ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

ANALYSIS OF VIBRATION TRANSMISSION AT THE CORNER INTERFACE OF TWO PLATES FOR REDUCTION OF STRUCTURE-BORNE SOUND

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Abstract

For build-up structure such as that of a ship, which should be not only statically stable but also watertight, blocking mass other than elastic interlayer could be used to reduce transmission of structure-borne sound. In this paper, a simplified model of corner interface of two infinite plates rigidly jointed at arbitrary angles with blocking mass is taken for theoretical analysis by wave approach. Two local coordinate systems are introduced to deduce formulations of transmission and reflection coefficients as well as transmission loss. The effects of different parameters of blocking mass on transmission loss are investigated numerically. Five samples were tested in experiment. Discussions were carried out based on the comparison between prediction and experiment in terms of insertion loss of blocking mass. It is concluded that blocking mass acts like a "low-pass filter", effective for vibration attenuation above certain frequency. The value of TL in "attenuation band" depends mainly on mass per unit length and band width of "attenuation band" on mass moment of inertia per unit length of blocking mass.

1. INTRODUCTION

This paper is concerned with vibration transmission in built-up structures such as those of ships. Owning to the complexity of build-up structures, statistical energy analysis (SEA) method [3] has been frequently used for vibration energy distribution calculation. However, SEA gives only statistical results of energy level of each component in the structure averaged in time, space and modes simultaneously, and is not suitable for analysis at low frequencies because of low modal density. At present, numerical methods such as FEM are more widely used in vibration analysis of built-up structures, but large number of elements increasing with frequency needed in calculation as well as linear assumption still cause much inconvenience at high frequencies[2]. So in a word, analytical analysis by wave approach supplemented with experiments is still meaningful to reveal the mechanism of vibration energy transmission in build-up structures.

R. M. Grice and R. J. Pinnington carried out an analysis of this kind for plate-stiffened beam [1]. Their research is focused mainly on the energy transmission in beams and that from beam to plate. However, energy transmission from plate to plate is more close to the case in ship structures. Furthermore, longitudinal waves are still of significance in the sense of their effects on boundary conditions and will cause secondary bending waves at the next joints, though they themselves do not induce much sound radiation.

An earlier analysis on wave transmission in plates was seen in L. Cremer and M. Heckl's book [4] where discussion is restricted to the case of right angle joint. The analysis in this paper is based on it and extended to the case of vibration transmission at the corner interface of two infinite plates rigidly joined at arbitrary angles. The effects of blocking mass attached to the joint for attenuation of vibration transmission are investigated.

2. DESCRIPTION OF MODEL

Consider a corner interface of two infinite thin plates, possibly of different material and different thickness, joined together at arbitrary angles θ ($0 < \theta \le \frac{\pi}{2}$), as depicted in Fig. 1 where x_1oy_1 and x_2oy_2 are two coordinate systems and subscripts 1, 2 indicate plate 1 and plate 2 respectively. For structure-borne sound attenuation, a blocking mass, which is not shown in left plot of Fig. 1, is attached to the joint with the assumption that the geometric size of the blocking mass is small enough comparing with the wavelength of the longitudinal and bending waves propagating in the two plates.

Plane bending wave propagating along plate 1 (primary plate) towards plate 2 (secondary plate) through the joint as well as the longitudinal waves generated at the joint are considered.

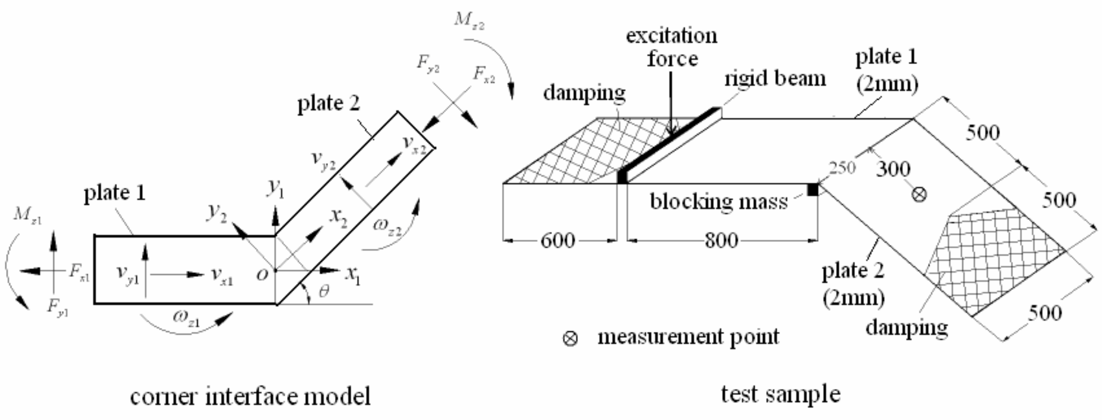

Fig.1 Corner interface joined at arbitrary angles

3. FORMULATION

Wave expressions

For a given frequency, this incident wave can be expressed in terms of transverse velocity as

$$\tilde{v}_{y1+} e^{-jk_1 x_1} e^{j\omega t},$$

where $\tilde{v}_{y1+}$ is the amplitude of incident bending wave velocity, k_1 bending wave number in plate 1 , ω the radian frequency and $j = \sqrt{-1}$. For simplicity, the time dependence $e^{j\omega t}$ has been omitted in the following expressions.

Upon impinging on the joint, a part of incident wave is reflected and the other part transmitted into plate 2. The total transverse velocity in plate 1 is therefore written as

$$v_{y1}(x_1) = \tilde{v}_{y1+} \left(e^{-jk_1 x_1} + r e^{+jk_1 x_1} + r_j e^{+k_1 x_1} \right), \quad \text{for } x_1 \le 0 \tag{1}$$

where r and r_j are the reflection coefficient for reflected travelling wave and evanescent wave respectively. And the total transverse velocity in plate 2 is written as

$$v_{y2}(x_2) = \tilde{v}_{y1+} \left(t e^{-jk_2 x_2} + t_j e^{-k_2 x_2} \right), \quad \text{for } x_2 \ge 0 \tag{2}$$

where t and t_j are the transmission coefficient for transmitted travelling wave and evanescent wave respectively.

Apart from bending wave, the transverse force in cross section of plate 1 induces a longitudinal wave in plate 2 for $\theta > 0$, and vice versa.

Transmission coefficients

The reflection and transmission coefficients in (1) and (2) are determined from boundary conditions at the joint which are expressed as follows:

a) Continuity of rotational, transverse and longitudinal velocity at the joint gives

$$w_{z1}\big|_{x_1=0} = w_{z2}\big|_{x_2=0}, \tag{3}$$

$$v_{x1}\big|_{x_1=0} = v_{x2}\big|_{x_2=0} \cos\theta - v_{y2}\big|_{x_2=0} \sin\theta, \tag{4}$$

$$v_{y1}\big|_{x_1=0} = v_{x2}\big|_{x_2=0} \sin\theta + v_{y2}\big|_{x_2=0} \cos\theta, \tag{5}$$

where v_{x1} and v_{x2} are the longitudinal velocities.

b) Dynamic equilibrium of bending moment, axial and transverse force gives

$$M_{z1}\big|_{x_1=0} - M_{z2}\big|_{x_2=0} = j\omega\Theta' w_{z2}, \tag{6}$$

$$F_{y2}\big|_{x_2=0} \sin\theta - F_{x2}\big|_{x_2=0} \cos\theta - F_{x1}\big|_{x_1=0} = j\omega M' v_{x1}, \tag{7}$$

$$F_{y1}\big|_{x_1=0} - F_{y2}\big|_{x_2=0} \cos\theta - F_{x2}\big|_{x_2=0} \sin\theta = j\omega M' v_{y1}, \tag{8}$$

where F_{x1}, F_{x2} are the longitudinal forces, and F_{y1}, F_{y2} the shear forces, M' is mass per unit length and Θ' mass moment of inertia per unit length of the blocking mass.

By substituting velocity expressions into boundary conditions as well as the differential relations between force, moment and velocity, the following formulation for r, r_j, t and t_j is deduced as

$$Ax = b, \tag{9}$$

where

$$A = \begin{pmatrix} j & 1 & j\chi & \chi \\ -1 & 1 & \psi + jQ_0 & Q_0 - \psi \\ (\gamma_1 + j\eta_1 + 1)\cos\theta & (\gamma_1 + j\eta_1 + 1)\cos\theta & -(\gamma_1\cos^2\theta + \beta_1\sin^2\theta + j\eta_1 + 1) & -(\gamma_1\cos^2\theta + j\beta_1\sin^2\theta + j\eta_1 + 1) \\ \beta_2 + j\eta_2 + 1 & j\beta_2 + j\eta_2 + 1 & (\gamma_2 - 1)\cos\theta & (j\gamma_2 - 1)\cos\theta \end{pmatrix}$$

$b = \left(j, \quad 1, \quad -(\gamma_1 + j\eta_1 + 1)\cos\theta, \quad \beta_2 - 1 - j\eta_2\right)^T, \qquad x = \left(r, \quad r_j, \quad t, \quad t_j\right)^T,$

in which $\chi = \frac{k_2}{k_1} = \frac{\lambda_1}{\lambda_2}$, $Q_0 = \frac{\omega^2\Theta k_2}{B_1 k_1^2}$ and $\psi = \frac{\sqrt{m_2'' B_2'}}{\sqrt{m_1'' B_1'}} = \frac{k_2^2 B_2'}{k_1^2 B_1'}$ where mass per unit surface area $m'' = \rho h$, bending stiffness of the plate $B' = \frac{EI'}{1-\mu^2}$ (E is Young's modulus, I' moment of inertia per unit width and μ Poisson's ratio), and $\beta_1 = \frac{c_{B2} m_2''}{c_{LI1} m_1''}$, $\beta_2 = \frac{c_{B1} m_1''}{c_{LI2} m_2''}$, $\gamma_1 = \frac{c_{LI2} m_2''}{c_{LI1} m_1''}$, $\gamma_2 = \frac{c_{B2}}{c_{LI2}}$, $\eta_1 = \frac{\omega M'}{m_1'' c_{LI1}}$, $\eta_2 = \frac{\omega M'}{m_2'' c_{LI2}}$ where c_{LI1} and c_{LI2} are longitudinal wave speeds, c_{B1} and c_{B2} bending wave speeds for plates.

It is worth to note that for the particular case of $\theta = \frac{\pi}{2}$ (i.e. corner interface at right angle) and $M' = \Theta' = 0$ (without blocking mass) equation (9) reduces exactly to the results given in Cremer and Heckl's book [4].

From (9) it is ready to have

$$x = A^{-1} b, \tag{10}$$

hence r, r_j, t and t_j are solved.

It should be noted that the previously discussed components in Fig. 1 are restricted to thin plates so as to ensure pure bending wave assumption which requires $\lambda_B > 6h$. This also sets an upper frequency limit

$$f_{\max} = \frac{1.8 c_{LI} h}{\lambda_B^2} < \frac{c_{LI}}{20h}. \tag{11}$$

This limit does not add much inconvenience for the reason that thin plate assumption can be satisfied in most cases of engineering. For example, the common thickness of steel plate used in ship structure is about 20mm for which $f_{\max}$ will be 13.5 kHz and that is high enough for general purpose of structure-borne sound investigation.

The assumptions of plane wave and infinite width make plates behave like beams. Thus formulation for r, r_j, t and t_j for beams is the same as that for plates, except B', I' and m'' for plate should be replaced by B, I and m' for beam.

Transmission loss

It is more meaningful to consider vibration transmission in terms of energy other than velocity in engineering. Furthermore, bending wave transmission other than longitudinal one should be paid more attention to because it is just bending wave that causes secondary sound radiation at destination. So that (energy) transmission efficiency of bending-to-bending wave τ_{BB} is induced. Transmission loss of bending wave is then given by

$$TL = 10 \cdot \log\left(\frac{1}{\tau_{BB}}\right) = 10 \cdot \log\left(\frac{1}{\chi \psi |t|^2}\right), \text{dB} \qquad (12)$$

4. NUMERICAL INVESTIGATION

Shown in Fig. 2 are some of calculated results of TL for the joints with blocking mass of different parameters comparing with those without blocking mass. In calculation, the two components are both steel plates of 2mm thickness. Mass per unit length M' and mass moment of inertia per unit length Θ' of the reference blocking mass are equal to those of a steel beam with cross section measured 20mm×20mm.

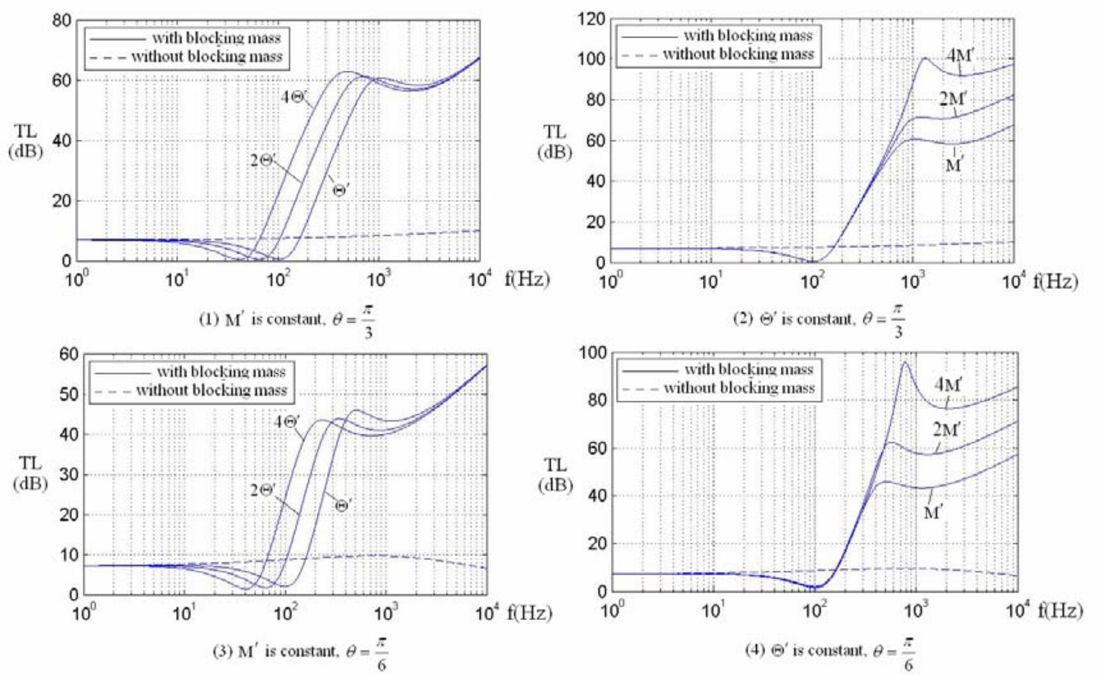

Fig. 2 Variations of TL with different parameters of blocking mass

The following observations are drawn from Fig. 2:

1) Each TL curve with blocking mass has a vale and afterwards increases rapidly to a highland. That is to say blocking mass acts like a kind of “low pass filter”, which divides the whole frequency domain into “transmission band” and “attenuation band” with a transition part in-between, for transmission of bending wave.
2) In “transmission band” TL of joint with or without blocking mass makes almost no difference. That is to say that blocking mass has very little effects on attenuation of bending wave transmission at low frequencies.
3) As M' is kept constant (see (1), (3)), TL curves for different Θ' are alike, and every single TL curve shifts leftwards, which broadens the bandwidth of “attenuation band”, with increasing Θ'.
4) As Θ' is kept constant (see (2), (4)), the value of TL in “attenuation band” increases with M' and TL curves at low frequencies have almost no difference from each other.

5. EXPERIMENT

There are 5 test samples used in the experiment. Parameters of the samples are listed in Table 1. Sketch and dimensions of test samples are shown in right plot of Fig. 1.

In order to ensure plane wave produced in plate 1 by point excitation, a steel beam of cross section 20mm×20mm, perpendicular to the direction of wave propagation, is attached. The excitation point is allocated at the midpoint of the beam. Some parts of plate are coated by damping sheet to suppress the returning waves from plate edges. Furthermore, damping is arranged in such pattern as shown in Fig. 1 to prevent giving rise to a sharp transition of impedance from undamped area to damped area which is not good for reflection reduction.

Table 1 Five test samples

No.	#1	#2	#3	#4	#5
θ	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$
section of blocking mass (mm×mm)	40×40	20×20	20×20	None	None

The samples are suspended by three elastic ropes to guarantee free boundary condition. The experiment site, measurement and analysis system are shown in Fig. 3. Samples are excited by B&K4810 shaker which is connected to B&K1407 sinusoidal signal generator and B&K2706 power amplifier. A B&K8200 force transducer is installed between the shaker and the beam to ensure equal magnitude of excitation force for each test. The measurement point is on the central line of the plate 2, 300mm away from the joint so as to get rid of the effect of evanescent waves generated at the joint. The signal picked-up by Kistler8791A250 accelerometer at plate 2 is conditioned by Kistler5134 coupler and processed by China-made DH5935N signal processor.

The insertion loss IL is calculated by

$$IL = 20 \cdot \log\left(\frac{v_{20}}{v_2}\right), \text{dB} \tag{13}$$

where v_2 and v_{20} represent the velocity responses at the measurement point for test sample with and without blocking mass respectively.

Fig. 3 Experiment site (left) / measurement and analysis system (right)

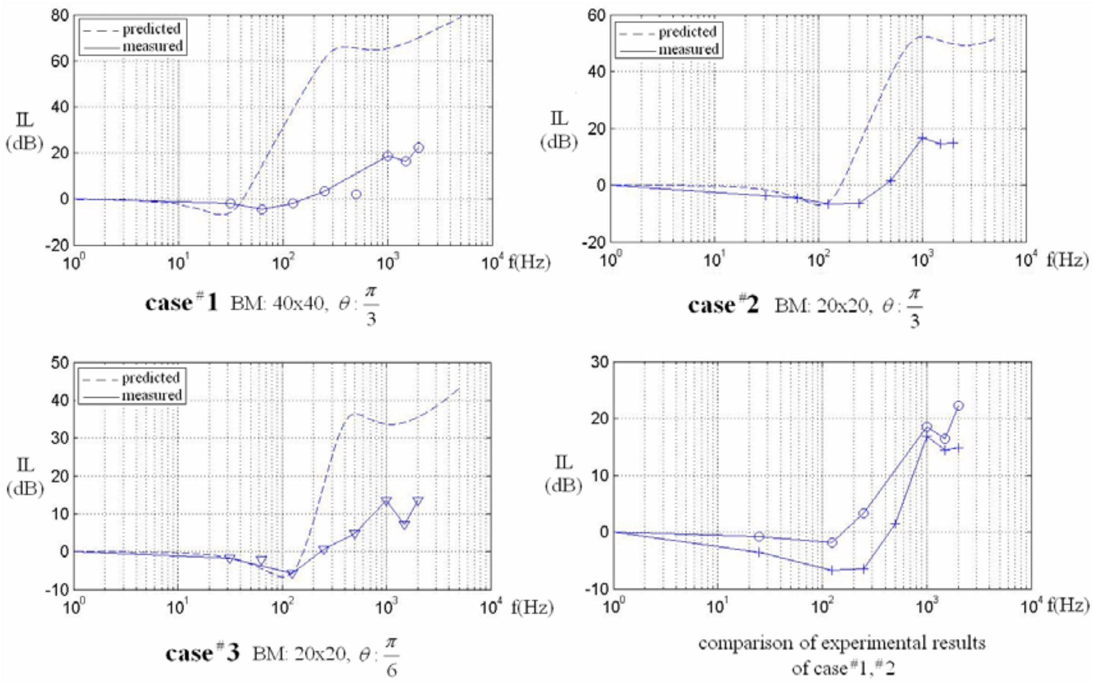

Fig. 4 Comparison of prediction and experimental results for different blocking mass size (BM: mm×mm) and θ

Shown in Fig. 4 are comparisons of IL results of numerical prediction and experiment.

Discussions about Fig. 4 are as follows:

1) Each IL curve of measurement has a vale with negative value and afterwards increases with frequency, similar to that of prediction.
2) The discrepancy of experiment and prediction is small at low frequencies and increases with frequency. Because plane wave assumption is no longer valid by point excitation on the beam at frequencies upper than the first mode (or resonance) frequency of the beam.
3) For the reason that returning waves can not be entirely absorbed to prevent the structure from resonance although damping sheet is attached, insertion loss at frequencies of resonance will be much lower than predicted such as the result at 500Hz in case #1 and 1500Hz in case #3.
4) Form the comparison of measurement curves in case #1 and case #2 for the same angle (lower-right plot), it can be seen that the value of IL with larger blocking mass is higher. Moreover the frequency corresponding to the vale decreases with mass moment of inertia, which is in accordance with theoretical analysis.

6. CONCLUSION

The results of numerical investigation and experiment show the similar trend of the effects of blocking mass on attenuation of vibration transmission. The following could be concluded:

1) Blocking mass attached at corner interface of two steel plats acts as a kind of "low-pass filter", effective for vibration attenuation above certain frequency.
2) The value of TL in "attenuation band" is mainly depends on mass per unit length of the blocking mass, and the bandwidth of "attenuation band" depends on mass moment of inertia per unit length of the blocking mass.
3) The simplified model combined with wave approach is useful to reveal the mechanism of vibration energy transmission in built-up structures although some discrepancy introduced by the complexity of practical wave fields.
4) More refined model is needed for further research on the phenomenon of vibration transmission of built-up structures.

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