ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

ON SUPPRESSION OF TRANSMISSION OF MECHANICAL ENERGY IN STRAIGHT ELASTIC TUBES BY USE OF A LIMITED NUMBER OF EQUALLY SPACED IDENTICAL INERTIAL ATTACHMENTS

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Abstract

The talk addresses theoretical and experimental analysis of the energy transmission in a straight elastic tube bearing three eccentrically attached identical masses in relatively low frequency range, where a classical beam theory is applicable.

The theoretical model is formulated as a system of boundary equations, which describe propagation of flexural, axial and torsion waves within each segment of a tube between inclusions, and continuity conditions at the points, where masses are attached. The presence of masses couples propagation of waves of all these types. An exact solution of this system is obtained and it is found that appropriate location of three identical equally spaced masses can dramatically decrease the power input into the system in some frequency 'stop bands' regardless the excitation conditions. This effect, well known for an infinite chain of periodic attachments, is demonstrated here for only three 'periodicity cells'. Theoretical analysis is expanded to parametric studies of location of 'stop bands' in a given frequency range and to sensitivity analysis of wave attenuation inside the 'stop bands' to possible imperfections of mounting and spacing of attachments.

INTRODUCTION

Control of vibroacoustic energy transmission in pipelines is an important and challenging problem in various industrial and civil applications. The ability to carry out 'low noise design' of, for example, heating system in houses as well as the noise and vibration control of oil- and gas-transporting industrial pipelines is necessary to meet modern regulations. The transmission of vibro-acoustic energy may be suppressed by various tools of 'anechoic termination' (see, for example, [2]) and it is not the goal of this paper to survey these. Rather, the attention here is focused on one particular tool - utilising the phenomenon of ‘frequency band gaps’ in periodic structures. Although the existence of this phenomenon is predicted by Floquet theory in unbounded periodic structures [1], recent publications [3-4] have shown that the presence of a small number of inclusions is capable to produce a substantial attenuation of the transmitted waves. In these references, the methodology of boundary integral equations has been applied in the framework of a general theory of thin shells to analyse wave propagation in compound shell with continuous inclusions. In this paper, the same problem is considered in the framework of a beam theory and the results of experimental investigation are compared with theoretical predictions.

THE THEORETICAL MODEL

The energy transmission in straight elastic beams bearing concentrated masses can conveniently be described within the framework of boundary integral equations method. In the general case of spatial vibrations, these equations should be formulated for the longitudinal waves, for torsion waves and for flexural waves. These waves propagate independently upon each other in a homogeneous beam, but they interact in a compound beam of spatial configuration or in a beam bearing inclusions.

$$u(\xi)=\left[EAu'(x)U(x,\xi)-EAu(x)\frac{\partial U(x,\xi)}{\partial x}\right]_{x=0}^{x=l}$$

$$\varphi(\xi)=\left[GI_t\varphi'(x)\Phi(x,\xi)-GI_t\varphi(x)\frac{\partial \Phi(x,\xi)}{\partial x}\right]_{x=0}^{x=l}$$

$$v(\xi)=\left[EI_z v'''(x)V(x,\xi)-EI_z v(x)\frac{\partial^3 V(x,\xi)}{\partial x^3}-EI_z v''(x)\frac{\partial V(x,\xi)}{\partial x}+EI_z v'(x)\frac{\partial^2 V(x,\xi)}{\partial x^2}\right]_{x=0}^{x=l}$$

$$\frac{dv(\xi)}{d\xi}=\left[EI_z v'''(x)\frac{\partial V(x,\xi)}{\partial \xi}-EI_z v(x)\frac{\partial^4 V(x,\xi)}{\partial x^3\partial \xi}-EI_z v''(x)\frac{\partial^2 V(x,\xi)}{\partial x\partial \xi}+EI_z v'(x)\frac{\partial^3 V(x,\xi)}{\partial x^2\partial \xi}\right]_{x=0}^{x=l}$$

$$w(\xi)=\left[EI_y w'''(x)W(x,\xi)-EI_y w(x)\frac{\partial^3 W(x,\xi)}{\partial x^3}-EI_y w''(x)\frac{\partial W(x,\xi)}{\partial x}+EI_y w'(x)\frac{\partial^2 W(x,\xi)}{\partial x^2}\right]_{x=0}^{x=l}$$

$$\frac{dw(\xi)}{d\xi}=\left[EI_y w'''(x)\frac{\partial W(x,\xi)}{\partial \xi}-EI_y w(x)\frac{\partial^4 W(x,\xi)}{\partial x^3\partial \xi}-EI_y w''(x)\frac{\partial^2 W(x,\xi)}{\partial x\partial \xi}+EI_y w'(x)\frac{\partial^3 W(x,\xi)}{\partial x^2\partial \xi}\right]_{x=0}^{x=l}$$

Here $U(x,\xi)$, $\Phi(x,\xi)$, $V(x,\xi)$ and $W(x,\xi)$ are Green’s functions, which describe the shape of forced vibrations of an infinitely long beam excited at a given frequency by a unit concentrated axial force, torque or transverse force, respectively. These equations are written for each segment of compound structure at the edges, $\xi=0+\varepsilon$

and $\xi = l - \varepsilon$, $\varepsilon \to 0$. They should be solved with continuity conditions at the interfaces between segments of a beam and appropriate boundary conditions.

The layout of a structure, which is analysed theoretically and experimentally, is presented in Figure 1

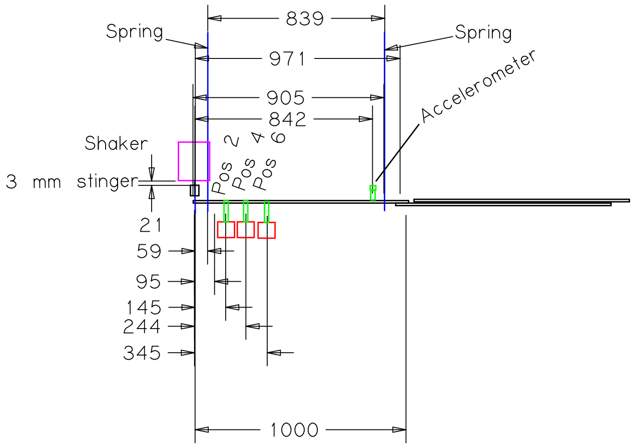

Figure 1. Analysed structure: Aluminum beam with three masses located periodically and excentric to the neutral axis. Measures in mm.

It may be modelled either as a semi-infinite (if waves reflected from the right edge of the structure are neglected) or as a structure of finite length (if appropriate impedance conditions are formulated at the point $x = 1000mm$ is introduced). In both cases, this structure consists of four segments. Displacement vector, rotation vector, force resultant and moment resultant at each edge are introduced for each segment of a compound structure.

The total power flow contains four components,

$$N_{\Sigma} = N_U + N_{\varphi} + N_V + N_W .$$

They present energies transported by axial, torsion and flexural waves:

$$N_U = \frac{1}{2} EA\omega \,\mathrm{Re}\left[u' \cdot \overline{iu} \right], \quad N_{\varphi} = \frac{1}{2} EI_t \omega \,\mathrm{Re}\left[\varphi' \cdot \overline{i\varphi} \right]$$

$$N_V = \frac{1}{2} EI_z \omega \,\mathrm{Re}\left[v'' \cdot \overline{iv'} - v''' \cdot \overline{iv} \right], \quad N_W = \frac{1}{2} EI_y \omega \,\mathrm{Re}\left[w'' \cdot \overline{iw'} - w''' \cdot \overline{iw} \right]$$

As discussed in the introduction, the presence of equally spaced inertial inclusions (the attachments shown in Figure 1) generates the band gap effect in an infinite structure bearing an infinitely large number of these inclusions (a periodic structure) for all power flow components. The practical issue explored in this paper is an assessment of a possibility to reduce the energy transmission by means of three inertial inclusions.

For the first test specimen is chosen a flat aluminium beam with a thickness of 3 mm, a width of 30 mm. The length is seen in Figure 1: 1000 mm. The reason for this type of beam is in the experimental setup to reduce the flexural waves to one (vertical) plane, thereby simplifying the experimental verification. The beam is terminated in a sandwich damped structure.

RESULTS OF COMPUTATIONS

Calculations are done for the model specified in the previous section of the paper. As discussed, it is sufficient to consider only flexural motion in a vertical plane. As a validity check, eigenfrequencies of a freely suspended beam composed of four segments in the absence of attached masses were calculated by use of the methodology of boundary equations. The obtained results were identical to those given by an elementary formula for eigenfrequencies of a uniform free-free beam. See Table 1.

Table 1. Eigenfrequencies (in Hz) of a beam with and without three attachments. Bold: experimentally obtained values with no inclusions.

Theory no inclusions	17	46	90	149	222	310	413	530	664	810
Theory three inclusions	11	31	52	83	120	142	170	185	224	363
Experiment no inclusions	**20**	**44**	**96**	**152**	**224**	**308**	**416**	**580**		**808**

The power flow is calculated for the beam with three inertial elements. Two boundary conditions are used at the right edge of the beam:

- 'perfect impedance matching' thin line in Figure 3
- Impedance coefficient Z = 0.1, dotted line in Figure 3

As is clearly seen, the frequency band gap effect is generated at around $f = 190$ Hz and it is extended up to approximately 700 Hz, where the second pass band is located.

As is seen, the energy transmission is suppressed starting from the same frequency independent of the impedance condition. The magnitude of impedance coefficient Z very weakly influences the magnitude of the lower boundary of the frequency band gap. However, it totally determines the energy input into the structure.

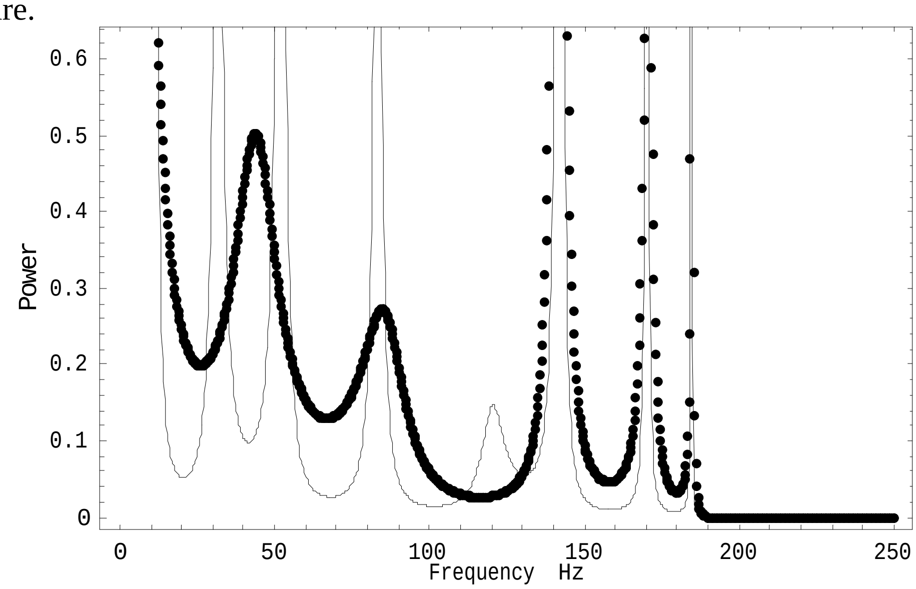

Figure 3. The scaled power flow in a beam with three inclusions versus excitation frequency

EXPERIMENTAL RESULTS

The experimental results are obtained by excitation of the beam shown, in Figure 1, with a white noise force generated by a shaker. The response is measured in the position shown as accelerometer.
The Modal Assurance Criteria comparison to the theoretically calculated eigenfrequencies in table 1 and the measured is acceptable, see Figure 4.

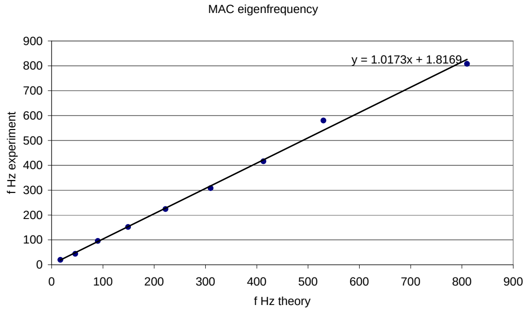

Figure 4. Modal Assurance Criteria, comparing eigenfrequencies, no inclusions

The transfer accelerance (m/s^2)/N measured as the transfer between the force and the vertical accelation is show in figure 5 for the beam without inclusions and with three inclusions with a mass of 675 g.

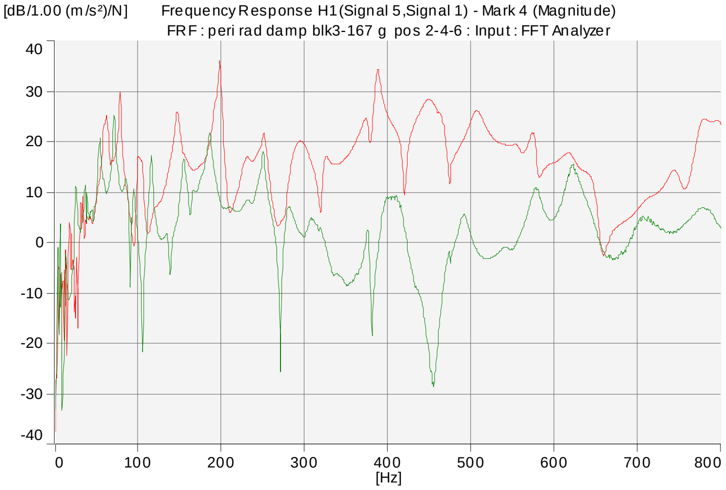

Figure 5. Vertical acceleration/Force. Red: no inclusions, blue: 3 inclusions

Four frequency regions are noted for the difference between the beam without inclusions and with three inclusions:

- Onset of band gap: 270 Hz - is higher than predicted 190 Hz
- Band gap between 270 and 660 Hz reductions of 10 to 20 dB are seen
- 660 – 700 Hz a region with little or no difference

- Band gap above 700 Hz as predicted

It is noted that a band gap is observed in the frequency region 270 – 660 Hz, corresponding to the prediction. The next band gap predicted as starting at 700 Hz is observed. The experimental setup is being refined to attempt explaining the difference in band gap onset frequency.

CONCLUSIONS

The results reported in this paper suggest that the frequency band gap effect may be achieved by use of small number of inertial elements within practically meaningful frequency range. The location of this band gap is not strongly influenced by the impedance conditions at the point, where a compound beam is connected to the outer part of oscillatory system. A fair comparison between the theory and experiment is observed. Further investigations using different dimensions of pipe and inclusions are planned.

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