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Method of Fundamental Solutions for the purpose of coupling Boundary Elements to a Raytracing Procedure

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Abstract
The coupling of boundary elements and a raytracing procedure using the method of fundamental solutions is presented here. As results the Boundary Element analysis gives the nodal values of the primary variable (e.g. sound pressure) and of the secondary variable (e.g. sound flux) on the boundary and on demand - using a postprocessing step - the sound pressure or sound flux in the domain. A raytracing procedure to be coupled on the Boundary Element Method expects intensities of sound sources as loading input data. The required transformation can be done by the Singular Indirect Boundary Element Method [8]. It couples both numerical procedures in an intermediate step between the Boundary Element Method and the raytracing algorithm. This method works well but suffers from the disadvantage needing a defined interface. Here, an alternative way to couple boundary elements and a raytracing procedure is presented. The method of fundamental solutions is used to find intensities of point sources which produce the previously determined sound pressure. The method of fundamental solutions is a meshfree method so it does neither demand the discretization of the domain nor of the boundary. A defined interface is not needed. The presented hybrid numerical method is applied on outdoor sound propagation problems.

INTRODUCTION

The Method of Fundamental Solutions (MFS) has so far been applied to various acoustic problems. A review of the developments and application of the MFS for scattering and radiation problems is given by Fairweather et al [5]. The performance of the MFS for acoustic wave scattering is analysed by Alves et al [1]. In the literature, many different names have been used for this method, as e.g. multipole radiator synthesis or equivalent source method.

The advantages of the MFS are basically its properties as a meshless method: It does not require the discretization of the model and no integration has to be performed. Optionally, an optimization algorithm can be used to optimize the position of the sources in order to minimize the residual at the prescribed boundary points, see Fairweather et al [4] and Cisilino et al [3]. The Method of Fundamental Solutions can either be applied for fixed source positions or with an optimization algorithm to find the source positions for which the residual is minimal. The second approach is usually referred in the literature as the MFS with moving sources.

The methods of computational acoustics can basically be divided into two groups, the wave-based methods and methods of geometrical acoustics. The first type consider the characteristic of sound propagation as traveling waves, and so include all wave phenomena like diffraction and interference. They are based on any kind of wave equation, which can be the scalar wave equation in the time domain or the Helmholtz equation in the frequency domain. These methods are usually implemented using the Finite Element Method (FEM) or the Boundary Element Method (BEM). Whereas in the geometrical acoustics approach the wave character is neglected and sound propagation is considered as propagation of sound particles. The travel path of a sound particle is called sound ray. Most of these methods require point sources as input data.

For an application with a noise barrier or a noise protection dam around the source and receivers at the far-field, the BEM is used for the near field around the source, where the geometry might be complex and where diffraction and multiple reflection occur. For the far-field over large propagation range, a ray method is applied which includes the effect of refraction in the atmosphere due to a vertical profile of sound speed. This sound speed profile can either result from a temperature profile or - using the effective sound speed approach - also from a wind speed profile. Thus, to make use of both kind of methods in one problem calculation, it is necessary to transform the sound field values (e.g. pressure) as output from the wave-based method into equivalent point sources as input for the ray method. This will be done by using the MFS in the following.

NUMERICAL PROCEDURE

Boundary Element Method

The boundary element method (BEM) is an effective numerical method for acoustic problems especially for exterior unbounded domains. It reduces the discretisation effort by one dimension compared to domain discretisation methods. Furthermore, the important SOMMERFELD radiation condition is implicitely fulfilled. This condition ensures that there are no reflections from infinity into the domain. The acoustic BEM in the frequency domain gives an approximate solution of the Helmholtz equation.

$$\Delta p + k^2 p = 0 \tag{1}$$

which corresponds to the scalar wave equation with a time-harmonic approach. p denotes the complex pressure and $k = \omega/c$ is the wave number. The differential equation (1) implies all wave phenomena such as diffraction and interference. Weighting the Helmholtz equation with a fundamental solution and applying Green's theorem twice yields the boundary integral equation [2]

$$\frac{1}{2}p(\vec{\xi}) + \int_{\Gamma} \frac{\partial p^*}{\partial \vec{n}}(\vec{x}, \vec{\xi})p(\vec{x})d\Gamma_x = \int_{\Gamma} p^*(\vec{x}, \vec{\xi})\frac{\partial p}{\partial \vec{n}}(\vec{x})d\Gamma_x, \tag{2}$$

with the 3D fundamental solution $p^*(\vec{x}, \vec{\xi}) = \frac{e^{ik|\vec{x}-\vec{\xi}|}}{4\pi|\vec{x}-\vec{\xi}|}$.

Discretisation of the boundary leads to a linear system of equations to be solved:

$$\mathbf{G}\frac{\partial \mathbf{p}}{\partial \mathbf{n}} - \mathbf{H}\mathbf{p} = \mathbf{0} \tag{3}$$

The application of the boundary element method is limited by the high demand of computation time, when larger systems of equations are to be solved, and by the restriction to homogeneous material in the domain. The second is due to the fact that a fundamental solution has to be known for the problem, which is normally just the case for homogeneous domains. However, there are possibilities to include certain inhomogeneities such as an exponential sound speed profile over a flat infinite ground: The so-called *Conformal Mapping* [9], [6] transforms the domain to a special coordinate system so that the problem is referred back to the homogeneous case.

Ray model

The main difference of a ray model compared to wave-based methods is that it describes sound propagation as a transportation of particles rather than as a travelling wave. The wave front is substituted by discrete particles. The propagation path of each particle is called a sound ray. These sound rays are traced until they reach the receiver.

Here, the semi-analytical ray model of SALOMONS [11] is implemented and coupled with the boundary element method. It is particularly suitable for this purpose because it computes the sound field in the frequency domain and so it matches well to the described BEM model with its time-harmonic approach. The ray model considers a monopole point source and a receiver point over a homogeneous ground of finite impedance. The model represents 3D-propagation with an axisymmetry with respect to the vertical axis through the source. The domain of interest is the vertical plane through the source and receiver point. It takes into account a downward refracting atmosphere which corresponds to an increasing effective sound speed profile with height. The profile can either have a linear or a logarithmic shape. As the ray theory neglects the wave character of sound propagation, the model doesn't include wave phenomena like diffraction at edges.

The algorithm of the model basically works as most other ray tracing models in two steps: In the first step, all sound rays are determined which connect the receiver with the source depending on the source and receiver height, the horizontal distance, and the layered atmosphere. In the second step, the contributions of all m sound rays are added up to get the pressure at the receiver, i.e.,$p = \sum_m A_m exp(i\phi_m)$, where $\phi_m = \omega t_m$ is the phase and A_m is the complex amplitude of ray m. For the special case of homogeneous media one can substitute $\omega t = kr$, and inserting $\phi_m = kr$ leads to an expression similar to the fundamental solution of BEM. It can be shown that this ray method provides the analytical solution for a non-refracting medium over an infinite flat ground.

The MFS for Acoustic Problems

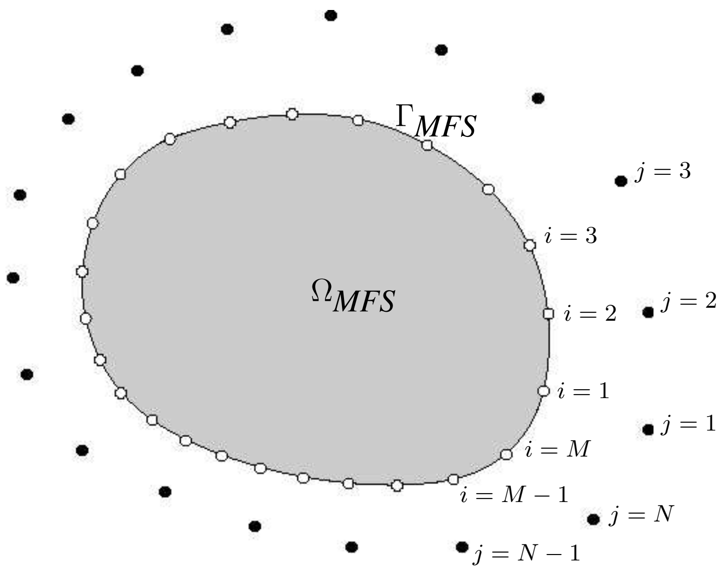


Figure 1: General sketch for the interior MFS problem.

Acoustic problems in the frequency domain are governed by the Helmholtz equation 1. In the following, 2D-problems are considered. A 2D fundamental solution for eq (1) is known to be

$$G(x_i, \xi_j) = -\frac{i}{4} H_0^{(1)}(kr), \tag{4}$$

which describes the pressure at x_i caused by a unit source at ξ_j. $H_0^{(1)}$ is the Hankel function of zero order and first kind. Here, i denotes the imaginary unit (not to mix with the index i for the field points!), k is the wave number and r is the distance from the source point S_j at position $\xi_\mathbf{j}$ to the field point R_i at position $\mathbf{x}_i$. If a half-space over rigid ground is considered, the fundamental solution changes into

$$G(x_i, \xi_j) = -\frac{i}{4} H_0^{(1)}(kr) - \frac{i}{4} H_0^{(1)}(kr'), \tag{5}$$

where r' is the distance of the mirror source S'_j to the field point R_i. The idea behind the MFS is to place a number of sources around the domain of interest Ω_{MFS}, with their positions and intensities set in order to fulfil the given boundary conditions along the boundary Γ_{MFS}

of the domain Ω_{MFS} (see Fig. 1). Each source j at position ξ_j outside the domain Ω_{MFS} contributes a pressure field which is described by the fundamental solution $G(x_i, \xi_j)$. The approximate solution is yield by collocation on a number of points x_i on the boundary Γ_{MFS}. For a given boundary point x_i the pressure value $p(x_i)$ is given by the linear superposition of all contributions $j = 1, 2, \ldots, N$, weighted by the intensity coefficients a_j for each source:

$$\sum_{j=1}^{N} G(x_i, \xi_j) \cdot a_j = p(x_i), \quad x_i \in \Omega, \quad \xi_i \in \bar{\Omega}. \tag{6}$$

This equation is used as boundary conditions by inserting the known boundary values $\bar{p}_i$ at boundary points i on the right hand side. Doing this for all M boundary points results in a system of linear equations

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b}, \tag{7}$$

where the matrix entries A_{ij} consist of the fundamental solutions $G(x_i, \xi_j)$ at point i due to a source with unit intensity at point j, so

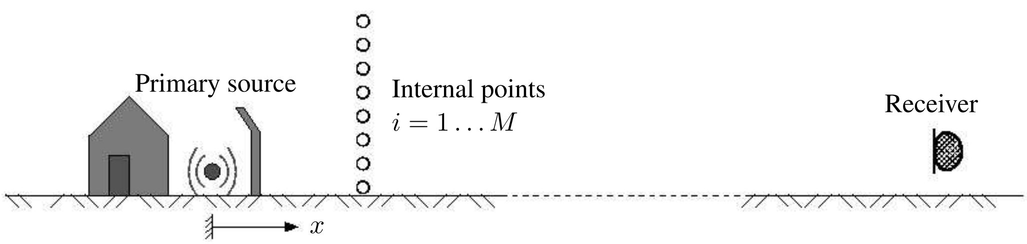
$$A_{ij} = G(x_i, \xi_j), \tag{8}$$

the solution vector $\mathbf{x}$ contains the unknown source intensity coefficients a_j, and vector $\mathbf{b}$ contains the known boundary values.

The number of sources N does not have to be equal to the number of prescribed boundary points M, but can also be smaller. In this case a non-square matrix arises from the equations shown above. This system of equations represents a linear least-squares problem which can be solved using a Single Value Decomposition (SVD) algorithm. For further information about SVD and its numerical implementation see e.g. Press et al [10].

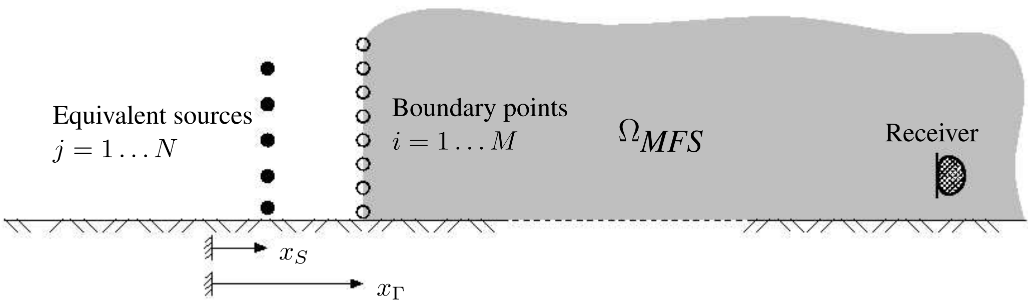
COUPLING

Here, the MFS with fixed source positions is applied to the described coupling problem. In addition, an optimization algorithm is implemented to find optimal positions of sources [7].



(a) Configuration for BEM.

Fig. 2(a) shows the configuration for the BEM calculation, with the primary source and obstacles in the nearfield. This nearfield is approximated in the MFS by a number of equivalent



(b) Configuration for MFS.

Figure 2: Sketch of the configurations for the considered coupling problem with fixed source positions.

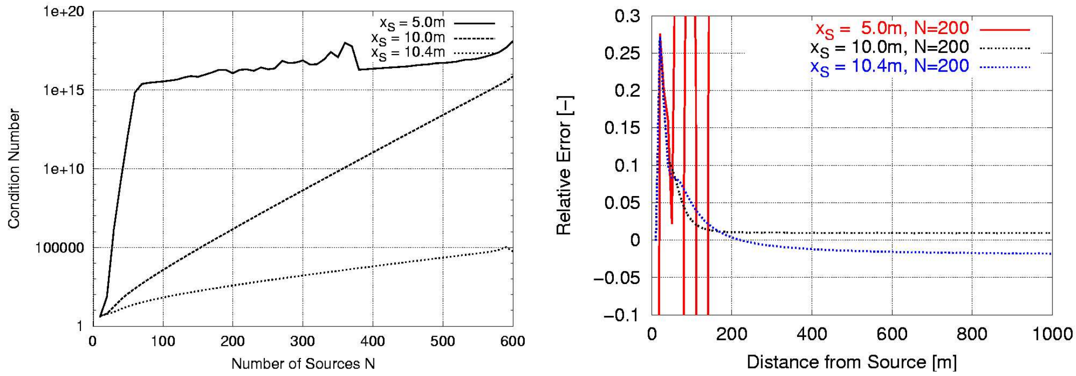
(or secondary) sources (•), see Fig. 2(b). The denotation in this figure is chosen according to Fig. 1. For the MFS the pressure at the boundary points (○) has to fulfil the pressure values yielded by the BEM calculation. The vertical line of boundary points can be considered as the left border of the MFS domain where the ray tracing calculation will be used. The x-position of these vertical lines is x_S for the sources and x_Γ for the boundary points, respectively.

APPLICATION

The described numerical procedure is applied a noise protection dam with a point source on its left hand side (figure 3).

Figure 3: Noise protection dam

Fig. 4(a) shows the condition number of the Matrix **A** the number of sources for three different source positions. The number of boundary points is fixed at $M = 600$, and they are placed in a vertical line at $x_\Gamma = 10.5\,m$ and height up to $60\,m$. Obtained results show that the positions of the sources x_S have a strong influence on the matrix condition. If the sources are too far away from the boundary points compared to the distance between two sources or two boundary points - which is the case for $x_S = 5.0\,m$ -, the system will be ill-conditioned (i.e. high condition numbers). On the other hand, if the sources are too close to the boundary, the matrix entries will become infinite as the fundamental solution G in eq. (8) is singular for $r \to 0$. In brief, the condition number of matrix A depends on the number of sources and their relative positions with respect to the boundary points. Fig. 4(b) shows the relative error as a



(a) Condition number of the system matrix (b) Relative error at internal points

Figure 4: Condition number and relative error at internal point for three different source positions $x_S = 5.0\,m$, $10.0\,m$ and $10.4\,m$; $x_\Gamma = 10.5\,m$.

function of the receiver distance for the case of an homogeneous domain. The relative error was computed by comparing the pressure results obtained using the two-step methodology (BEM-ray method) to the exact solution given by a BEM calculation. The number of boundary points and the source positions are the same as in Fig. 4(a) in order to evaluate the effect of the matrix condition number on the actual quality of the result. The number of source points was chosen to $N = 200$. It can be seen that the error curves associated to source positions close to the boundary (i.e. $x_S = 10.0\,m$ and $x_S = 10.4\,m$) show a nice behaviour. On the other hand, the error for the source position $x_S = 5.0\,m$ diverges and delivers unusable results. This is a consequence of the very high condition number of about 10^{16} (see Fig. 4(a)). Results in Fig. 4(b) also show that the error is always zero at $x_\Gamma = 10.5\,m$, since the boundary conditions in the MFS were imposed in this position. Just beyond this position the error increases because the approximated pressure field is not smooth in the near-field. At some distance from the source, the pressure field smoothes for $x_S = 10.0\,m$ and $x_S = 10.4\,m$ and the proposed procedure approaches the exact solution very good. The error is almost constant over the whole range and even up to $10\,km$ (not shown in Fig. 4(b)) the relative error is lower than two percent.

SUMMARY

It is shown in this paper that the MFS can be successfully applied to couple a wave-based method and a ray methods for solving outdoor sound propagation. The pressure distribution, which is a result from a wave-based method, e.g. the BEM, can be well-approached by a number of equivalent point sources, which are required as input data for most ray methods. The results of error analyses encourage to use MFS for this coupling purposes. The presented method is an alternative way to couple BEM and a ray model. Compared to coupling via the Singular Indirect Boundary Element Method [8] it is more flexible. No defined interface is needed an the number of boundary points and of equivalent sources may differ.

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