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MODE LOCALIZATION IN DOUBLY COUPLED CANTILEVER BEAMS IN CYCLIC CONFIGURATIONS.

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Abstract

The dynamic behaviour of a chain of cantilever beams, coupled by two rows of linear springs in a cyclic configuration is investigated. This simulates turbo-machinery blades which are too long and have to be laced by two rows of stiffeners. A theoretical approach based on Green's functions is used to obtain the exact solutions of the differential equations of motion, which leads to the determination of the natural frequencies and mode shapes of the beams. The displacements of the constraint points in free vibration are calculated and used as a measure of mode localization. The finite element method (FEM, ABAQUS program) is also applied to the system, and the results are in excellent agreement with the theoretical. Using 12 beams and 24 springs of known mistuned properties, it is observed that for the doubly coupled cantilever beams, the first pass-band mode shapes (mode 1 up to mode 12) are weakly localized, but for the second and higher pass-bands, there is very strong localization. This is a new finding which should be of interest to turbo-machinery designers.

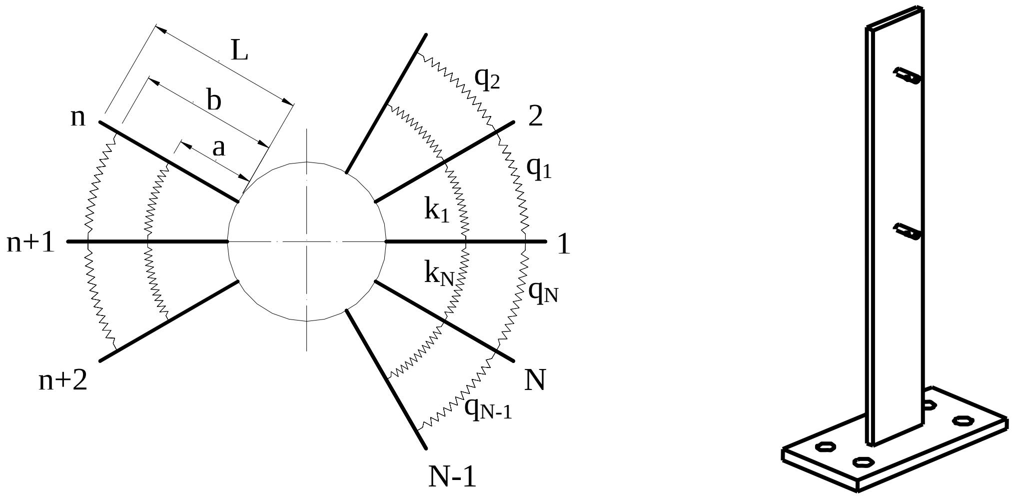
INTRODUCTION

Periodic structures, which are made of repeating sub-structures, ideally identical in every respect(along one or more directions), are a common occurrence in engineering design. Typical examples of periodic structures are aircraft fuselages, atomic structure in all matter, truss beams, ship hulls, railway lines (rail road tracks), antenna dishes, and bladed-disk assemblies such as the rotor of a turbine. For mathematical analysis, the assumption of identical properties of the sub-structures in design leads to exact solutions. But, due to manufacturing tolerances, assembly errors, minute differences in material properties and operational wear, the minute variations from sub-structure to sub-structure leads to a totally different dynamic behaviour of the complete structure. In this case the structure will be known as a nearly periodic (mistuned or disordered) structure instead of a periodic (tuned or ordered) structure. The focus here is on bladed-disk assemblies arranged cyclically as in turbo-machinery blades. The blades, in practice, may be laced by one or two rows of stiffeners depending on length. The coupling effect of such stiffeners may result in mode localization where input energy is confined to a small section of the blade assembly leading to possible failure. The literature on this subject is extensive, starting with Anderson [1] who first linked minute dissimilarities in atomic structure to mode localization. Anderson's findings were first applied to engineering structures by Hodges [2], who showed that the presence of disorder in a nearly periodic structure may invalidate the results of the tuned analysis. Mohamad [3] introduced an exact method based on Green's function to obtain the solution for the dynamic response of beams with general mass and spring attachments, and Mohamad and Al-jawi [4, 5, 6] applied this method to tuned and mistuned linear and cyclic systems. They calculated the natural frequencies for mistuned structures numerically by using both the theoretical method based on Green's function and the finite element approach (ABAQUS. Pierre and Dowell [7] investigated the underlying physical mechanisms of mode localization by perturbation methods for simple structures such as a chain of coupled oscillators. The free localized response of nearly periodic structure with cyclic symmetry was examined by Pierre, Tang and Dowell [8]. In the cited material, single rows of couplers were considered. For the subject of this paper, a double row of coupler linear springs is in effect. Double rows of stiffeners are encountered where the blade lengths are large. It is found that mode localization may or may not occur depending on frequency.

SYSTEM MATHEMATICAL MODEL

Figure 1a shows the schematic of a turbo-machinery bladed system with two rows of stiffeners. The system consists of N beams and two sets of springs (series k and q). An arbitrary beam was selected as beam number 1 and all elements numbered in the counter-clockwise direction. Figure 1b shows the blade in 3D. The differential equation of motion of beams 1, n and N can easily be written down, and after suppressing the harmonic dependence on time, the resulting mode-shape equations satisfy

$$
\begin{aligned}
Y_1(x) = {} & G_1(x,a:z)\left[\frac{k_N Y_N(a)}{EI_1} - \frac{(k_1 + k_N)Y_1(a)}{EI_1} + \frac{k_1 Y_2(a)}{EI_1}\right] \\
& + G_1(x,b:z)\left[\frac{q_N Y_N(b)}{EI_1} - \frac{(q_1 + q_N)Y_1(b)}{EI_1} + \frac{q_1 Y_2(b)}{EI_1}\right]
\end{aligned}
\tag{1}
$$



(a) System configuration *(b) Blade view in 3D*

Figure 1-Blade system model

$$
\begin{aligned}
Y_n(x) = G_n(x,a:z)&\left[\frac{k_{n-1}Y_{n-1}(a)}{EI_n} - \frac{(k_n + k_{n-1})Y_n(a)}{EI_n} + \frac{k_n Y_{n+1}(a)}{EI_n}\right] \\
+ G_n(x,b:z)&\left[\frac{q_{n-1}Y_{n-1}(b)}{EI_n} - \frac{(q_n + q_{n-1})Y_n(b)}{EI_n} + \frac{q_n Y_{n+1}(b)}{EI_n}\right]
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
Y_N(x) = G_N(x,a:z)&\left[\frac{k_N Y_1(a)}{EI_N} - \frac{(k_N + k_{N-1})Y_N(a)}{EI_N} + \frac{k_{N-1}Y_{N-1}(a)}{EI_N}\right] \\
+ G_N(x,b:z)&\left[\frac{q_N Y_1(b)}{EI_N} - \frac{(q_N + q_{N-1})Y_N(b)}{EI_N} + \frac{q_{N-1}Y_{N-1}(b)}{EI_N}\right]
\end{aligned} \tag{3}
$$

In the above equations, G is the green's function after Mohamad [5], z is a frequency parameter and Y the mode shape functions evaluated at a general point x, a or b. EI represents flexural stiffness. To proceed to find the natural frequencies, equations (1), (2) and (3) are evaluated at x = a and at x = b for all the beams. The resulting equations are cast in matrix form as:

$$
\begin{bmatrix} P & R \\ S & Q \end{bmatrix} \begin{Bmatrix} Y_a \\ Y_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{4}
$$

where P, R, S and Q are N x N tri-cyclic sub-matrices while Y_a and Y_b are N x 1 matrices of the modal displacements of points a and b. Natural frequencies are obtained by setting the determinant of the coefficient matrix of equation (4) equal to zero, that is,

$$\left| Q - SP^{-I} R \right| = 0 \text{ , or alternatively, } \left| P - RQ^{-I} S \right| = 0 \tag{5}$$

Upon evaluation of the frequencies from equation (5), the constraint point displacements $Y_n(a)$ and $Y_n(b)$ are found, for each frequency, from equation (4) with $Y_1(a)$ set to unity. The natural frequencies obtained, corresponding to the system data in Tables 1 and 2 are shown in Table 3.

Table 1. Beam Geometrical Properties (mm)

Beam	Length	Width	Thickness
1	400.667	39.847	5.212
2	400.733	39.858	5.205
3	401.300	39.890	5.170
4	400.967	39.897	5.189
5	401.167	39.861	5.177
6	401.137	39.740	5.199
7	401.300	39.867	5.161
8	400.467	39.741	5.160
9	400.767	39.927	5.182
10	401.010	39.936	5.175
11	401.120	39.819	5.198
12	401.400	39.893	5.180

Table 2. Spring Stiffnesses (N/m)

Spring	Spring k,	Spring q,
1	71.932	62.230
2	72.618	61.250
3	72.618	60.662
4	72.618	61.642
5	73.010	62.524
6	71.932	61.936
7	72.618	62.524
8	73.402	58.506
9	72.814	60.074
10	70.756	59.584
11	70.560	60.466
12	72.716	60.760

Table 3. The first twenty four natural frequencies for the cyclic chain of twelve beams, Hz.

Mode	THEORETICAL	FEM	Mode	THEORETICAL	FEM
1	26.84721	27.01277	13	167.58079	170.02782
2	26.90683	27.07084	14	167.87300	170.32403
3	26.92373	27.08862	15	168.11316	170.56627
4	27.01258	27.16985	16	168.21116	170.66574
5	27.03253	27.18849	17	168.24681	170.70136
6	27.15648	27.30238	18	168.27672	170.73474
7	27.23338	27.38390	19	168.71293	171.17628
8	27.33169	27.46887	20	168.77157	171.23398
9	27.33445	27.47348	21	168.93402	171.39589
10	27.42579	27.55633	22	168.95322	171.41288
11	27.43886	27.56782	23	169.48649	171.95730
12	27.55152	27.68335	24	169.77341	172.24725

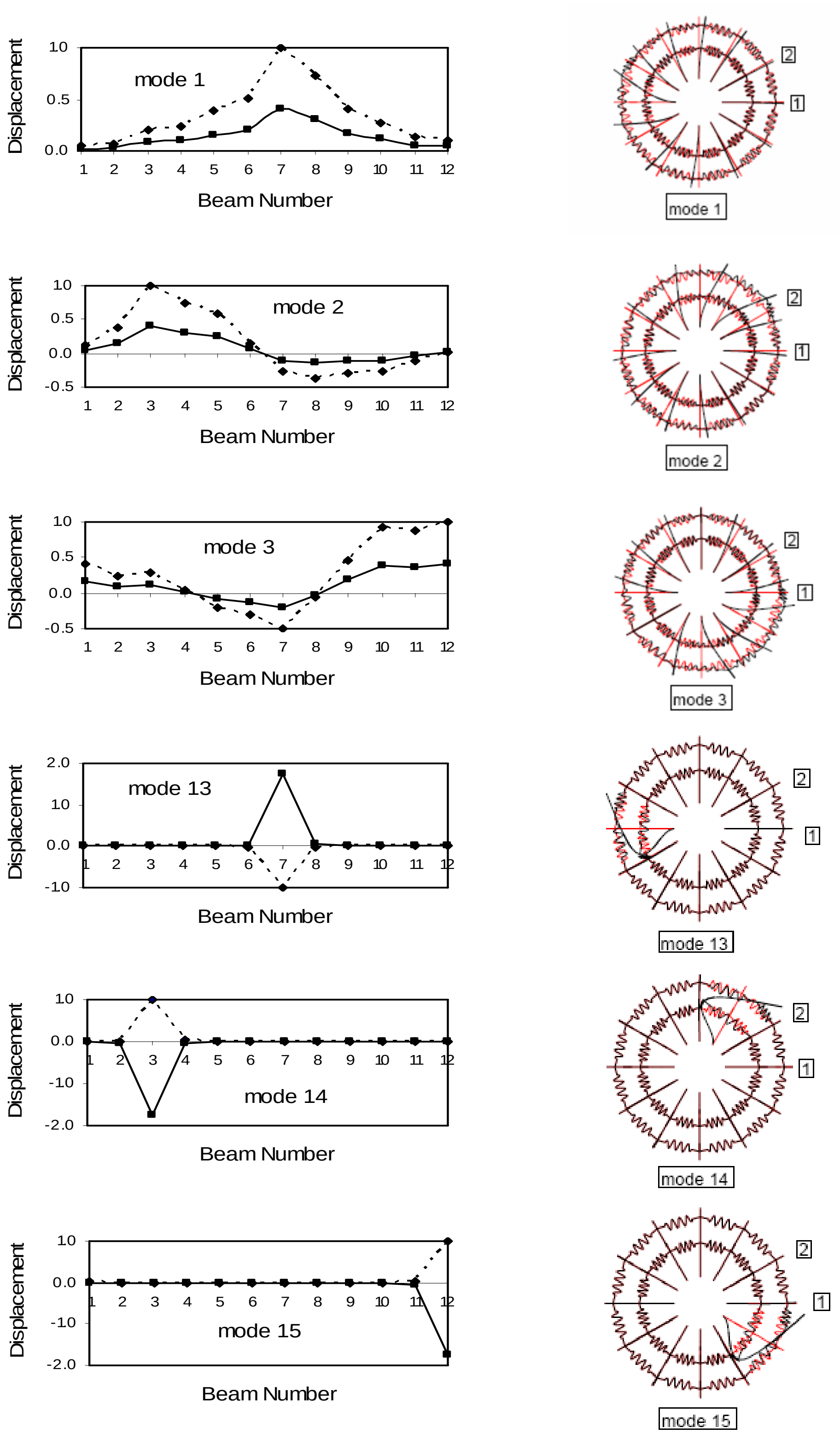
In Table 3, the Finite Element Method (FEM) results were obtained using ABAQUS, and the Young's Modulus for steel and density were assumed to be 208 GPa and 7800 kg/m^3. In addition, the spring attachment points were a /L= 0.500 and b/L = 0.875. The constraint point displacement vectors

$$\left[1 \;\; Y_2(a) \;\; Y_3(a) \ldots\ldots Y_{12}(a)\right]^T \tag{6}$$

and

$$\left[Y_1(b) \;\; Y_2(b) \;\; Y_3(b) \ldots\ldots Y_{12}(b)\right]^T \tag{7}$$

corresponding to each frequency are plotted in Figure 2. The finite element method is also used to plot the mode shapes as shown in the Figure.



(a)--- ------- Y(a), ———— Y(b) *(b)*

Figure 2-Mode shapes. (a) Theoretical, (b) FEM

CONCLUSIONS

For the system configuration used in this study, it is possible to qualitatively discuss energy localization by examination of Figure 2a (constraint point displacements) and Figure 2b (full mode shapes from the finite element method). For modes 1, 2 and 3 which belong to the first group of 12 modes in the first pass-band, it is observed that localization is practically non-existent. Both the theoretical and finite models depict an extended modal behaviour where all the beams are in motion. At all twelve modes in this pass-band, there is no localization. In the second group (modes 13-24), a very strong localization phenomenon is observed. Reference to the Figures shows that in mode 13 for example, strong localization occurs at beam 7 for both theoretical and finite element models. Similarly, for modes 14 and 15, strong localization is found at beams 3 and 12 respectively. Strong localization also occurs at the third group frequencies (modes 25-36, not shown).

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