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A DETAILED EVALUATION OF ANALYTICAL, NUMERICAL AND EXPERIMENTAL METHODS FOR A SMART PIEZOELECTRIC BEAM IN A HIGHER FREQUENCY RANGE

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Abstract

In the design of smart structures analytical models can contribute to the general understanding and analysis of the structural behaviour. In the paper techniques for the advanced analytical modelling of a cantilever beam are presented and experimentally verified. The beam is attached with piezoelectric patches that are used as actuators and sensors in a collocated and non-collocated manner. The structure is described first. Then the strain and stress distributions within the active and passive beam layers are derived. A novel methodology is introduced to model analytically the mechanical coupling between collocated actuator and sensor patches. The effective electromechanical properties of the patches and the effect of modal truncation is discussed. The analytical frequency response functions (FRF) are compared and verified with numerical and experimental data in the frequency range up to 5 kHz including the first 10 bending modes and the first longitudinal mode. The observed differences between the simulated and measured eigenfrequencies are less than 0.5% except for the first bending and longitudinal mode. The average modelling error in the amplitudes is less than 1dB for collocated patch combinations in the frequency range up to 3 kHz.

INTRODUCTION

As the demand increases for thin and lightweight mechanical structures, structures are becoming more and more limited by their dynamic behaviour [8]. For high performance goals and large dynamic uncertainties passive solutions are often inadequate, especially in the lower frequency range, such that active vibration control

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based on distributed parameter systems has to be used.

The use of piezoelectric patches as elements of smart structures and the analytical modelling of system transfer functions were investigated in many studies [5], [4], [13] and [14]. Alvarez-Salazar and Iliff [2] inferred the mechanical coupling between collocated actuator-sensor patches from measured data and included it in the FRFs as an additional constant, called feedthrough. The effect of model reduction errors, termed residual mode, on the modelling of FRFs was discussed by Gao and Randall [9]. In general, the experimental validation of FRFs for analytical models that can be found in the literature is carried out only for the first few bending modes [1].

In this paper an approach is presented to model analytically the mechanical coupling between collocated actuator and sensor patches. Furthermore the effective electromechanical coupling and the modelling error due to model reduction is discussed. Finally a new advanced analytical beam model is proposed in modal form. The intention is not only to present a new model but to extend the validation of theory to higher modal orders and frequency ranges.

ASSUMPTIONS

A thin, rectangular and isotropic cantilever beam, as shown in Fig. 1, is considered. Four identical piezoelectric patches are supposed to be perfectly and symmetrically surface mounted in a collocated and non-collocated manner on both sides of the beam. The patches can be used as actuators or sensors. In the following the super- or subscript b denotes the beam and the super- or subscript p the patches.

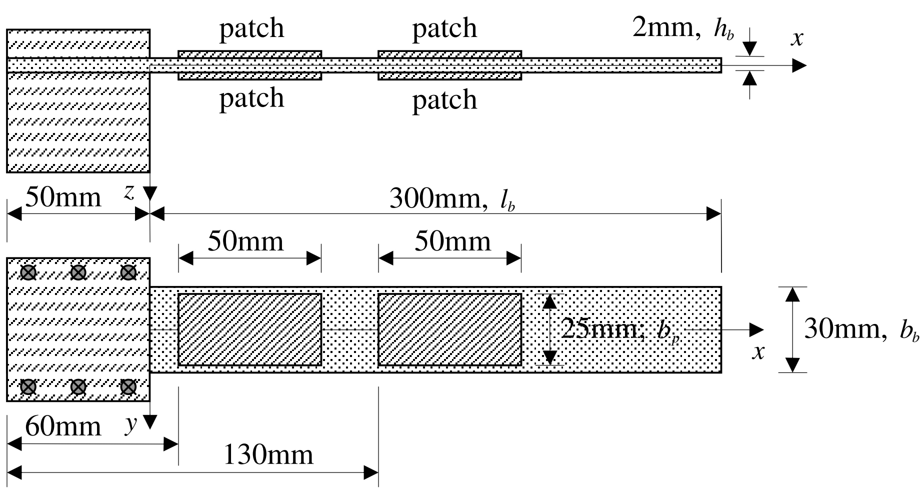

Fig. 1 - Geometry and layout of the cantilever beam with surface mounted patches.

The beam is made of steel St05Z and the piezoelectric patches are made of Lead-Zirkonate-Titanate (PZT) ceramics Sonox P53. The principal characteristics of both materials are listed in Tab. 1.

It is assumed that the mass and the stiffness of the piezoelectric patches can be neglected in comparison with the mass and the bending stiffness of the beam [10]. Because of the length to the thickness ratio of the patches the edge effects of the patches can be ignored. In the analysis the bonding layer is supposed to be plane and thin such that it has not to be considered [5]. An Euler-Bernoulli model is used, neglecting the rotary inertia and shear deformation of the beam.

Tab. 1 Characteristics of steel St05Z and Sonox P53 [3].

	steel St05Z	Sonox P53
$l \times b \times h$ [10^{-3} m]	300 x 30 x 2	50 x 25 x 0.2
ρ_p [kg/m^3]	7850	7830
E [10^9 N/m^2]	198	67
ν_{13}, ν_{23}	0.33	0.34
d_{31}, d_{32} [10^{-12} m/V]	---	-233
e_{31}, e_{32} [N/Vm]	---	-15.3
ε_r	---	1630

ACTUATOR SENSOR COUPLING

The stress and strain distributions within the beam are derived for a patch configuration as shown in Fig. 2. The approach is based on the work of Fuller et. al. [7] but includes the additional layer of the sensing patch. The super- or subscripts pa and ps denote the actuating and sensing patch respectively.

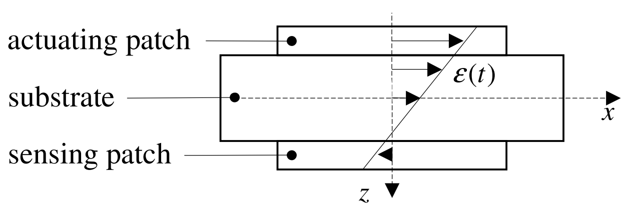

Fig. 2 - Asymmetric strain distribution.

Applying voltage $V(t)$ across an unconstrained patch causes it to strain in both in-plane directions by $\varepsilon_{pa}(t) = d_{31} \cdot V(t)/h_p$, where d_{31} is the piezoelectric constant and h_p is the height of the patch. A bonded patch however is constrained by the stiffness of the beam. If the actuator patch is provided with an electric voltage, as shown in Fig. 2, the beam will both expand and bend which results in an asymmetric strain distribution in the xz plane.

The patches are transversally isotropic and the beam has nearly the same Young's modulus E_b in both in-plane directions. Furthermore, the Poisson's ratio ν of the patches and of the beam are almost the same. Based on the Euler-Bernoulli assumption the strain distribution $\varepsilon(z,t)$ can be composed from a linear function and a constant term as

$$\varepsilon(z,t) = C(t) \cdot z + \hat{\varepsilon}(t) \tag{1}$$

where $C(t)$ is the slope and $\hat{\varepsilon}(t)$ is the constant part. Using Hooke's law the stress $\sigma_{pa}(z,t)$ within the actuating patch, the stress $\sigma_b(z,t)$ within the beam substrate and the stress $\sigma_{ps}(z,t)$ within the sensing patch are specified as

$$\sigma_{pa}(z,t) = \frac{E_p}{1-\nu^2} \cdot [(1+\nu) \cdot (C(t) \cdot z + \hat{\varepsilon}(t)) - (1-\nu) \cdot \varepsilon_{pa}(t)] \tag{2}$$

$$\sigma_b(z,t) = \frac{E_b}{1-\nu^2} \cdot [(1+\nu) \cdot (C(t) \cdot z + \hat{\varepsilon}(t))] \tag{3}$$

$$\sigma_{ps}(z,t) = \frac{E_p}{1-\nu^2} \cdot [(1+\nu) \cdot (C(t) \cdot z + \hat{\varepsilon}(t))] \tag{4}$$

where E_p is the Young's modulus of the patch. Employing the force equilibrium in x direction, the moment equilibrium about the centre of the beam and integration allows the derivation of $C(t)$ and $\hat{\varepsilon}(t)$, which are given by

$$\hat{\varepsilon}(t) = \frac{E_p \cdot h_p}{2 \cdot E_p \cdot h_p + E_b \cdot h_b} \cdot \varepsilon_{pa}(t) = K_u \cdot \varepsilon_{pa}(t) \tag{5}$$

$$C(t) = \frac{6 \cdot E_p \cdot h_p \cdot (h_b + h_p)}{E_b \cdot h_b^3 + 6 \cdot E_p \cdot h_b^2 \cdot h_p + 12 \cdot E_p \cdot h_b \cdot h_p^2 + 8 \cdot E_p \cdot h_p^3} \cdot \varepsilon_{pa}(t) = K_w \cdot \varepsilon_{pa}(t) \tag{6}$$

where h_b is the height of the beam. The mechanical coupling K_D between the patches caused by the strain $\varepsilon(z,t)$ produces an electric displacement $D(x,t)$ in the sensing patch. Integrating $D(x,t)$ we obtain the charge $Q(t)$. Assuming that the patch is similar to an ideal parallel plate capacitor the voltage $V_{ps}(t)$ of the sensing patch can be expressed as

$$V_{ps}(t) = \frac{Q(t)}{C} = \underbrace{-2 \cdot \frac{1}{\varepsilon_0 \cdot \varepsilon_r} \cdot e_{31} \cdot d_{31} \cdot \left(-K_w \cdot \frac{h_b + h_p}{2} + K_u \right)}_{K_D} \cdot V_{pa}(t) \tag{7}$$

where ε_0 is the permittivity of free space, ε_r is the relative permittivity of the piezoelectric layer and e_{31} is the piezoelectric charge density.

EFFECTIVE COUPLING FACTOR

The coupling coefficient of a PZT ceramic decreases with time due to a time depending reduction of the polarisation. A characteristic reduction of the coupling coefficient of low-voltage PZTs is in a magnitude of about 0.5% to 2% per unit time decade [12]. Mishandling by exceeding the electrical, mechanical or thermal limitations accelerates the aging process. In addition, both the thickness h_p and the dielectric properties d_{31} and e_{31} of the patch show manufacturing tolerances that are within $\pm 10\%$ of the nominal values [11]. The actual electromechanical properties are considered by an effective coupling factor K_{eff}, which gives

$$d_{31}^{eff} = K_{eff} \cdot d_{31} \quad \text{and} \quad e_{31}^{eff} = K_{eff} \cdot e_{31} \tag{8}$$

RESIDUAL MODE

The beam is a system of infinite modal order. The FRF can be written as [9]

$$H(\omega) = \underbrace{\sum_{i=1}^{n_p} \frac{R_i}{(j \cdot \omega - s_{pi}) \cdot (j \cdot \omega - s_{pi}^*)}}_{H_{in}(\omega)} + \underbrace{\sum_{i=n_p+1}^{\infty} \frac{R_i}{(j \cdot \omega - s_{pi}) \cdot (j \cdot \omega - s_{pi}^*)}}_{H_{out}(\omega)} \tag{9}$$

where R_i is the residue of the mode i, s_{pi} is the corresponding complex pole in the s plane and * means the complex conjugate. Neglecting the modes above the frequency range of interest results in a reduced order model $H_{in}(\omega)$. The out-of-band FRF $H_{out}(\omega)$ is called residual mode $K_R(\omega)$.

The anti-resonances of the reduced order model are the zeros of the FRF $H_{in}(\omega)$. In general they differ from the anti-resonances of the full order model $H(\omega)$ determined by the zeros of the sum $H_{in}(\omega) + H_{out}(\omega)$.

In addition, the residual mode $K_R(\omega)$ contributes to the magnitude and phase of the FRF. The ratio of zeros and poles determines the frequency dependency of the contribution within the in-band range. At best the actuating and the sensing patch are arranged in a collocated manner resulting in a FRF with alternating zeros and poles. Thus the contribution of a truncated pole s_{pi} is compensated for by the subsequent truncated zero s_{zi} such that the residual mode $K_R(\omega)$ is nearly constant.

ADVANCED BEAM MODEL

The traditional beam model considers only the modal input and output gain of the patches. Based on the previous discussion we can derive an advanced beam model incorporating the mechanical coupling K_D, the effective coupling factor K_{eff} and the residual mode $K_R(\omega)$. The model can be expressed in modal form as [6]

$$H(x_{pa}, x_{ps}, \omega) = K_{eff}^2 \cdot \left[\sum_{i=1}^{n} \frac{N_i(x_{pa}) \cdot V_i(x_{ps})}{\omega_i^2 - \omega^2 + 2 \cdot j \cdot \xi_i \cdot \omega_i \cdot \omega} + K_D \right] + K_R(\omega) \tag{10}$$

where x_{pa} and x_{ps} are the positions of the actuator and sensor patch respectively, ω_i

is the eigenfrequency of the mode i while $\xi_i \cdot \omega_i$ is the modal damping δ_i. The modal input gain $N_i(x_{pa})$ and the modal output gain $V_i(x_{ps})$ are calculated as given in [6].

EXPERIMENTAL VERIFICATION

A cantilever beam, as shown in Fig. 1, is used to validate the beam model. The experimental setup is given in Fig. 3. The actuating patch is driven by a white noise generated by the measurement hardware (HW) platform (Onosokki, DS-2000). The signal is amplified to 20Volt route mean square (Vrms) by an audio voltage amplifier (KME, SPA 3200MP) and a 50 Volt DC offset is added by a DC power supply (Gossen Konstanter, 14K60R). The voltage applied to the actuating patch is measured by a differential probe (Testec, SI-50) and is used as reference channel for the measurement HW platform. The generated piezoelectric voltages of the sensing patches are used as sensor signals. The FRFs are determined by averaging the response of 100 single measurements.

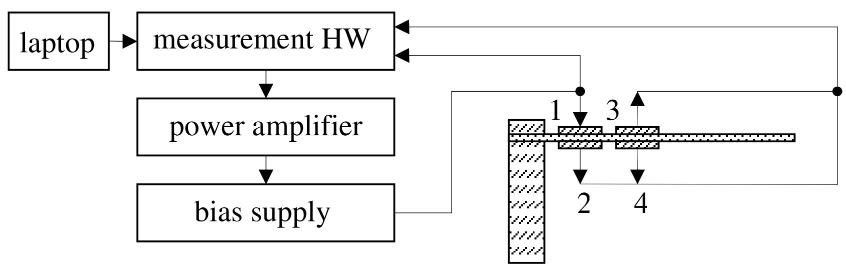

Fig. 3 - Experimental setup.

Eigenfrequencies

The experimental eigenfrequencies f_i of the beam are inferred from measured data. The analytical eigenfrequencies f_i are computed by the results of the longitudinal and bending vibration analysis presented in [6].

Moreover a FE model of the beam is developed based on 3-dimensional finite elements including the patches with their fully coupled electromechanical fields. The finite element software COSAR (see www.femcos.de) was used to carry out the simulations as well as the calculation of the FRFs.

The experimental, analytical and numerical (FE model) eigenfrequencies show errors less than 0.5% with respect to the measured ones. Exemptions are the first bending and longitudinal mode. The analytical model gives errors of -7.3% and -4.5% respectively whereas the FE model yields smaller errors of 3.53% and –5.24%.

Feedthrough, effective coupling factor and residual mode

The mechanical coupling K_D is calculated analytically to 0.033, see equation (7). The effective coupling factor K_{eff} and the residual mode K_R are inferred by minimizing the mean absolute amplitude modelling error of the analytical FRFs with respect to the measured FRFs in the frequency range from 15Hz to 3 kHz.

The effective coupling factor K_{eff} and the residual mode K_R depend on the patch combination used. The former varies between 0.81 and 0.86 due to the individual mechanical, electrical or thermal aging and manufacturing tolerances of the single patches. The later extends over the range from -0.0080 to 0.0009 as a result of the different modal input and output gains of the truncated modes associated with the single patch locations.

A comparison of measured data with the FRF of a traditional beam model and the presented advanced one is given in Fig. 4. The feedthrough K_D and the residual mode K_R considerably improve the correspondence of the measured and analytical FRFs. The analytical anti-resonances are shifted left to lower frequencies such that they match the measured ones. The effective coupling factor K_{eff} shifts the FRF horizontally. Thus the average amplitude modelling error of the analytical FRF is minimised from 10.4dB to 0.9dB in the range up to 3 kHz.

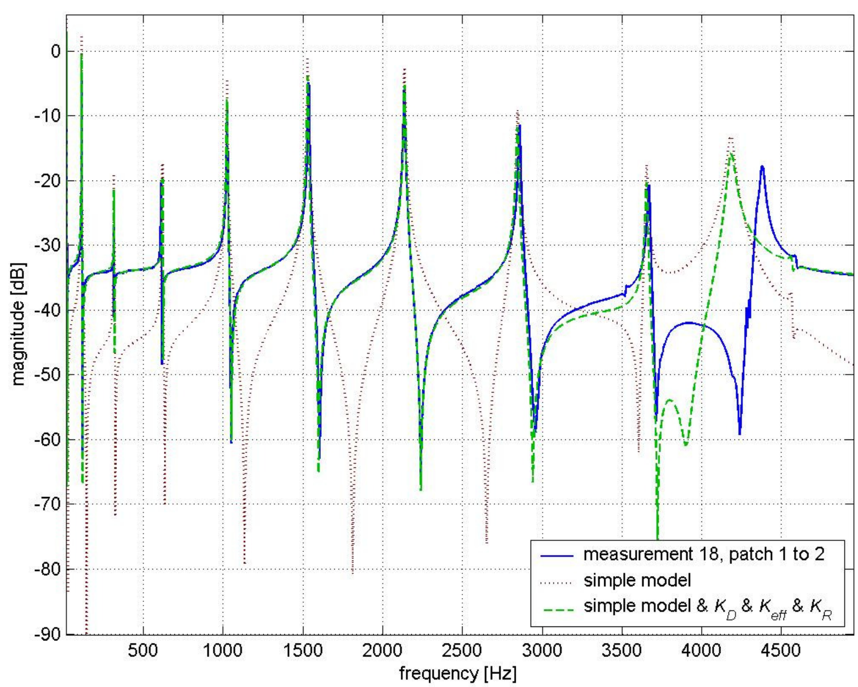

Fig. 4 - Collocated FRFs of simple and advanced analytical beam model.

The complete simulation results and experimental validation of the model is given in [6]. The paper presents in detail the influence of the mechanical coupling K_D, of the effective coupling factor K_{eff} and of the residual mode K_R on the FRFs. Furthermore non-collocated actuator and sensor patch combinations are investigated.

CONCLUSION

An analytical formulation of the mechanical coupling of collocated actuator and sensor patches has been presented. The effective electromechanical coupling and the modelling error due to model reduction has been discussed. Moreover, a new advanced analytical beam model incorporating the mechanical coupling, the effective electromechanical coupling and the residual mode has been proposed. The presented analytical and measured FRFs show excellent agreement. The study demonstrates the possibility to extend the validation of the beam theory up to highest modal orders.

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