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SOUND FIELD REPRODUCTION IN CLOSED SPACES USING THE BOUNDARY ELEMENT METHOD

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Abstract
In this paper the Boundary Element Method (BEM) is used for determining the transfer matrix that relates the internal field pressures to the source excitation inside an arbitrarily shaped enclosure with a known impedance boundary condition. Given a distribution of point sources inside an enclosure the transfer matrix is evaluated applying the BEM, and the optimal solution for the production of a desired sound field is then obtained by minimising, in the least square sense, the difference between the reproduced and the desired sound field at a finite number of receiving positions. Simulation results for the production of a plane wave inside an L-shaped and a regular hexagonal room are presented to validate the feasibility of the process.

INTRODUCTION

With the relatively recent arrival of digital representation and multiple channels of audio, new possibilities of sound-field control and sound reproduction have been investigated. In a broad sense, most of these techniques try to increase the listening area and to create a desirable 3D or 2D auditory scene. Sound field reproduction has been investigated in both free field [1] and in-room conditions [2],[3]. In the latter case, the reproduction system has to compensate for the room's natural dynamics which can vary in an unpredictable manner because of differences in the geometry or because of reflections.

In this paper BEM is used to evaluate the transfer matrix that relates the internal field pressures to the source excitation in an enclosure of known specific impedance boundary conditions. Conceptually, the reproduction process can be investigated in any enclosure independently of its shape or its boundary properties.

THEORETICAL MODEL

This section introduces the theoretical model on which the simulations are based. The discussion begins by assuming that the transfer matrix that relates the internal field pressures to the source excitations is already known. How this relation is achieved will be shown after the oncoming section, where the BEM formulation will be explained and coupled to the sound field reproduction process.

Control Modelling

The reproduction system is characterized by M sound-pressure sensors surrounded by L sources inside the enclosure. The M sensors are properly distributed in the enclosure in the sense that they cover the listening zone of interest and their spacing is capable of sampling the properties of the generated field adequately. The investigation here focuses on the determination of the optimal source strength vector $\mathbf{S}=[S_1...S_j...S_L]^T$ that when applied on the sources an actual field is produced as close as possible to a desired one. The actual field, which is sampled at the M receiving positions at $\{\mathbf{r}_1,\mathbf{r}_2,...,\mathbf{r}_M\}$, is expressed by the pressure vector $\hat{\mathbf{p}}_f$ which is related to the source excitation by

$$\hat{\mathbf{p}}_f = [\hat{p}(\mathbf{r}_1)...\hat{p}(\mathbf{r}_m)...\hat{p}(\mathbf{r}_M)]^T = \mathbf{ZS}\,. \tag{1}$$

Here $\mathbf{Z}$ is an MxL transfer matrix which is obtained by BEM. In what follows, the number of sensors is considered to be greater than the number of sources ($M>L$), and thus the system in Eq. (1) is over-determined. Let now $\mathbf{p}_f$ be the vector of the desired values of pressures over the above positions. If $\hat{\mathbf{p}}_f$ is replaced by $\mathbf{p}_f$ in Eq. (1), Singular Value Decomposition (SVD) can be used to solve for the optimum complex source strength vector $\mathbf{S}$. The effectiveness of the reproduction process is measured by the fitting in a least square sense between the desired field pressure and the reproduced field, where the error at the m^{th} receiving point is defined as the difference between the actual and the desired sound pressure. The error is averaged over all the sensor positions and normalised to the desired pressure amplitude, leading to the definition of the cost function defined by

$$J = \frac{\mathbf{e}^H\mathbf{e}}{\mathbf{p}_f^H\mathbf{p}_f} = \frac{1}{\mathbf{p}_f^H\mathbf{p}_f}\sum_{m=1}^{M}\left|\hat{p}_f(\mathbf{r}_m) - p_f(\mathbf{r}_m)\right|^2 , \tag{2}$$

where $\mathbf{e}$ is a Mx1 vector containing the errors at the receiving positions and $p_f(\mathbf{r}_m)$ is the desired pressure at the m^{th} receiving position. Obviously, a small value of J denotes a good fit between the desired and the reproduced field pressures.

Relating the Field Pressure to the Source Excitation

For a time harmonic acoustic field in a closed region D with boundary S and exterior E the well known Kirchhof-Helmholtz integral equation can be modified to include the spherical wave potential of L point sources radiating inside the enclosure and expressed as follows [4]

$$c(\mathbf{r})p(\mathbf{r}) = \int_S \left[p(\mathbf{r}_0)\frac{\partial G(\mathbf{r}-\mathbf{r}_0)}{\partial \hat{\mathbf{n}}_0} - \frac{\partial p(\mathbf{r}_0)}{\partial \hat{\mathbf{n}}_0} G(\mathbf{r}-\mathbf{r}_0) \right] dS + \sum_{l=1}^{L} S_{\mathrm{p}} \frac{e^{-ik|\mathbf{r}-\mathbf{r}_l|}}{4\pi|\mathbf{r}-\mathbf{r}_l|}, \quad (3)$$

where $\hat{\mathbf{n}}_0$ is the surface unit normal vector at the surface point $\mathbf{r}_0$, $p(\mathbf{r}_0)$ is the pressure at that point, $p(\mathbf{r})$ is the pressure at the field point $\mathbf{r}$, $\mathbf{r}_l$ and S_l denote the position vector and the strength of the l^{th} point source respectively, and G is the free-field Green's function, defined as $G(\mathbf{r}-\mathbf{r}_0) = \exp(-ikR)/4\pi R,$ where k is the wave number and $R = |\mathbf{r}-\mathbf{r_0}|$ denotes the distance between the field and the surface point. In this paper the constant boundary element formulation is considered, and thus the acoustic pressure and the normal velocity are considered to be constant over each triangular element and equal to their respective values at the centre of each element. In this way a 'smooth' boundary is always achieved.

Let the enclosure be bounded by surfaces of known specific normal acoustic impedance, $\mathrm{z} = p/u_n = -i\omega\rho_0 p/(\partial p/\partial\hat{\mathbf{n}}_0)$, where p is the surface pressure, ρ_0 is the density of the medium, ω is the angular frequency, and $\hat{\mathbf{n}}_0$ is the unit vector normal to the surface. By dividing the boundary into N constant triangular elements and taking the field points at the element's centre Eq. (3) is modified to the following matrix form

$$\mathbf{Hp} = \mathbf{Gv} + \mathbf{c}. \quad (4)$$

Here $\mathbf{p}$ and $\mathbf{v}$ are Nx1 complex vectors denoting the acoustic pressure and the normal component of the particle velocities on the surface respectively, $\mathbf{H}$ and $\mathbf{G}$ are the NxN influence matrices, and $\mathbf{c}$ is the Nx1 vector

$$\mathbf{c} = [c_n] = \left[\sum_{l=1}^{L} S_l \frac{e^{-ik|\mathbf{r}_n-\mathbf{r}_l|}}{4\pi|\mathbf{r}_n-\mathbf{r}_l|} \right], \quad (5)$$

where $\mathbf{r}_n$ and $\mathbf{r}_l$ are the position vectors of the n^{th} element and the l^{th} point source respectively. Considering now M field points inside the enclosure, the matrix expression of Eq. (3) takes the form

$$\mathbf{p}_f = \mathbf{H}_f\mathbf{p} + \mathbf{G}_f\mathbf{v} + \mathbf{c}_f\,. \quad (6)$$

Here $\mathbf{H}_f$ and $\mathbf{G}_f$ are MxN matrices and $\mathbf{c}_f$ is similar to Eq. (5), but this time $\mathbf{r}_n$ is replaced by $\mathbf{r}_m$ which is the position vector of the m^{th} field point.

According to the impedance boundary condition the pressure and the normal velocity vector on the surface are related by

$$\mathbf{v} = \mathbf{R}\mathbf{p}, \tag{7}$$

where $\mathbf{R} = diag(1/z_1, 1/z_2, ..., 1/z_N)$. Inserting Eq. (7) into Eqs. (4) and (6) one obtains

$$(\mathbf{H} - \mathbf{G}\mathbf{R})\mathbf{p} = \mathbf{c} \text{ and } \mathbf{p}_f = (\mathbf{H}_f + \mathbf{G}_f\mathbf{R})\mathbf{p} + \mathbf{c}_f. \tag{8), (9}$$

If $(\mathbf{H} - \mathbf{G}\mathbf{R})^{-1}$ exists then $\mathbf{p}$ can be solved from Eq. (8), and with proper factorisation of the vectors $\mathbf{c}$ and $\mathbf{c}_f$ Eq. (9) can be written as

$$\mathbf{p}_f = \mathbf{Z}\mathbf{S}, \tag{10}$$

where $\mathbf{Z}$ is the MxL transfer matrix that relates the M internal field points to the L point source excitations and $\mathbf{S}$ is the Lx1 vector containing the strengths of the point sources. The optimal source strength vector $\mathbf{S}$ can be calculated using the analysis of the previous section.

SIMULATION RESULTS

To validate the feasibility of the process simulation results for the case of generating a plane wave inside an L-shaped and a regular hexagonal room are presented.

Generation of a Plane Wave in an L-Shaped Room

The first example for the validation of the reconstruction process is an L-shaped room with dimensions L_x = 3.5 m, L_y = 1.5 m, and L_z = 0.3 m, depicted in Figure 1. The boundary of the room is divided into 1712 constant triangular elements which are given a fixed impedance of z=300+300i, corresponding to a very hard and reflecting surface. Ninety sensors are equally distributed over the two sensor planes that are represented by the small dots in Figure 1. Twelve point sources, represented in the figure by the small squares, are positioned in three sets of four sources with the first set being at x=0.1 m with y=d/8, 3d/8, 5d/8 and 7d/8, the second set being at x=y=2+d/8, 2+3d/8, 2+5d/8 and 2+7d/8 on the symmetry line, and the last set being at y=0.1 m with x=d/8, 3d/8, 5d/8 and 7d/8, where d is the actual width of the enclosure being equal to 1.5 m. Since the height of the room is much smaller than the other two dimensions the resulting field is two-dimensional in the frequency range considered here, and thus the vertical positioning of the sources and the sensors does not affect the result. Hence all the sensors and sources are assumed to be at the plane at z=0.15

m. The room geometry as well as the source and sensor configuration is symmetric with respect to the *x*=*y* line.

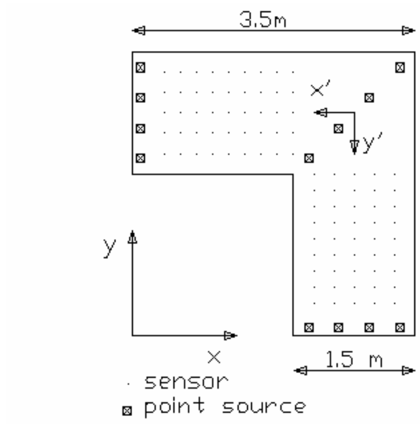

Figure 1 - The distribution of 90 sensor positions and 12 point sources inside the L shaped room. The height of the enclosure is 0.3 m

The challenge in the first simulation is to produce two perpendicular plane waves that travel simultaneously over the x' and y' direction defined by the secondary set of coordinates shown in Figure 1. The amplitude of the desired sound pressure is set to a constant value for all the sensors. The phase of the sound pressure though varies as kx' for the error sensors at the first plane and as ky' for the error sensors at the second plane, so that two propagating plane waves are simulated. The number of four sources along the axis perpendicular to the direction of propagation was found to be the minimum number required for proper plane wave reproduction in the given frequency range. All the elements of the enclosure are given the same high value of acoustic impedance corresponding to a hard and reflecting surface.

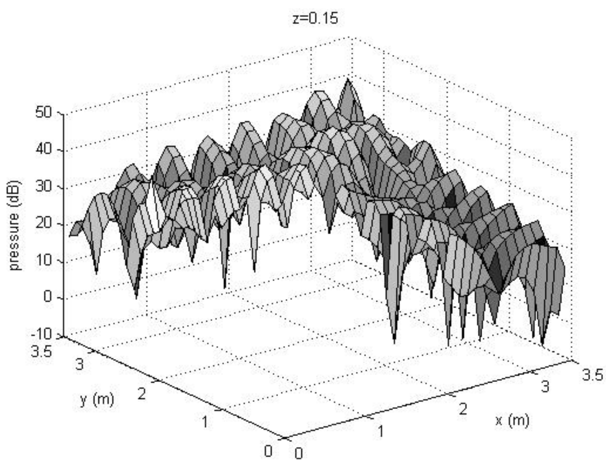

Figure 2 - Amplitude of the sound pressure in the L-shaped room caused by excitation of a point source at (0.1, 0.1875) m at 415 Hz

It is interesting to compare the sound pressure of the resulting sound field to the sound pressure of the sound field generated by exciting the enclosure with a single source as shown in Fig. 2. As can be seen from Fig. 3, a proper adjustment of the sources strengths can remove the spatial fluctuations of the sound pressure so that a uniform sound pressure in almost the entire listening area is achieved.

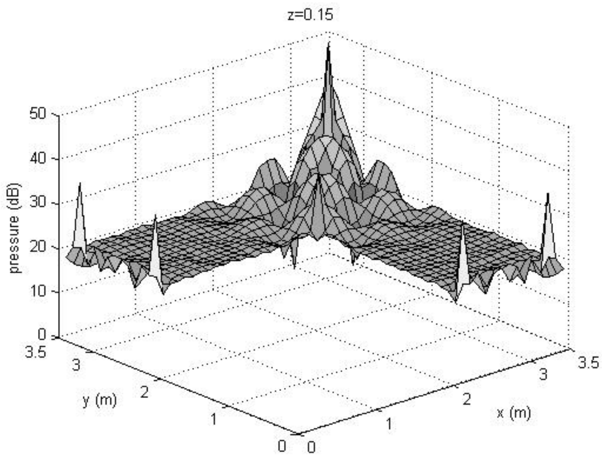

Figure 3 - Amplitude of the sound pressure as a result of two plane waves at 415 Hz

Generation of a Plane Wave in a Regular Hexagonal Room

A regular hexagonal room of height 0.3 m and side length of 1.2 m is selected for this simulation. Eighteen piston sources are placed co-planar to the six side walls in order to control the internal field pressure. The radius of each piston is 0.1 m, and three pistons are placed at each side wall as shown in Figure 4. A total number of 3384 constant triangular elements is used for meshing the enclosure, 720 of which are used for the piston sources and the rest 2664 are given a fixed impedance of z=300+300i, corresponding to a very hard and reflecting surface. The input signals for the pistons are determined through the reproduction process in terms of complex velocity values under the assumption that each piston vibrates as a rigid body.

A rectangular grid of 13x11 error sensors is placed on the x-y plane at the height of z=0.15 m, covering a large portion of the enclosure as shown in Figure 4. The distance between adjacent sensors is 0.1 m along both x- and y- axes allowing for a proper spatial sampling in all the considered frequency range. The image field is a plane wave of unit amplitude travelling parallel to the y-axis. The amplitudes of the sound pressure generated when the room is excited by just one piston source at 346 Hz and when the plane wave is generated inside the room are shown in Figure 5 and 6 respectively. It is evident that a proper source strength adjustment in this kind of enclosure can compensate for the undesired spectral coloration over an area that covers almost the entire volume of the enclosure.

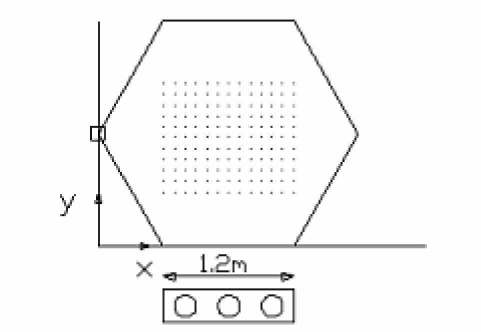

Figure 4 - Distribution of 143 sensor positions and 18 piston sources inside the regular hexagonal room

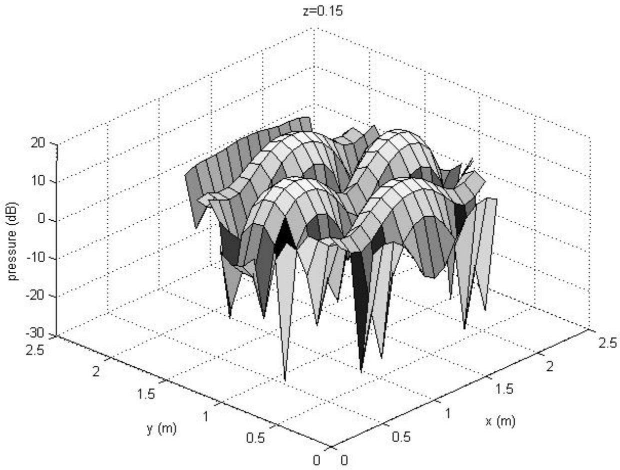

Figure 5 - Sound pressure amplitude caused by the excitation of the hexagonal room with just one piston source at 346 Hz

CONCLUSIONS

In this paper the theoretical possibilities for sound field reproduction using a transfer matrix that is based on algebraic manipulation of the equivalent boundary element problem has been investigated. BEM can be applied to sound field reproduction with satisfactory quantitative reproduction results if the acoustic impedance of the boundary is specified. This method can be applied to arbitrarily shaped enclosures where an analytical solution is not known. Furthermore, sources of more complicated shape and radiation behaviour than the simple monopole can be modelled.

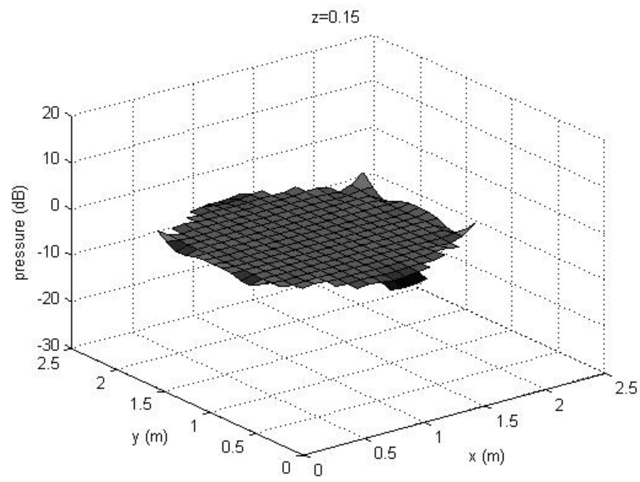

Figure 6 - Sound pressure amplitude as a result of the reproduction of a plane wave at 346Hz

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