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SOUND INSULATION PROVIDED BY A LATERAL CONFINED SINGLE LAYER USING A BEM APPROACH

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Abstract

The sound insulation provided by a single layer partition that is infinite along its plane and divides an infinite acoustic medium is analysed in this paper. The partition is confined laterally (on two sides) and the solution is obtained by means of a Boundary Element Method formulation in the frequency domain, where only the discretization of the restricted surfaces of the panel is required since Green's functions are used for the layered medium. The confinement is achieved by ascribing null displacements to the two boundaries defined by the width of the panel. These boundaries may define panels of varying sizes. The responses are calculated assuming that the incident wave field is generated by cylindrical line loads placed in the acoustic medium. Material losses are taken into account by using a complex Lamé constant and a complex Young's modulus. The analysis is performed in the frequency domain, taking the solution for no confinement as a reference. Simulations are performed for a partition made of various materials, with different lengths and thicknesses. Wave propagation features, including the influence of the vibration modes of a confined partition and the coincidence effect, are analysed.

INTRODUCTION

The transmission of sound through a partition has been widely investigated over recent years. One approach that has been used to predict sound transmission assumes that the partition behaves like a group of infinite juxtaposed masses with independent displacement and null damping forces, and that the source is a single or random plane wave. The well-known theoretical Mass Law is based on these assumptions. However the calculated results often fail to match the predictions so it has been assumed that, to reduce these discrepancies, a limited maximum incidence angle (i.e. diffuseness) should be considered. This improves the results, but differences are still found. In fact the finiteness of the partition and the boundary conditions may also determine the acoustic response provided by the panel. In finite sized partitions additional reflected waves are produced in the boundaries, causing interference with the incident waves propagating inside the panel, and this may result in transverse panel motion. In addition, diffraction via the aperture that contains the panel may also determine the final response and therefore this factor should also be addressed.

In the seventies, Sewell [1] derived an expression to predict sound insulation which assumes the resonance transmission, valid for frequencies bellow the critical frequency.

More recently, M. Villot et al. [2] have proposed a technique based on the wave approach to predict transmission loss provided by single and multilayer partitions of finite size. Their model is based on a spatial windowing to introduce the diffraction effect associated with the finite size. A paper by Jong-Hwa et al. [3] revisits the problem of resonant transmission related to the sound insulation of rectangular finite panels in an infinite baffle, at frequencies below the critical frequency by using the general modal expansion method followed by Sewell. They investigated the validity of neglecting the resonant transmission components in the prediction of transmission loss by calculating the differences between the total transmission loss and the non-resonant transmission loss.

In this work a contribution to this problem is addressed by developing and applying a Boundary Element Method (BEM) model to assess the acoustic behaviour of single partitions confined laterally (on two sides of the partition). The responses are calculated for low, medium and high frequencies, assuming that the incident wave field is generated by cylindrical line pressure loads. Simulations are performed for a partition of varying materials, length and thickness, and the analysis takes the solution for no confinement as a reference. Wave propagation features, including the influence of the vibration structural modes of a confined partition and the coincidence effect, are analysed.

The next section outlines the problem. The Green's functions and the BEM model formulation are then described. The simulations and the discussion of the results complete this paper.

PROBLEM FORMULATION

Consider a single elastic layer of thickness h, infinite along its plane (x and z directions), confined laterally on two sides. The confinement is defined by ascribing null displacements to the two boundaries defined by the width of the panel (see Figure 1). The rigid boundaries may define a partition with a length L along the x direction. Note that these boundaries only limit the panel's dimension along the x direction and that in the z direction no confinement has been defined. This layer divides an infinite acoustic medium with a mass density ρ_f, a Lamé constant λ_f and permits a dilatational wave velocity $c = \sqrt{\lambda_f / \rho_f}$. The material properties of the elastic medium are the density ρ, Poisson's ratio ν and a Young's modulus E. In this medium the propagation occurs following compressional waves with a velocity

$c_L = \sqrt{\frac{E(1-\nu)}{\rho(1-2\nu)(1+\nu)}}$ and shear waves with a velocity $c_S = \sqrt{\frac{E}{2\rho(1+\nu)}}$. The internal material losses are considered by using a complex shear modulus and a complex Lamé constant. The Young's modulus is computed as $E = E_r(1+\mathrm{i}\eta)$, where E_r corresponds to the classic modulus and η is the loss factor. The Lamé constant is written in the same form as the Young's modulus.

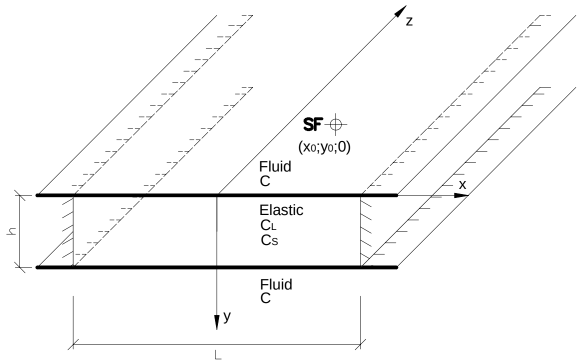

Figure 1 – Geometry of the problem

When the system is excited by a harmonic line load oscillating with a frequency ω and acting in the acoustic medium at (x_0, y_0), the incident pressure field at a point (x, y) can be obtained in the frequency-wavenumber domain, by the following expression,

$$\sigma^{full}(\omega, x, y, k_z) = \frac{-\mathrm{i}A}{2}\mathrm{H}_0^{(2)}\left(k_c\sqrt{(x-x_0)^2+(y-y_0)^2}\right)\mathrm{e}^{-\mathrm{i}k_z z}, \qquad (1)$$

in which A is the wave amplitude and $\mathrm{i}=\sqrt{-1}$; $k_c = \sqrt{(\omega/c)^2 - k_z^2}$, with $\mathrm{Im}(k_c) \le 0$ and k_z being the axial wavenumber. Note that when k_z equals zero expression (1) gives the incident pressure field provided by a cylindrical line load.

GREEN'S FUNCTIONS FOR SINGLE LAYERED MEDIUM

This section briefly describes the procedure used to obtain the 2.5D Green's functions for a single homogeneous elastic layer bounded by two fluid media, when excited by sinusoidal harmonic line loads with different k_z values. These solutions have already been derived by Tadeu et al. [4] and can be expressed as the sum of the source terms equal to those in a full space (which can be calculated according to expression (1), above) and the surface terms generated by the fluid/solid interfaces. The calculation of the surface terms requires the knowledge of the solid layer displacement potentials and the pressure potentials generated by the solid/fluid surfaces. These potentials are written as a superposition of plane waves by means of a discrete wavenumber representation (after applying a Fourier transform in the x direction). The integrals of the expressions are transformed into a summation by considering an infinite number of virtual plane sources distributed along the x direction at equal intervals, L_x. In the fluid medium, pressure potentials are defined at the interfaces, whereas in the elastic medium, the wave field is expressed by means of pressure potentials and shear potentials. Details for the potentials when the load is applied in acoustic and in the elastic medium can be found in [4].

To fully define the potentials it is necessary to determine a set of coefficients by deriving the potentials in order to calculate stresses and displacements, and then establishing the appropriate boundary conditions: continuity of normal displacements and stresses and null tangential stresses at the interfaces.

Once the unknown coefficients have been calculated, the displacements and stresses associated with the surface terms can be determined by applying partial derivatives to the potentials. The Green's functions for the solid/fluid formation are then obtained from the sum of the source terms and the surface terms.

BEM FORMULATION

When the layer is assumed to be confined laterally the corresponding scattered field produced by the presence of the boundaries inside the layer is solved in the frequency domain by using the Boundary Element Method (BEM). The model used in this work includes the above-defined Green's functions for a single-layer medium, thus only the two boundaries need to be discretized. The confinement is taken to be rigid, therefore null displacements are ascribed to the boundaries.

The basic BEM equations can be found in [5], so their details are not given here. If we consider a virtual load $\delta(x - x_P)$, acting in the elastic medium, at point $\underline{x}_P$ of the boundary, in the k direction, and by imposing null displacements on the lateral boundaries, the simplified Boundary Integral equations, may be written as:

$$\sum_{l=1}^{3} \int_S t_{k,l}\left(\underline{x}, \nu, \omega\right) G_{k,l}^{surf}\left(\underline{x}_{\mathrm{P}}, \underline{x}, \omega\right)\, dS + u_k^{inc}\left(\underline{x}_0, \underline{x}_P, \omega\right) = 0\,. \tag{2}$$

In these equations $t_{k,l}\left(\underline{x}, \omega\right)$ describe the stresses in direction l at point $\underline{x}$ of the boundary S; $G_{k,l}^{surf}\left(\underline{x}_{\mathrm{P}}, \underline{x}, \omega\right)$ are the Green's functions for displacements in the single layered medium (obtained as described in the previous section) at point $\underline{x}$ in direction l caused by a sinusoidal line load acting at source point $\underline{x}_P$ in direction k; $u_k^{inc}\left(\underline{x}_0, \underline{x}_P, \omega\right)$ is the incident displacement field when the source is placed at $\underline{x}_0$, obtained from the Green's functions described in the previous section; ν is the unit outward normal for the boundary S; the subscripts $k, l = 1, 2, 3$ denote the normal (n), tangential (t) directions relative to the boundary surface and z directions; $C_{k,l}$ is a constant that equals $\delta_{k,l}/2$ for a smooth boundary, where $\delta_{k,l}$ is the Kronecker delta function. Standard vector transformation operators are used to transform the Green's functions from the Cartesian coordinate system.

The Boundary Integral equations are solved after discretization into N constant boundary elements. The resulting integrations are calculated using a Gaussian quadrature scheme, except for the integrations of the source terms regarding the Green's functions for the single solid layer, which are carried out analytically when the element to be integrated is the loaded element.
Solving the resulting system makes it possible to find the nodal stresses. The scattered wave field produced by the lateral rigid boundaries at any point of the domain can then be calculated by applying the Boundary Integral equation.

SIMULATIONS

The simulations performed refer to a single layer made of glass ($c_L = 5734.3$ m/s; $c_S = 3435.6$ m/s; $\rho = 2500$ kg/m^3; $\eta = 4 \times 10^{-3}$) or concrete ($c_L = 3498.6$ m/s; $c_S = 2245.0$ m/s; $\rho = 2500.0$ kg/m^3; $\eta = 4 \times 10^{-3}$), dividing an infinite acoustic medium which assumes the air properties ($\rho_f = 1.2$ kg/m^3 e $c = 340$ m/s). A cylindrical pressure source was placed at $(0.0\text{m}; -0.09\text{m})$ and the responses were calculated at a line of receivers equally spaced 0.15 m apart, placed in the acoustic receiving medium 0.05 m from the layer's surface, as in Figure 2.

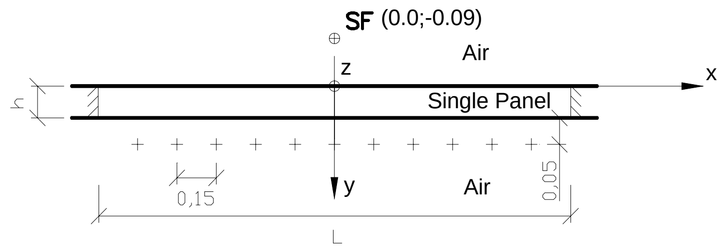

Figure 2 - Geometry of the simulations

The plots shown in this paper refer to sound insertion loss which is obtained by finding the difference between the average pressure responses obtained without and with the presence of the panel. First the analysis is performed for a laterally confined glass panel with length $L = 1.8$ m and thicknesses of $h = 0.008$ m and $h = 0.004$ m. Then the influence of the length of the panel is assessed for a concrete layer $h = 0.04$ m thick with $L = 1.8$ m and $L = 5.0$ m.
Figure 3 presents the insertion loss for a single glass panel $h = 0.008$ m thick without the lateral confinement (labelled in the plots grid #1). This Figure shows a second curve which was obtained for a grid of receivers equally spaced at 2.0 m intervals along the x direction, defining a length of 30 m (labelled in the plots grid #2). A detailed analysis of this curve can be found in [6]. Analysis of this Figure shows that that the curve provided by grid #1, is almost smooth near the critical frequency (labelled 'fc' in the plots) which is generated by the propagation of bending waves. However, in the curve provided by grid #2, it is possible to clear identify a sharp dip at this frequency. In fact, in the absence of boundaries to limit the panel's dimension

along the x direction, the bending waves propagate along the layer, and to record their effect the receivers must be placed further away from the source. When the panel is confined these bending waves are reflected, and so the receivers placed near the source are able to record their presence.

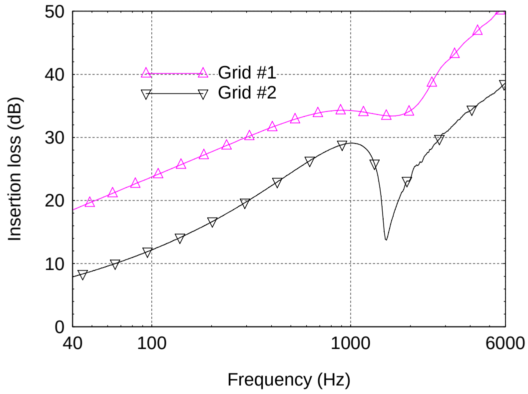

Figure 3 - Insertion loss provided by a single panel $h = 0.008$ m thick of infinite extent

Figure 4 plots the responses provided by a single glass panel $h = 0.008$ m thick (Figure 4a) and by the same panel $h = 0.004$ m thick (Figure 4b).

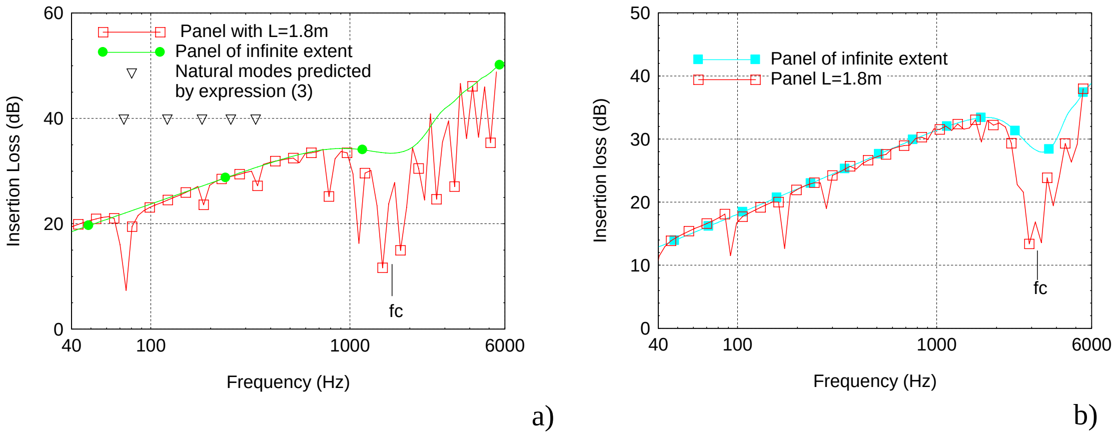

Figure 4 - Insertion loss provided by a single glass panel of infinite extent vs laterally confined panel with $L = 1.8$ m : a) $h = 0.008$ m thick; b) $h = 0.004$ m thick

Analysis of Figure 4a reveals that when the panel is confined laterally the resulting structural modes generate dips in the insertion loss curve at low frequencies.
In this Figure five frequencies corresponding to the natural modes of a clamped-clamped beam are added. These frequencies can be predicted according to the following expression [7]

$$f_n = \frac{\pi}{8}\sqrt{\frac{B}{m}}\frac{(2n-1)^2}{L^2}, \qquad (3)$$

where B is the bending stiffness; m is the mass per unit length and $n \geq 2$. Notice that the dips predicted by the BEM model agree with this expression. However not all the dips obtained by expression (3) occur in the curve provided by the BEM model. This is due to the position of the load (located at a symmetry axis of the panel) that does not allow the modes with nodal lines at the centre of the panel to be excited.
Analysis of Figures 4a and 4b, also shows that there is a dip in the curves provided by the BEM model in the vicinity of the critical frequency (labelled 'fc' in the plots). This dip was in fact present in the response provided by the single panel of infinite extent, but when the panel is laterally confined it becomes more pronounced. When the panel is assumed to be laterally confined, the bending waves reflected at the edges are combined with the incident waves producing standing wave patterns which may result in transverse panel motion. As a result, a decrease in the insertion loss curve in the vicinity of the critical frequency is found to occur. Note also, that the response provided by the BEM model displays a set of sharp dips at higher frequencies, which are also related to this effect. These tend to become more pronounced with increasing panel thickness, due to the size of the boundaries which allow more energy to be reflected. The responses provided by experimental results do not usually reveal the presence of this feature. This is likely to be because, at higher frequencies, bending waves may be transmitted to the outer medium and less energy is reflected, thus the stationary field tends to be attenuated.

The insertion loss provided by a concrete panel $h = 0.04$ m thick with a length of $L = 1.8$ m and $L = 5.0$ m is plotted in Figure 5.

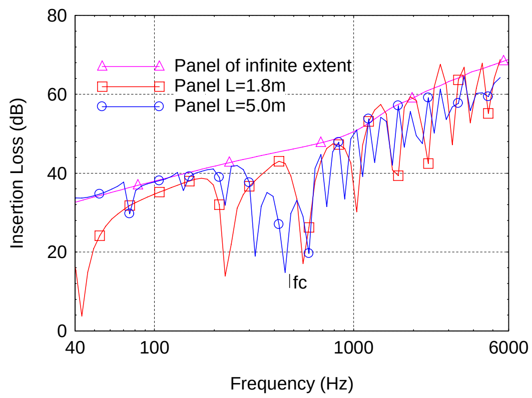

Figure 5 -Insertion loss provided by a single concrete panel $h = 0.04$ m *thick*

Again, the presence of dips related to the natural modes of the clamped panel and with the stationary field provided by the interaction between the incident and reflected bending waves, are found in the responses. When the length of the panel changes from $L = 1.8$ m to $L = 5.0$ m, more dips are found and these tend to be less sharp as the frequency increases. Moreover an additional dip is being formed when the panel takes a length of $L = 5.0$ m, and this is related to the coincidence effect.

CONCLUSIONS

In this paper a BEM model has been developed to predict sound insulation provided by laterally confined single partitions. Simulations were performed and the results were compared against those provided by a single panel of infinite extent. The results showed that at low frequencies the model was able to predict the structural modes for a laterally confined panel. Furthermore it was concluded that when the panel is thin the panel's size plays an important role in the prediction of the critical frequency. It was also found that, at higher frequencies, this model predicts dips related to the stationary field provided by the interaction between the incident and the reflected bending waves. These tend to be more pronounced as the thickness of the panel increases, since the size of the boundaries allows more energy to be reflected. The results provided by real partitions do not indicate the presence of these dips. An explanation for this may be that part of the energy is transmitted to the elastic surrounding medium. This issue will be discussed in future work.

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