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OPTIMAL SHOCK-WAVE STRUCTURES AND NEW IDEAS ABOUT SUPERSONIC GAS JET NOISE GENERATION

Vladimir N. Uskov[*1], Mihail V. Chernyshov[2], Valery K. Erofeev[1], and Pavel Genkin[1]

[1]Department of Plasma and Gas Dynamics, Baltic State Technical University
1 1st Krasnoarmeyskaya Str., 190005 St. Petersburg, Russia
[2]Special Materials Ltd., 28A Bolshoy Sampsoniyevsky Ave., 190044 St. Petersburg Russia
mvcher@newmail.ru

Abstract

New approximate analytical model for the prediction of the frequency and other parameters of the shock-wave noise of supersonic jets is suggested. Some special features of shock-wave structure (triple configurations of stationary shocks) are studied in connection with shock-wave noise problem.

INTRODUCTION

Generation of intensive discrete noise by supersonic gas jets depends on their shock-wave structure. For example, auto-oscillations evidently occur at small supersonic Mach numbers where solutions for both stationary regular and Mach reflections do not exist [2].

Incident shock geometry is supposed to be key parameter defining shock-wave noise frequency. Unlike popular Powell's scheme, proposed physical model of discrete noise generation is based on treatment with jet barrels like volumes with elastic walls and corresponding natural frequencies. Shock-wave structures are treated like three-dimensional Helmholtz resonators or systems with two or three resonance frequencies, etc. Natural frequencies can be calculated according to classical works of Rayleigh, Helmholtz and others. Comparison of the numerical (modified Godunov-type schemes were used) and experimental data reveals that natural frequencies were successfully (with 5-10% error) estimated analytically.

Another possible reason of the oscillations at the discrete frequencies is the enormous difference of some parameters (stagnation pressures, gas velocity) of flows separated by the slipstream emanating from the triple point of Mach reflection is also expected to be source of auto-oscillating effects. This analytical study also defines extreme differences of flow parameters downstream triple configurations.

ANALYTICAL MODEL OF SUPERSONIC GAS JET NOISE GENERATION

Except of immediate studies of gas dynamics and geometrical parameters of supersonic gas jet flows, aeroacoustical processes (i.e. the generation of the acoustical fields by these jets) draw the scientific attention traditionally (see, for example, [6]). Several approximate models trying to clarify the physical mechanism of generation of the acoustical waves by gas streams are worked out nowadays. Gasodynamical regimes of the stream (supersonic or subsonic, incalculability, flow stucture, etc.) are primarily sufficient in noise generation.

Lighthill’s model of acoustical analogy, Powell’s model of discrete-frequency shock-wave noise, model of Mach stem oscillations are accepted universally.

Basing on our studies of gas dynamics of imperfectly expanded supresonic jets, we suppose now new physical and mathematical model of shock-wave discrete-frequency noise generation which differs from Powell’s model, being free from a number of its disputable assumptions and restrictions.

We regard the shock-wave structures (the “barrels”) of incalculable supersonic jets as volumes with elastic boundaries. As any physical volume, these structures have own natural frequencies with harmonics that depend on its dimensions, geometry, presence of orifices, etc.

Different mechanisms of natural frequency excitation in these structures can be presented by such models of noise generation as Helmholtz volume resonator with one or two orifices, quarter-wave and half-wave noise emitters, two-resonant and three-resonant systems, and so on. Their natural frequencies can be computed using the following relations:

$$f_0 = \frac{a_0}{2\pi}\sqrt{\frac{d_\Gamma \varepsilon}{V_0}}\,, \tag{1}$$

$$f_0 = \frac{a_0}{2\pi}\sqrt{\frac{K}{V_0}} = \frac{a_0}{2\pi}\sqrt{\frac{S_\Gamma}{l_\Gamma V_0}}\,, \tag{2}$$

$$f_{1/4} = \frac{a_0}{4H}\,, \tag{3}$$

$$f_{1/2} = \frac{a_0}{2H}\,, \tag{4}$$

$$f_{02} = \frac{a_0}{2\pi}\sqrt{\frac{d_1\varepsilon_1 + d_2\varepsilon_2}{V_0}}\,, \tag{5}$$

$$\left(n_\Sigma^2 - \frac{k_2 a^2}{V_2}\right)\cdot\left(n_\Sigma^2 - \frac{k_2 a^2}{V_2}\right) - \frac{k_2 a^2}{V_1} n_\Sigma^2 = 0\,. \tag{6}$$

Here d_Γ and l_Γ are nozzle throat diameter and length, correspondingly; $V_{0,1,2}$ are volumes of the given resonators; $n_\Sigma = 2\pi f_\Sigma$ is the cyclic frequency; $K = S_\Gamma / l_\Gamma$ is throat conductivity; d_a and d_{MD} are nozzle exit section and Mach disc diameters, correspondingly; a_0 is stagnation sound velocity. Formula (1) relates to three-dimensional Helmholtz resonator with plane throat; (2) relates to three-dimensional Helmholtz resonator with long throat; (3) – to quarter-wave generator (oscillator) with higher harmonics; (4) – to half-wave generator with higher harmonics; (5) – to three-dimensional Helmholtz resonator with two orifices; relation (6) describes the two-resonant system.

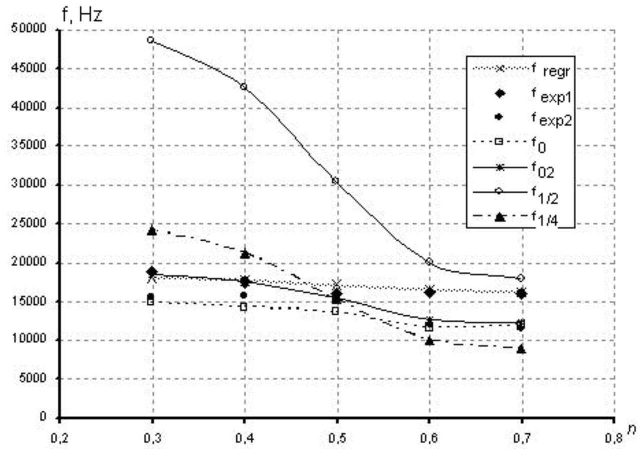

Figure 2 – discrete frequencies as functions of jet incalculability

Physical model written earlier allows us to estimate the range of discrete frequencies emitted depending on the geometrical parameters of the shock-wave systems of supersonic jets.

Experimental research of amplitudes and frequencies of acoustical fields generated by supersonic air jets at Mach numbers $M_a = 1 - 3$, range of incalculabilities $n_a = 0.3 - 1.2$, and stagnation temperature $T_0 = 274$ K have shown us that, in fact, all basic discrete tones can be calculated using the formulas of natural frequencies for the volumes of the first and the second barrels.

Depending of occasional exterior conditions, discrete frequencies can be rebuilt. They change, but physical mechanisms of their generation preserves. High-speed imaging of gasodynamical structures convinced us in the existence of discontinuity surface pulsations with the frequencies characteristic for prescribed discrete tones.

We computed the flow field and, consequently, the geometry of shock-wave structures using the method of characteristics and several modifications of well-known Godunov's method. Mach number, nozzle apex half-angle (θ), and incalculability (n) were used as variable parameters. Different combinations of the parameters allowed us to fulfil the statistics of experimental and computational results. Dependencies for noise frequencies and geometrical parameters of shock-wave structures were interpolated from the statistics achieved.

Geometrical parameters of the "barrels" calculated by the methods of computational gas dynamics were used for the estimations of the natural frequencies of these “barrels”. The comparison of achieved data revealed that the difference between experimental quantities of discrete natural frequencies and computationally achieved ones is not more than 5-10%. For example, the following results are received for the nozzle at Mach number $M_a = 2.5$, apex half-angle $\theta = 7.5^\circ$, and diameter of the critical section $d_{cr} = 4.8$ mm computationally and experimentally: $n = 0.5$: $f_{comp} = 15480$ Hz, $f_{\exp} = 15000$ Hz; $n = 0.6$: $f_{comp} = 11970$ Hz, $f_{\exp} = 11950$ Hz; $n = 0.7$: $f_{comp} = 11600$ Hz, $f_{\exp} = 11400$ Hz.

Models of monopole spherical of cylindrical sources with equivalent volume of surface area were used for the approximate estimations of the amplitudes of the discrete tones. Depending of the characteristic lengths of noise-emitting surfaces of jet barrels, of the discrete frequencies and corresponding wavelengths, realizations of “piston” and “monopole” noise sources are possible. Experimental and computational investigations were also done at the following conditions: exit Mach number $M_a = 1-3$; range of incalculabilities $n = 0.3-1.0$; air as jet medium; ratio of the specific heats $\gamma = 1.4$; stagnation temperature $T_0 = 294$ K. Such quantities as discrete frequencies and the intensity of acoustical pressure were measured several times.

Table 1

n	*f, kHz*	*T, μs*	*S, mm²*	ΔP, *bar*	*k*	*r, mm*	*W, Wt*	*J, Wt/m²*	L_p, *dB*	L_e, *dB*
0.4	17.578	56.9	52.8	0.598	330	2.05	235	18.7	133	129
0.5	16.015	62.4	79.6	0.499	330	2.52	246	19.6	133	131
0.6	16.250	61.5	93.1	0.398	305	2.72	183	14.6	132	130
0.7	16.093	62.1	105.0	0.296	302	2.89	114	9.10	130	130

Numerical studies using the method of characteristics and Godunov-type methods were also fulfilled for parameters and geometry of shock-waves systems.

Several results achieved during these investigations is presented below. Most general plot of experimentally and computationally achieved dependencies of natural frequency vs. jet incalculability is shown at Fig.1.

For comparison, usage of the analogous dependencies based on Powell’s model

$$1/f_p = L(1/a_0 + 1/w),$$

provides us with the sufficiently overestimated quantities of the own frequency at small incalculability.

It is necessary to remark that several discrete frequencies of the different intensity or only alone (energy-carrying) discrete frequency can exist at the same initial conditions (regime of gas dynamics). As a rule, all existing discrete frequencies can be computed using the formulas (1-6) written earlier with the accuracy of 5-10%. It seems that the realization of the definite set of supposed discrete frequencies depends on many factors such as surrounding media parameters.

Results of numerical and experimental studies necessary for the computation of amplitudes of given discrete frequencies are given at the Table 1. Here n is jet incalculability; f is the frequency of the discrete tones; T is the period of the oscillations; ΔP_{Σ} is the averaged static pressure difference of the surface of discontinuity; S is the area of pulsating peripheral surface of the "barrel"; r is the length scale characteristic for this pulsating surface; k is wave number; W is the acoustical power, Wt; L_c is noise intensity calculated; L_e is noise intensity experimentally defined.

As the first approach, supposing $2r \approx \lambda$, one can calculate the acoustical power emanating using the relation for the "piston" emitter: $W = 0.5 \cdot \rho a_0 v^2 S$, where v is the velocity of oscillations at the pulsating surface; ρ is the density of the gaseous media. Table 1 contents the results of our computations and experimental data for the noise power and intensity at the distance of 1 m.

Comparison and analysis of achieved results allows us to conclude that supposed simple physical model of the generation of shock-wave noise as well as the methods of the calculation of amplitudes and frequencies can be used for the approximate estimations of discrete-tone noise generated by the supersonic jets. But it is not enough for the theoretical establishing of supposed mathematical model, and these studies must be continued.

OPTIMAL SHOCK-WAVE STRUCTURES (TRIPLE CONFIGURATIONS)

Shock-wave systems consisted of three stationary shocks with common (triple) point T (Fig. 2,a-e) are called triple configurations. The slipstream (τ) emanates from the triple point and divides the streams that have gone through the sequence of shocks 1-2 and through the alone (main) shock 3 at another side of the triple point.

Three sorts of the triple configurations are distinguished according to the direction of flow deflection on the shocks 1-3. Flow deflection direction on the shock 1 differs from the analogous directions on the shocks 2 and 3 at the triple configurations of the first type (TC-1, Fig. 2,a). Flow deflection direction at the shock 2 differs from others at TC-2 (Fig. 2,b), and flow deflections on all shocks forming TC-3 (Fig. 2,c) are at the same direction. Stationary Mach configuration (SMC, Fig. 2,d) with normal main shock is intermediate between the first type and the second one, and the configuration with normal shock 2 (ITC, Fig. 2,e) is intermediate between the second and the third type.

Our interest was drawn to the triple configurations due to the large differences of flow parameters at the slipstream emanating from the triple point. Many flow parameters (such as full (stagnation) pressure p_0, temperature T, acoustic impedance $z = \rho a$, functions $q = \rho v$, $d = \rho v^2$, $j = p + \rho v^2$) behind the shocks 2 and 3 can differ in dozens and hundreds times. It is theoretically proven that shock-wave structures with extreme relations of all mentioned quantities exist [10, 11]. We call the configurations with extreme relations of flow parameters downstream optimal.

Large distinction of flow parameters (of stagnation pressures, especially) downstream the triple point assists to the development of different instabilities and, among them, to the excitation of self-oscillations in supersonic gas jets [4, 5]. Connection between self-oscillation excitation and extreme triple configurations is already discussed in [10, 11]. At the same time, excitation of self-oscillating regimes of jet-obstacle interaction can be extremely undesirable (when rocket starts) and, on the contrary, can be used in technological processes (in metallurgy, when we create the pulsating flow out of the nozzles [9]).

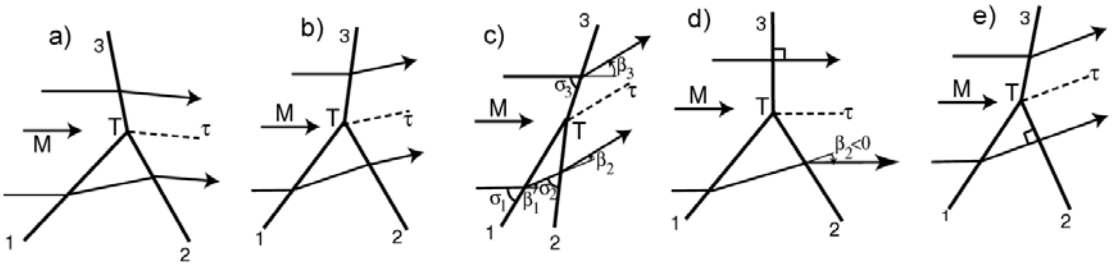

Figure 2 – Classification of the stationary triple configurations

Stationary shocks forming some triple configurations can have the special strength (J_i ($i=1..3$) is the strength of *i*-th shock that is equal to the relation of static pressures behind and before it): the strength $J_m(M_k)$ of the normal shock at the stream with Mach number M_k; the strength $J_*(M_k)$ of stationary shock with Mach number behind equal to unity; the strength $J_l(M_k)$ of the shock with maximum deflection angle; and also the strength $J_c(M_k)$ (Crocco point) and $J_p(M_k)$ (constant pressure point) connected with the differential parameters of the flowfield [1]. Crocco and constant pressure points relate to the shocks that curves strongly even at the presence of small gradients of flow parameters downstream, so it can relate to shock instability and, consequently, shock-wave noise. Flow behind mentioned shocks is subsonic ($J_* < J_c < J_l < J_p < J_m$ at the same Mach number). Triple configurations with the shocks of special strengths are described at [10, 11].

Studied relations of downstream flow parameters $I^{(f)} = f_2/f_3$ can be exressed through shock strength and flow Mach number, Among them,

$$I_0 \equiv I^{(p_0)} = \frac{p_{02}}{p_{03}} = \left(\frac{E_3}{E_1 E_2}\right)^{\frac{1+\varepsilon}{2\varepsilon}},$$

where $E_i = (1+\varepsilon J_i)/(J_i+\varepsilon)$, $\varepsilon = (\gamma-1)/(\gamma+1)$, γ is the ratio of the specific heats. Ratios of many gas parameters (densities, temperatures, sound speeds, acoustic impedances, etc.) are simply power functions of $I^{(p_0)}$. It is evident that the configurations optimal for $I^{(p_0)}$ are also optimal for the set of other thermodynamic parameters. But configurations optimal for parameters dependent on flow velocity do not coincide with the optimal for $I^{(p_0)}$ in common case:

$$I^{(v)} \equiv \frac{v_2}{v_3} = \frac{M_2}{M_3}\sqrt{\frac{E_1 E_2}{E_3}},\ I^{(q)} = \frac{M_2}{M_3}\sqrt{\frac{E_3}{E_1 E_2}},\ I^{(d)} = \frac{M_2^2}{M_3^2},\ I^{(j)} = \frac{1+\gamma M_2^2}{1+\gamma M_3^2}$$

These relations represent the objective functions in our problem. Computational results for optimal triple configurations are given further at the value of $\gamma = 1.4$.

The fullest description of optimal triple configurations and their parameters can be found in [11]. Now we can only remark that optimal configuration exist, as well as the optimal sequences of stationary shocks at whole [3, 7, 8], at almost every supersonic Mach number. The configurations of the third type (TCs-3) are optimal at the region of small Mach numbers (beginning from $M = 1.245$). Intermediate configurations ITCs are optimal for $I^{(p_0)}$ at $M = 1.596$ ($I^{(p_0)} = 1.076$ there); for $I^{(v)}$ at $M = 1.567$ ($I^{(v)} = 1.085$); for $I^{(q)}$ at $M = 1.571$ ($I^{(q)} = 1.107$); for $I^{(d)}$ at $M = 1.569$ ($I^{(d)} = 1.201$); for $I^{(j)}$ at $M = 1.584$ ($I^{(j)} = 1.090$). After these points, optimal configurations belong to the second type.

Configurations TC-2 are optimal up to the Mach number $M_a = \sqrt{4 - 3\varepsilon + \varepsilon^2}\big/(1-\varepsilon) = 2.254$ where SMC-configuration becomes optimal for all objective functions. Strengths of the incident (1) and the reflected (2) shocks are equal at this SMC: $J_1 = J_2 = 2/(1-\varepsilon) = 2.4$. It is proven [7] that the equality of the strengths leads to the maximum of stagnation pressure behind shock sequence when the production of these strength is fixed. Though this production (equal to strength J_3) is not constant here, optimal SMC subordinates to this theorem. Studied ratios downstream this SMC are the following: $I^{(p_0)} = 1.448$, $I^{(v)} = 1.649$, $I^{(d)} = 3.024$, $I^{(j)} = 1.587$, and so on.

First-type configurations are optimal at $M > M_a$. Though, as we have seen earlier, maximal relations $I^{(f)}$ are not large at small and moderate Mach numbers, optimal values of these objective functions increase slowly and finitely and reach large quantities at hypersonic flow velocities. For example, optimal relations at $M \to \infty$ strive to the following limits:

$$I^{(p_0)} = \varepsilon^{-\frac{1+\varepsilon}{2\varepsilon}} = 529.1,\ I^{(v)} = 5.261,\ I^{(d)} = 155.8,\ I^{(q)} = 30.41,\ I^{(j)} = 30.22\,.$$

But the configurations TCs-3 exist also where the relations of the parameters strive to the same or close values:

$$I^{(p_0)} = \varepsilon^{-\frac{1+\varepsilon}{2\varepsilon}} = 529.1,\ I^{(v)} = \sqrt{\frac{K + (1-\varepsilon)H}{2\varepsilon^2}} = 5.033,$$

$$I^{(d)} = \frac{K + (1-\varepsilon)H}{2\varepsilon^3} = 152.0,\ I^{(q)} = I^{(j)} = \frac{1 - \varepsilon + H}{2\varepsilon^2} = 30.20$$

($H = (1-\varepsilon)\sqrt{(1-\varepsilon)^2 + 4\varepsilon^3}$, $K = 1 - 2\varepsilon + \varepsilon^2 + 2\varepsilon^3$). All these types of configurations with extremely large difference of flow parameters not exist at jet flow with Mach reflections: TCs-1, as a rule, appear at the intersection of two shocks of the different direction, TCs-3 – of the same direction. Maximal relations of flow parameters downstream the triple point of irregular shock reflection can be estimated at the example of SMCs in its limiting ($M \to \infty$) case. Indeed, all functions $I^{(f)}$ in SMCs increase monotonously but reach significantly smaller values: $I^{(p_0)} = 69.72$, $I^{(v)} = 5.059$, $I^{(q)} = 17.01$, $I^{(d)} = 86.05$, $I^{(j)} = 15.17$. So the optimization of triple configurations significantly changes the parameters downstream.

Flow downstream the shock 3 at the majority of optimal configurations is subsonic. This promotes the reverse influence on the system from the obstacle upstream and do not prevent the appearance of self-oscillating regimes. Practical importance of not relations, but differences of such flow parameters as stagnation pressures, temperatures, acoustic impedances [11] increase at large Mach numbers.

CONCLUSIONS

Study of the connection between supersonic gas jet noise of discrete frequencies and special (optimal) features of shock-wave flow structures seems to be at the initial stage but provides us with simple and rather accurate approximate models.

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REFERENCES

[1] Adrianov A.L., Starykh A.L., Uskov V.N., *Interaction of discontinuities in gas dynamics.* (Nauka, Novosibirsk, 1995 [in Russian]).

[2] Bulat P.V., Zasuhin O.N., Uskov V.N., “Jet formation on the smooth work of de Laval nozze”, Gas Flow in Channel and Nozzles, **10**, 3-22 (1993 [in Russian]).

[3] Erofeev V.K., Omelchenko A.V., Uskov V.N., “The analysis of acoutical impedance in stationary supersonic flows”, J. Engr. Phys. Therm. Phys., **71**, 4, 663-668.

[4] Glaznev V.N., Zapraygayev V.I., Uskov V.N. et al., *Jets and non-stationary flows in gas dynamics.* (Nauka, Novosibirsk, 2000 [in Russian]).

[5] Gorshkov G.F., Uskov V.N., “Auto-oscillations in supersonic impact jet systems”, Adv. in Mechanics, **2**, 1, 47-67 [in Russian].

[6] Henderson B., Bridges J., Wernet M. “An experimental study of the oscillatory flow structure of tone-producing supersonic impinging jets”, J. Fluid Mech., **542**, 115-137 (2005).

[7] Omelchenko A.V., Uskov V.N., “Optimal shock systems”, Fluid Dyn., 6, 118-126 (1995).

[8] Omelchenko A.V., Uskov V.N., “Optimal shock-wave systems at the restrictions on the total flow deflection angle”, Fluid Dyn., 4, 142-150 (1996).

[9] Sizov A.M., *Gas dynamics and shock transfer in metallurgical processes.* (Mashinostroyenie, Moscow, 1987 [in Russian]).

[10] Tao Gang, Uskov V.N., Chernyshov M.V., “Optimal triple configurations of stationary shocks”, Proc. of the XXIV Int. Symp. on Shock Waves, **2**, 499-504 (Springer, 2005).

[11] Uskov V.N., Chernyshov M.V., “Special and extremal triple configurations of the shocks”, Appl. Mech. Tech. Phys., **47**, 4 (2006).