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LONGITUDINAL GUIDED WAVES IN TRANSVERSELY ISOTROPIC CIRCULAR CYLINDRICAL SHELLS

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Abstract
An analysis of the propagation of longitudinal waves in transversely isotropic cylindrical shells based on the three-dimensional theory of elasticity is outlined in this paper. The mathematical model developed for the scattering of acoustic waves from solid cylinders [F. Honarvar and A. N. Sinclair, J. Acoust. Soc. Am. 100(1), 57 (1996)] is extended to the case of cylindrical elastic shells. Stress-free boundary conditions are applied at the inner and outer cylindrical boundaries of the shell and the exact form of the frequency equation is derived in terms of the wave number, cylinder radii, and material elastic constants. Dispersion curves are plotted for a number of different transversely isotropic materials. A perturbation study on the effects of shell thickness and relative magnitude of elastic constants is conducted. The results indicate that the dispersion curves are relatively sensitivite to perturbations of the thickness and axial stiffness of the shell.

INTRUDUCTION

Transversely isotropic cylindrical shells, such as pipes, have wide applications in industry. Transverse isotropy is usually desirable since it provides higher strength to stiffness ratio along the cylinder axis. The wave phenomena are more complicated in anisotropic materials compared to isotropic materials. This necessitates a more rigorous study of these phenomena when using ultrasonic nondestructive testing for inspection of anisotropic materials.

Theoretical studies on wave propagation in anisotropic cylinders and shells have been pursued for many years. Mirskey [1] studied the propagation of free harmonic waves in transversely isotropic circular cylinders. Tsai et al. [2] and Tsai [3] investigated the cylindrically guided waves in transversely isotropic shafts and thick hollow cylinders and plotted the dispersion curves for a number of transversely isotropic rods and shells. Chan and Tsang [4] investigated the propagation of acoustic waves in a fluid-filled borehole surrounded by concentrically layered, transversely isotropic media. In all these studies the mathematical model is based on prediction of the form of displacement field and the final solution is somehow guessed.

Honarvar and Sinclair [5] used a mathematical model in which the scalar potential representing the horizontally polarized shear wave can be decoupled from the compressional and vertically polarized shear waves. The solution was used for scattering of acoustic waves from transversely isotropic cylinders.

Ahmad and Rahman [6] verified that transversely isotropic materials can show two different behaviors. They found that type I materials qualitatively behave similar to isotropic materials and have two critical angles whereas type II transversely isotropic materials possess an additional critical angle. They calculated the critical angles for several materials and determined the type of transverse isotropy. One of the authors [7], showed that the ratio c_{33}/c_{11} is very important in determining the type of transversely isotropic materials where in type I/type II, the stiffness along the axis is higher/lower than the other directions lying in the transverse plane.

In this paper, propagation of longitudinal waves in transversely isotropic shells is studied. The mathematical model is based on the approach used by Honarvar and Sinclair [5] for modeling the scattered pressure field from a transversely isotropic cylinder. The shell is considered to have free boundary conditions. Using this model, the dispersion curves for two types of transversely isotropic materials are plotted. A perturbation study is also conducted on the elastic constants and thickness of the shells.

THEORY

A cylindrical coordinate system (r,θ,z), is chosen with the z-direction coincident with the axis of the cylinder, see Fig. 1. Wave propagates through the shell of infinite length, outer radius a and inner radius b .

The wave equations are obtained by combining the constitutive equations and equations of motion for a transversely isotropic material. These equations are in terms of components of the displacement vector, i.e. u_r, u_θ and u_z. The displacement vector is written in terms of three scalar potential functions as follows [5]:

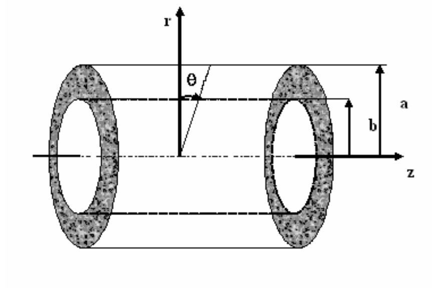

Figure 1- The coordinate system used in the derivation of equations

$$\boldsymbol{u} = \nabla\phi + \nabla\times(\chi\,\hat{e}_z) + l\,\nabla\times\nabla\times(\psi\,\hat{e}_z) \tag{1}$$

where $\hat{e}_z$ is unit vector in the z-direction. The constant l with the dimension of length is introduced for equidimensionality, and is set to be the inverse of the wavenumber in the z-direction, $l = k_z^{-1}$. The potential ϕ represents the P wave (compressional wave) and ψ and χ, respectively, represent the SV wave (vertically polarized shear wave) and SH wave (horizontally polarized shear wave). By solving the resulting partial differential equations, the scalar potentials for an infinite cylindrical shell are found to be of the form:

$$\begin{aligned}
\phi(r,\theta,z,t) &= \sum_{n=0}^{\infty}\{B_n J_n(s_1 r) + C_n Y_n(s_1 r) + q_2[D_n J_n(s_2 r) + E_n Y_n(s_2 r)]\}\cos n\theta\, e^{i(kz-\omega t)}, \\
\psi(r,\theta,z,t) &= \sum_{n=0}^{\infty}\{q_1[B_n J_n(s_1 r) + C_n Y_n(s_1 r)] + D_n J_n(s_2 r) + E_n Y_n(s_2 r)\}\cos n\theta\, e^{i(kz-\omega t)}, \\
\chi(r,\theta,z,t) &= \sum_{n=0}^{\infty}\{F_n J_n(s_3 r) + G_n Y_n(s_3 r)\}\sin n\theta\, e^{i(kz-\omega t)},
\end{aligned} \tag{2}$$

where $k = \omega / c$ is the wave number, c is the phase velocity, ω is the circular frequency and J_n and Y_n are first and second type Bessel functions of order n, respectively. Moreover, s_1, s_2, s_3, q_1 and q_2 are constants depending on the elastic constants of the material, frequency and phase velocity[5].

For axisymmetric problems, i.e. longitudinal waves travelling along the shell axis, the displacement field is independent of the θ-coordinate and of the form $(u_r, 0, u_\theta)$. This mode of wave propagation corresponds to $n=0$ in Eqs. (2) and results in $\chi = 0$ [9].

For an empty cylindrical shell in vacuum, the traction-free boundary conditions hold in both the inner and outer radii. Therefore,

$$\begin{aligned}
\sigma_{rr}^{s}(a,\theta,z) &= 0 \qquad & \sigma_{rr}^{s}(b,\theta,z) &= 0 \\
\sigma_{rz}^{s}(a,\theta,z) &= 0 \qquad & \sigma_{rz}^{s}(b,\theta,z) &= 0
\end{aligned} \tag{3}$$

By applying the boundary conditions, the system of linear algebraic equations will be as follows;

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{pmatrix} A_0 \\ B_0 \\ C_0 \\ D_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

where elements a_{ij} of the coefficient matrix are given in appendix A. In this case, the frequency equation is of the form,

$$det\lfloor a_{ij} \rfloor = 0 \tag{5}$$

This equation is also called the dispersion equation and shows the relationship between the frequencies and the phase velocities of various modes of longitudinal guided waves in a transversely isotropic cylindrical shell in terms of its elastic constants. Using this equation, the dispersion curves can be plotted for different modes.

NUMERICAL RESULTS

In order to illustrate the general behavior of the solution, numerical examples are considered in this section. Realizing the large number of parameters involved, no attempt is made to exhaustively evaluate the effect of varying each of them. Our purpose is merely to illustrate the kind of results to expect from some representative and physically realistic choices of values for these parameters. From these data some trends are noted and general conclusions are made about the relative importance of certain parameters.

The wave numbers of the longitudinal guided modes propagating in free transversely isotropic cylindrical shells can be calculated at any frequency by numerically searching for the zeros of the frequency equation. We shall present the dispersion relation in terms of the frequency-dependent phase velocity. The dispersion curves can be plotted using the roots of Eq. 5. The phase velocity is normalized with respect to the bar velocity, c_b where $c_b = \sqrt{E_a / \rho}$ and $E_a = c_{33} - 2c_{13}^2/(c_{11} + c_{12})$, E_a is the axial Young's modulus.

To study the effects of type of transversely isotropic material in a cylindrical shell, the dispersion curves are plotted for type I and type II shells with different elastic constants. The calculations are made for cobalt as a type I and cadmium as a type II transversely isotropic material with elastic properties given in [6].

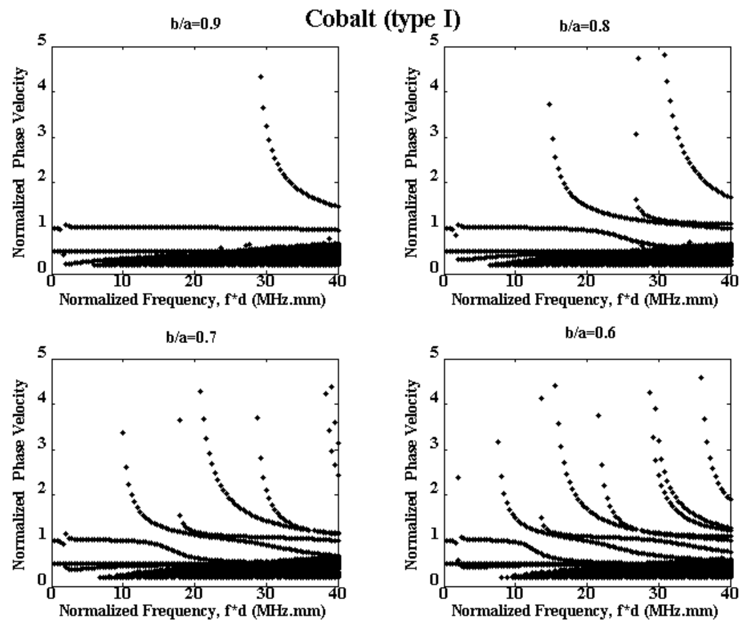

Figure 2- Calculated dispersion curves for cobalt (type I) circular cylindrical shell with:

$a = 0.002,\ \ b/a = 0.9, 0.8, 0.7, 0.6$

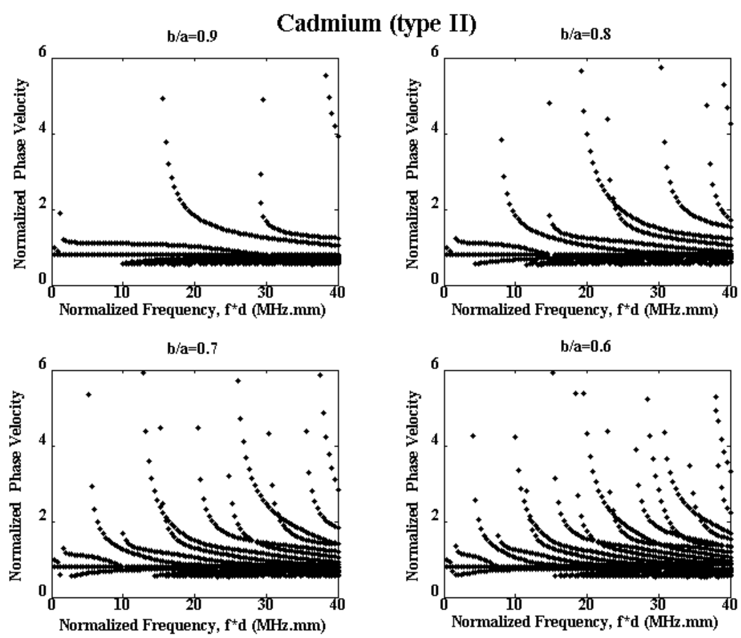

Figure 3- Calculated dispersion curves for cadmium (type II) circular cylindrical shell with:
$a = 0.002,\ \ b/a = 0.9, 0.8, 0.7, 0.6$

Figure 2, shows the calculated dispersion curves for different longitudinal modes of cobalt (Type I) shells having different thicknesses. As the normalized frequency tends to infinity, the phase velocity of the lowest mode approaches the velocity of Rayleigh wave. With increasing the thickness of the cylindrical shell, cut-off frequencies are observed at higher frequencies, which indicate that the mode does not exist below that value.

Figure 3, shows the calculated dispersion curves for different longitudinal modes of cadmium (Type II) circular cylindrical shells having different thicknesses. The dispersion curves for this material are different. Compared to Figure 2, differences can be observed especially at low frequencies and low phase velocities. It seems that in this region, there is severe dependence on the type of transversely isotropic material. The thinner the shell, the lower the number of modes propagating along it, and the larger the cut-off frequencies.

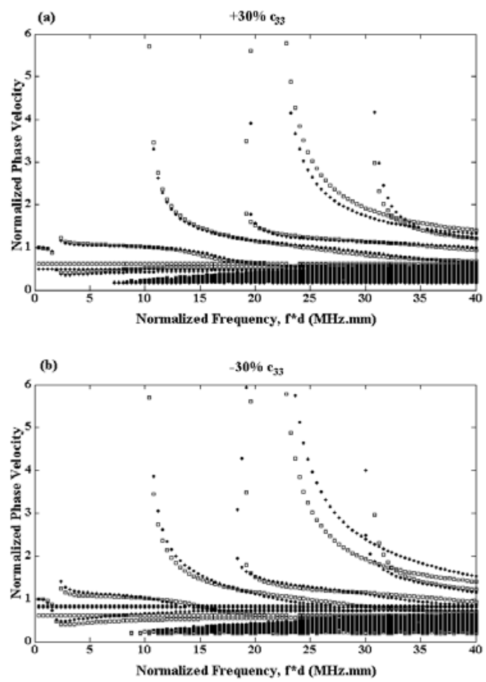

Figure 4 - Calculated dispersion curves for an aluminum ($a = 0.02$ and $b/a = 0.7$) circular cylindrical shell in two cases of axial stiffness perturbations: white square (□): isotropic aluminum and filled square (■): a) 30% increase in c33, b) 30% decrease in c33:

Aluminum is considered as isotropic material with Lame constants corresponding to a transversely isotropic material having the same elastic properties as aluminium [7]. Since c_{33} is a crucial parameter in determining whether a transversely isotropic material is of type I or II, this elastic constant of the aluminum shell ($a = 0.02$, $b/a = 0.7$), is perturbed by $\pm 30\%$.

The resulting dispersion curves are plotted in Figures 4(a) and (b). Comparing

Figs. 2 and 3 with Figs. 4(a) and 4(b), we observe that increasing/decreasing stiffness of aluminum along the z axis has resulted in dispersion curves similar to those of a type I/ II material. The dispersion curves for cobalt/cadmium as type I/type II transversely isotropic material is very similar to the dispersion curves for aluminum with 30% increase/decrease in c_{33}.

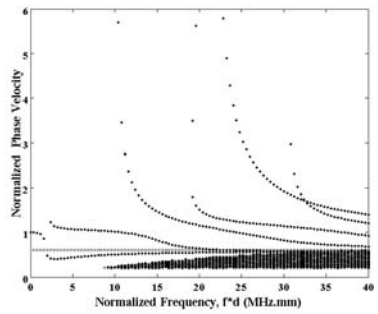

Figure 5 - Calculated dispersion curves for Aluminum (Isotropic material and $a = 0.02$ $b/a = 0.7$) circular cylindrical shell with white square (□): transversely isotropic modeling, filled square (■): isotropic modeling

The dispersion curves of Fig. 5 can be used to verify the validity of the mathematical model used in this paper. Curves shown by white squares (□) are obtained by the transversely isotropic modeling while curves shown by filled square (■) are obtained by the Helmholtz decomposition technique commonly used in the case of isotropic materials. As shown, there is good agreement between these two models except at lower frequencies where the model for transversely isotropic materials predicts the velocity of Rayleigh wave while the Helmholtz decomposition in high frequencies predict the velocity of Rayleigh wave.

CONCLUSIONS

In this paper, the propagation of longitudinal waves in circular cylindrical shells is studied. The normal mode expansion is used for longitudinal waves propagating along free transversely isotropic circular cylindrical shells. The displacement vector is written in terms of three scalar potential functions representing the compressional, vertically polarized and horizontally polarized shear waves. The formulation accommodates two types of transversely isotropic materials. Results indicate that resonances are sensitive to the type of transversely isotropic material and thickness of the shell.

This method can also be used for other modes of wave propagation in transversely isotropic cylindrical shells. For instance if $n \neq 0$, flexural vibration modes of the shell are obtained and if potential functions (Eqs. 2-4) are changed and appropriately selected, this method can be used for studying the propagation of torsional waves. This method of solution can be extended for solving the problem of multilayered solid cylinders and shells.

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APPENDIX A

Elements of the matrices given in Eq. 4:

$$
\begin{aligned}
a_{11} = \{&[c_{11} + i(c_{11} - c_{13})q_1][(n^2 - n - s_1^2 a^2) J_n(s_1 a) + (s_1 a) J_{n+1}(s_1 a)] \\
&+ [c_{12} + i(c_{12} - c_{13})q_1][n J_n(s_1 a) - (s_1 a) J_{n+1}(s_1 a)] \\
&+ [-c_{13} k_z^2 a^2 - c_{12} n^2 + i(c_{13} - c_{12}) n^2 q_1] J_n(s_1 a)\} \\
a_{12} = \{&[c_{11} + i(c_{11} - c_{13})q_1][(n^2 - n - s_1^2 a^2) Y_n(s_1 a) + (s_1 r) Y_{n+1}(s_1 a)] \\
&+ [c_{12} + i(c_{12} - c_{13})q_1][n Y_n(s_1 a) - (s_1 a) Y_{n+1}(s_1 a)] \\
&+ [-c_{13} k_z^2 a^2 - c_{12} n^2 + i(c_{13} - c_{12}) n^2 q_1] Y_n(s_1 a)\} \\
a_{13} = \{&[c_{11} q_2 + i(c_{11} - c_{13})][(n^2 - n - s_2^2 a^2) J_n(s_2 a) + (s_2 a) J_{n+1}(s_2 a)] \\
&+ [c_{12} q_2 + i(c_{12} - c_{13})][n J_n(s_2 a) - (s_2 a) J_{n+1}(s_2 a)] \\
&+ [(-c_{13} k_z^2 a^2 - c_{12} n^2) q_2 + i(c_{13} - c_{12}) n^2] J_n(s_2 a)\} \\
a_{14} = \{&[c_{11} q_2 + i(c_{11} - c_{13})][(n^2 - n - s_2^2 a^2) Y_n(s_2 a) + (s_2 a) Y_{n+1}(s_2 a)] \\
&+ [c_{12} q_2 + i(c_{12} - c_{13})][n Y_n(s_2 a) - (s_2 a) Y_{n+1}(s_2 a)] \\
&+ [(-c_{13} k_z^2 a^2 - c_{12} n^2) q_2 + i(c_{13} - c_{12}) n^2] Y_n(s_2 a)\} \\
a_{21} = \{&[q_1 (s_1^2 a/k_z - k_z a) + 2i k_z a][n J_n(s_1 a) - (s_1 a) J_{n+1}(s_1 a)]\} \\
a_{22} = \{&[q_1 (s_1^2 a/k_z - k_z a) + 2i k_z a][n Y_n(s_1 a) - (s_1 a) Y_{n+1}(s_1 a)]\} \\
a_{23} = \{&[(s_2^2 a/k_z - k_z a + 2i q_2 k_z a][n J_n(s_2 a) - (s_2 a) J_{n+1}(s_2 a)]\} \\
a_{24} = \{&[(s_2^2 a/k_z - k_z a + 2i q_2 k_z a][n Y_n(s_2 a) - (s_2 r) Y_{n+1}(s_2 a)]\}
\end{aligned}
$$

$$
\begin{aligned}
a_{32} = \{&[c_{11} + i(c_{11} - c_{13})q_1][(n^2 - n - s_1^2 b^2) J_n(s_1 a) + (s_1 b) J_{n+1}(s_1 b)] \\
&+ [c_{12} + i(c_{12} - c_{13})q_1][n J_n(s_1 b) - (s_1 b) J_{n+1}(s_1 b)] \\
&+ [-c_{13} k_z^2 b^2 - c_{12} n^2 + i(c_{13} - c_{12}) n^2 q_1] J_n(s_1 b)\} \\
a_{33} = \{&[c_{11} + i(c_{11} - c_{13})q_1][(n^2 - n - s_1^2 b^2) Y_n(s_1 b) + (s_1 b) Y_{n+1}(s_1 b)] \\
&+ [c_{12} + i(c_{12} - c_{13})q_1][n Y_n(s_1 b) - (s_1 b) Y_{n+1}(s_1 b)] \\
&+ [-c_{13} k_z^2 b^2 - c_{12} n^2 + i(c_{13} - c_{12}) n^2 q_1] Y_n(s_1 b)\} \\
a_{34} = \{&[c_{11} q_2 + i(c_{11} - c_{13})][(n^2 - n - s_2^2 b^2) J_n(s_2 b) + (s_2 b) J_{n+1}(s_2 b)] \\
&+ [c_{12} q_2 + i(c_{12} - c_{13})][n J_n(s_2 r) - (s_2 b) J_{n+1}(s_2 b)] \\
&+ [(-c_{13} k_z^2 b^2 - c_{12} n^2) q_2 + i(c_{13} - c_{12}) n^2] J_n(s_2 b)\} \\
a_{35} = \{&[c_{11} q_2 + i(c_{11} - c_{13})][(n^2 - n - s_2^2 b^2) Y_n(s_2 b) + (s_2 b) Y_{n+1}(s_2 b)] \\
&+ [c_{12} q_2 + i(c_{12} - c_{13})][n Y_n(s_2 b) - (s_2 b) Y_{n+1}(s_2 b)] \\
&+ [(-c_{13} k_z^2 b^2 - c_{12} n^2) q_2 + i(c_{13} - c_{12}) n^2] Y_n(s_2 b)\} \\
a_{41} = \{&[q_1 (s_1^2 b/k_z - k_z b) + 2i k_z b][n J_n(s_1 b) - (s_1 b) J_{n+1}(s_1 b)]\} \\
a_{42} = \{&[q_1 (s_1^2 b/k_z - k_z b) + 2i k_z b][n Y_n(s_1 b) - (s_1 b) Y_{n+1}(s_1 b)]\} \\
a_{43} = \{&[(s_2^2 b/k_z - k_z b + 2i q_2 k_z b][n J_n(s_2 b) - (s_2 b) J_{n+1}(s_2 b)]\} \\
a_{44} = \{&[(s_2^2 b/k_z - k_z b + 2i q_2 k_z b][n Y_n(s_2 b) - (s_2 b) Y_{n+1}(s_2 b)]\}
\end{aligned}
$$