

where $V_p(x,y)$ is the velocity amplitude of the holed plate obtained from (12).

Scattering sound field

Scattering field is consequent on sound pressure passing through the hole on the plate. Kirchhoff-Helmholtz integral (13) is again employed for the result with $\vec{r}\,*$ a point located on the hole [7]. In the integral, the derivative $\frac{\partial P(\vec{r}\,*)}{\partial n}$ is expressed as

$$\frac{\partial P(\vec{r}*)}{\partial n} = -\frac{Q(t^*)}{4p} \frac{\partial}{\partial z} \left\{ R* - \frac{a\Omega}{c} \left[-(x - x_0)\sin(\Omega t^*) + (y - y_0)\cos(\Omega t^*) \right] \right\}^{-1} \tag{15}$$

and $P(\vec{r}*)$ is directly obtained from the rotating source. Choosing Green's function as $G(\vec{r}) = g(\vec{r})$ yields the scattering sound pressure. This concept is also referred as Fresnel' s theory of diffraction.

NUMERICAL RESULTS AND DISCUSSIONS

Consider a simply-supported aluminium plate with $b = 0.1\text{m}$ and $h = 8\times10^{-4}\text{m}$. The holes are distributed over the plate in a symmetric way as shown in Fig. 2.

Retarded time

The difference between the retarded time $t*$ and time t are given in Fig. 3 and 4 for varying radius a and frequency Ω , respectively. Here, the source location is $(x_0, y_0, z_0) = (0.05, 0.05, 0.03)$ and T represents $2p/\Omega$. From the numerical results, the time difference $t - t*$ is small, and furthermore it is found nearly a periodic function of t with frequency Ω. Therefore, in eq. (10), the pressure on the plate from the rotating source is also a periodic function formulated by a Fourier series. From Fig. 3, larger rotating radius a gives larger time difference. From Fig. 4, varying the rotating frequency Ω can only yield negligible effect on the time difference.

Sound pressure on the plate

Amplitudes of sound pressure due to the rotating source on different locations are illustrated in Fig. 5. Here, pressure amplitudes are found periodic functions with frequency Ω. Note that the pressure amplitude on $(x, y) = (0.05, 0.05)$ is constant since the center of the rotating source is exactly on top of this point.

Displacement of the holed plate excited by the rotating source

The displacement amplitudes of the plate are shown in Fig. 6 and Fig. 7 where $w = 2500\text{Hz}$, $\Omega = 500\text{Hz}$, 9 holes, and total hole area $0.030\times0.030m^2$. The displacement distribution is quite similar to the combination of mode shapes (2,3) and (3,2) (Fig 2) which have a common natural frequency 2523.7Hz [5,9]. Figure 7 yields larger displacements, compared to Fig. 6, since a larger rotating radius a gives more loading on the plate.

Corresponding sound field

Figures 8 and 9 present the scattering pressures P_{sh}, for different hole areas, where the sound pressure is solved at the point $z*$ above the center of the plate. From the numerical results, number of holes does affect the scattering pressure probability because fewer holes with large area can reinforce the constructive interfering effect. Furthermore, large rotating radius a or hole area can increase the sound pressure.
Figures 10 and 11 give the radiated pressures P_{sp}. The amplitudes of the radiated

pressure is generally a function of the vibrating pattern of the plate, and the difference between the excitation frequency and its nearest natural frequency.
Total sound field is the sum of the radiated and the scattering sound fields as shown in Fig. 12. In our cases, most contribution to the sound field comes from the scattering pressure. Large hole area and radius of rotating can enhance sound pressure.

SUMMARY

The dynamics of a holed plate, due to a rotating sound source, is studied by artificially applying "ectoplasm" on the plate. The sound field resulted from the sound source and the vibration plate is also investigated. The rotating effect of the source introduces a new time scale, retarded time, to the system. This, then, gives an additional harmonic frequency to the loading to the plate. The vibration amplitude of the holed plate is dependent on the rotating radius and the rotating frequency of the source as well as the hole arrangement of the plate. In our cases, most contribution to the sound field results from the scattering pressure although the hole area is less than the holed plate area.

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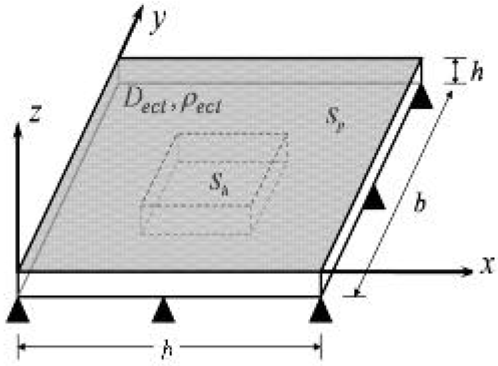

Fig. 1 *The holed plate and the ectoplasm*

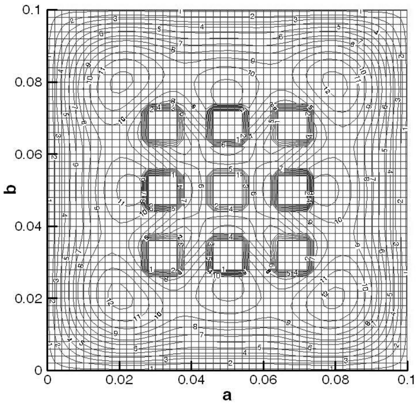

Fig 2. *Combined Mode shape of (2,3) & (3,2)*

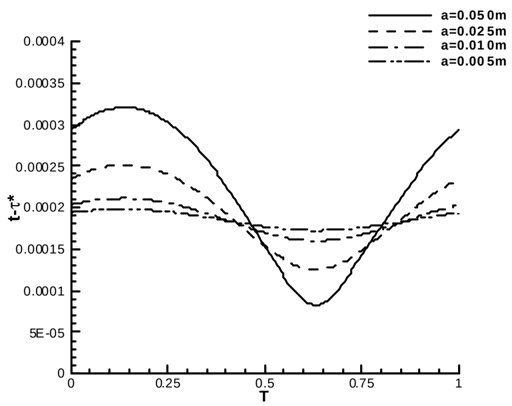

Fig 3. *Difference between t and* t^* ($\Omega = 50\text{Hz}$)

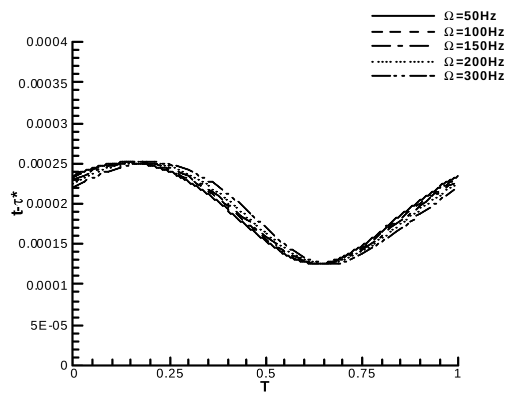

Fig 4. *Difference between t and* t^* ($a = 0.025\text{m}$)

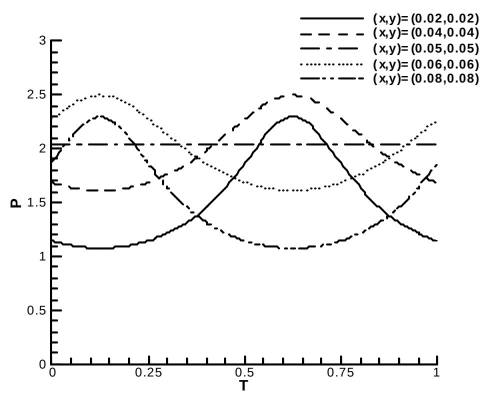

Fig. 5 *Sound pressures on the plate*

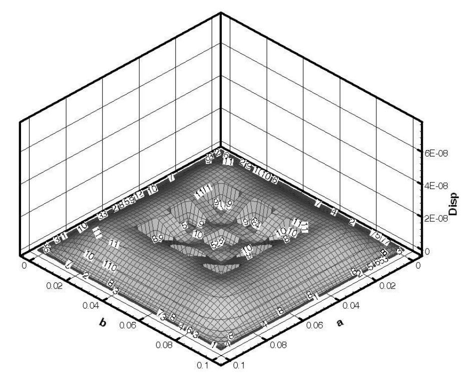

Fig. 6 *Displacement of the plate* ($a = 0.025\text{m}$)

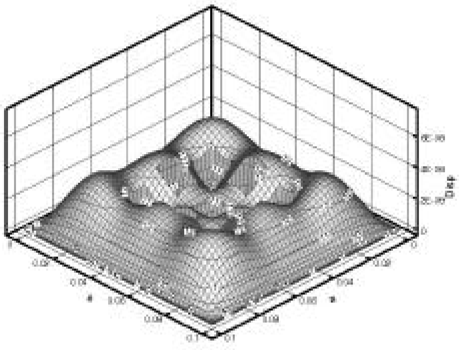

Fig. 7 *Displacement of the plate* $(a = 0.05\mathrm{m})$

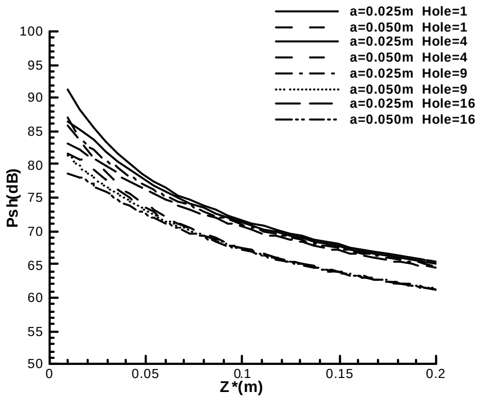

Fig. 8 *Scattering sound pressure (total hole area* $0.030\times0.030\mathrm{m}^2$ *)*

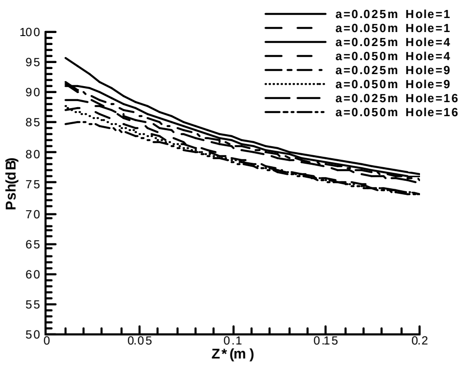

Fig. 9 *Scattering sound pressure (total hole area* $0.060\times0.060\mathrm{m}^2$ *)*

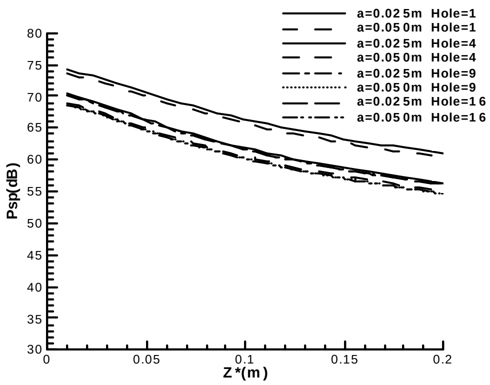

Fig. 10 *Radiated sound pressure (total hole area* $0.030\times0.030\mathrm{m}^2$ *)*

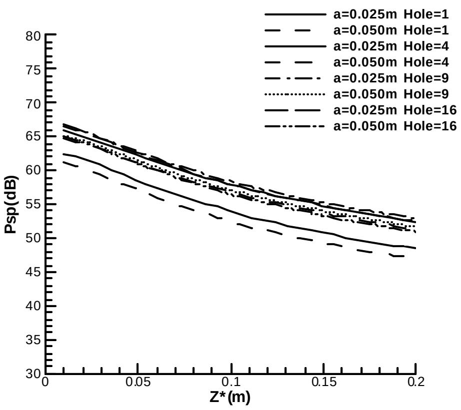

Fig. 11 *Radiated sound pressure (total hole area* $0.060\times0.060\mathrm{m}^2$ *)*

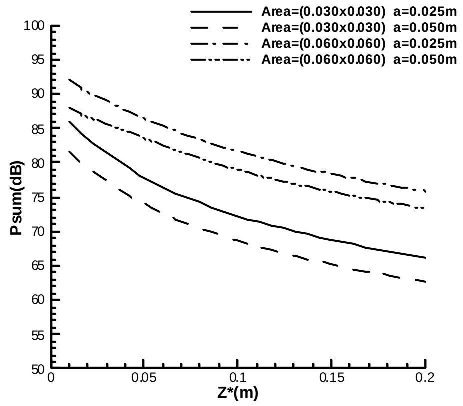

Fig. 12 *Total sound pressure*