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SMART MODAL CONTROL OF A FLEXIBLE PANEL USING SMART SENSORS / ACTUATORS

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Abstract

This paper is concerned with modal control of a rectangular plate using both smart sensors and smart actuators. Since filtering and actuation are related with duality, discussion on a smart sensor comes first, and then that of the counterpoint, smart actuator, follows in the work. This paper begins by discussing an ideal case of modal control comprising an extraction and activation of a target structural mode of a plate. Then, for the purpose of implementing a modal control system, three-dimensional shaped smart sensors and actuators are introduced, the design procedure being discussed. Taking into consideration the practicability, two-dimensional smart sensors and actuators for modal filtering and modal actuation respectively are then presented, the number, location and shaping function of the two-dimensional shaped modal sensors and actuators being clarified. Furthermore, the extraction and actuation of the (1, 3) mode of a simply supported rectangular panel using the smart sensors and actuators are shown. Finally, a modal feedback control system is constructed utilizing both the smart sensors and smart actuators, demonstrating the effectiveness for suppressing the particular structural mode of interest without causing spillover.

INTRODUCTION

Difficulty in modal control of a distributed-parameter structure lies in a large number of vibration modes (theoretically infinite) it has, and hence the corresponding number of sensors for modal filtering (sometimes termed modal analyzer) and actuators for modal actuation (sometimes dubbed modal synthesizer) are required for building a modal control system. Regarding the modal filtering, most papers reported in the past would resort to point sensors, mostly due to the availability of the point sensors. The other way to alleviate the burden is to exchange point sensors for distributed parameter sensors[7 ~ 10] such as PVDF film sensors. A distributed parameter sensor can be viewed as the limit of an infinite number of point sensors, and hence the use of the distributed sensor *per se* entails the spatial integration capability. Moreover, shaping a distributed sensor with appropriate functions is equivalent to using point sensors together with a large number of read, multiply and addition operations in the associated digital signal processing. In other words, the output of the shaped sensor is the desired control signal for the controller, and thus the direct sensing of the target mode becomes possible, with a corresponding large reduction in the digital signal processing requirement, hence smart sensor. Again the same is true for smart actuators based on the distributed piezo-electric laminar because of the duality.

This paper is concerned with modal control of a rectangular plate using both smart sensors and smart actuators. Since filtering and actuation are related with duality, discussion on a smart sensor comes first, and then that of the counterpoint, smart actuator, follows. This paper begins by discussing an ideal case of modal control comprising an extraction and activation of a target structural mode of a distributed plate. Then, for the purpose of implementing the modal control system, three-dimensional shaped smart sensors and actuators are introduced, the design procedure being discussed. Taking into consideration the practicability, two-dimensional smart sensors for modal filtering are presented, the number, location and shaping function of the two-dimensional shaped modal sensors being clarified. Furthermore, the extraction of the (1,3) mode of a simply supported rectangular panel using the smart sensors is shown. Moreover, the two-dimensional shaped smart actuators for modal actuation are presented, its number, location and shaping function being investigated. Furthermore, based upon the smart actuator the modal actuation on the (1,3) structural mode of the panel is discussed. Finally, a modal feedback control system is constructed utilizing both the smart sensors and the smart actuators, demonstrating the effectiveness for suppressing the particular structural mode of interest without causing spillover.

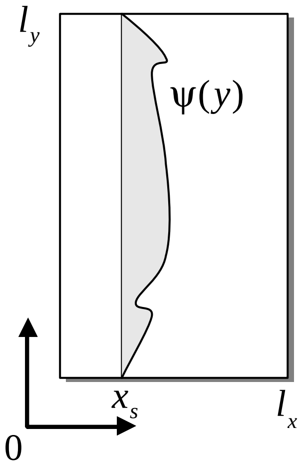

Fig. 1 Smart sensor

MODAL CONTROL USING SMART SENSOR / ACTUATOR

Suppose that a PVDF film shaped by a function $\Gamma_a(\mathbf{r})$ is placed at $x = x_s$ as shown in Fig. 1. The equation of motion of a plate is then given by

$$\mathrm{D}\ \nabla^2\nabla^2 w(\mathbf{r},\mathrm{t}) + \rho\, h\, \ddot{w}(\mathrm{\mathbf{r}},\mathrm{t}) = -\kappa\, \mathrm{e}(\mathrm{t})\left(\mathrm{e}_{3\,1}\frac{\partial^2\Gamma_a(\mathbf{r})}{\partial\, x^2} + \mathrm{e}_{3\,2}\frac{\partial^2\Gamma_a(\mathbf{r})}{\partial\, y^2}\right) \qquad 1$$

where

$$G_a(\mathbf{r}) = h_0\left\{u(x - x_s) - u(x - x_s - \psi(y))\right\} \qquad 2$$

where $u(x)$ is the step function. Note that the thickness of the area with hatched lines in Fig. 1 is h_0, otherwise zero. Furthermore, in order to activate the *pq*th structural mode, the basic function is written as

$$\psi(y) = a_{pq}\varphi_{pq}(x_s, y), \qquad a_{pq} << 1 \qquad 3$$

Substituting Eqs. 2 and 3 into Eq. 1, multiplying both sides of the formula by the *pq*th structural mode $\varphi_{pq}(\mathbf{r})$ and integrating it over the domain of the plate, the equation of motion of a modal coordinate system is given by

$$\begin{aligned}&\ddot{\eta}_{mn}(\mathrm{t}) + \omega_{mn}^2\eta_{mn}(\mathrm{t})\\ &= -h_0\kappa a_{pq}b_{mq}\int_0^{l_y}\varphi_{pq}(x_s, y)\varphi_{mn}(x_s, y)\, dy\, \mathrm{e}(\mathrm{t})\\ &= -h_0\kappa a_{pq}b_{mq}c_{pm}(x_s)\mathrm{e}(\mathrm{t})\delta_{qn}; \quad m,n = 1,2,3\cdots\end{aligned} \qquad 4$$

Equation 4 implies that the two-dimensional smart actuator excites the target mode as well as unwanted modes with the same modal index q in the y direction. To overcome this problem, multiple two-dimensional smart sensors are required to excite the targeted mode without spillover as will be discussed shortly.

Consider to activate the pqth structural mode of a simply supported rectangular plate using the two-dimensional PVDF actuator shaped by

$$\mathrm{G}_a(\mathbf{r}) = h_0\left\{u(x-x_s) - u(x-x_s-a_{pq}\sin\beta_q y)\right\}. \tag{5}$$

Then the displacement of the plate excited by the distributed actuator at is given by

$$w_s(\mathbf{r}) = \frac{h_0\kappa a_{pq} l_y}{2}\sum_{m=1}^{M}\varphi_{mq}(\mathbf{r})\frac{b_{mq}\ \sin\alpha_p x_s\ \sin\alpha_m x_s}{\omega_{mp}^2-\omega^2}\mathrm{e}_s \tag{6}$$

where e_s is the driving voltage of the actuator placed at x_s. Suppose that there are M structural modes with the modal index q in the y direction in the frequency range of interest. Then M shaped smart actuators are needed to perform the modal excitation. Then summing up all the M displacement responses $w_s(\mathbf{r})$ in Eq. 6 resulting from each distributed actuator located at driven by voltage e_s should be the target modal response $\eta_{pq}\varphi_{pq}(\mathbf{r})$, and hence

$$\begin{aligned}&\sum_{s=1}^{M} w_s(\mathbf{r})\\ &= \frac{h_0\kappa a_{pq} l_y}{2}\sum_{s=1}^{M}\sum_{m=1}^{M}\varphi_{mq}(\mathbf{r})\frac{b_{mq}\ \sin\alpha_p x_s\ \sin\alpha_m x_s}{\omega_{mq}^2-\omega^2}\mathrm{e}_s\,. \\ &= \eta_{pq}\varphi_{pq}(\mathbf{r})\end{aligned} \tag{7}$$

In order for the above equation to hold, the term with respect to each $\varphi_{mq}(\mathbf{r})$ except the target structural mode $\varphi_{pq}(\mathbf{r})$ must be zero, so that we have the following formula,

$$\mathbf{Ae} = \mathbf{b} \tag{8}$$

where

$$\mathbf{A} = \begin{pmatrix} \frac{b_{1q}\ \sin\alpha_p x_1\ \sin\alpha_1 x_1}{\omega_{1p}^2-\omega^2} & \frac{b_{1q}\ \sin\alpha_p x_2\ \sin\alpha_1 x_2}{\omega_{1p}^2-\omega^2} & \cdots & \frac{b_{1q}\ \sin\alpha_p x_M\ \sin\alpha_1 x_M}{\omega_{1p}^2-\omega^2} \\ \frac{b_{2q}\ \sin\alpha_p x_1\ \sin\alpha_2 x_1}{\omega_{2p}^2-\omega^2} & \frac{b_{2q}\ \sin\alpha_p x_2\ \sin\alpha_2 x_2}{\omega_{2p}^2-\omega^2} & \cdots & \frac{b_{2q}\ \sin\alpha_p x_M\ \sin\alpha_2 x_M}{\omega_{2p}^2-\omega^2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{b_{Mq}\ \sin\alpha_p x_1\ \sin\alpha_M x_1}{\omega_{Mp}^2-\omega^2} & \frac{b_{Mq}\ \sin\alpha_p x_2\ \sin\alpha_M x_2}{\omega_{Mp}^2-\omega^2} & \cdots & \frac{b_{Mq}\ \sin\alpha_p x_M\ \sin\alpha_M x_M}{\omega_{Mp}^2-\omega^2} \end{pmatrix}, \tag{9}$$

$$\mathbf{e} = \begin{pmatrix}\mathrm{e}_1 & \mathrm{e}_2 & \cdots & \mathrm{e}_M\end{pmatrix}^{\mathrm{T}}, \tag{10}$$

$$\mathbf{b} = \begin{pmatrix}0 & \cdots & 0 & \eta_{pq} & 0 & \cdots & 0\end{pmatrix}^{\mathrm{T}}. \tag{11}$$

The matrix $\mathbf{A}$ further expands to

$$\mathbf{A} = \hat{\mathrm{L}}\hat{\mathbf{B}}\mathrm{F}^{\mathrm{T}}\hat{\mathrm{y}} \qquad 12$$

where

$$\hat{\mathrm{L}} = \frac{h_0 \kappa a_{pq} l_y}{2}\begin{pmatrix} \frac{1}{\omega_{1p}^2 - \omega^2} & 0 & \cdots & 0 \\ 0 & \frac{1}{\omega_{2p}^2 - \omega^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\omega_{Mp}^2 - \omega^2} \end{pmatrix}. \qquad 13$$

$$\hat{\mathbf{B}} = \begin{pmatrix} b_{1q} & 0 & \cdots & 0 \\ 0 & b_{2q} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_{Mq} \end{pmatrix} \qquad 14$$

$$\mathrm{F} = \begin{pmatrix} \sin\alpha_1 x_1 & \sin\alpha_2 x_1 & \cdots & \sin\alpha_M x_1 \\ \sin\alpha_1 x_2 & \sin\alpha_2 x_2 & \cdots & \sin\alpha_M x_2 \\ \vdots & \vdots & \vdots & \vdots \\ \sin\alpha_1 x_M & \sin\alpha_2 x_M & \cdots & \sin\alpha_M x_M \end{pmatrix} \qquad 15$$

$$\hat{\mathrm{Y}} = \begin{pmatrix} \sin\alpha_p x_1 & & & \mathbf{0} \\ & \sin\alpha_p x_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \sin\alpha_p x_M \end{pmatrix} \qquad 16$$

Consequently, the driving voltage vector **e** for modal actuation, the solution to a set of equations in Eq. 8, is found to be

$$\begin{aligned}\mathbf{e} &= \mathbf{A}^{-1}\mathbf{b} \\ &= \hat{\mathrm{y}}^{-1}\left(\mathrm{F}^{\mathrm{T}}\right)^{-1}\hat{\mathbf{B}}^{-1}\hat{\mathrm{L}}^{-1}\mathbf{b}\,\cdot\end{aligned} \qquad 17$$

MODAL ACTUATION OF THE (1,3) MODE

Using the two-dimensional smart PVDF actuator, let us activate the (1,3) structural mode as was done for modal sensing.

A. Direction of the actuator placement

With the same reason used in the modal sensing design, the y direction is exploited for the two-dimensional smart sensor location.

B. Shaping function

Since the modal index of the y direction is 3, the basic function as shown in Eq. 3 will be of the form

$$\psi(y) = a_{13} \sin \frac{3\pi}{l_y} y . \qquad 18$$

As such, the shaping function in Eq. 44 may be written as

$$G_a(\mathbf{r}) = h_0 \left\{ u(x - x_0) - u\big(x - x_0 - \psi_0 \sin \frac{3\pi}{l_y} y\big) \right\}. \qquad 19$$

C. Number of the necessary actuators

Smart actuator designed with the shaping function in Eq. 18excites not only the targeted (1,3) mode but the (2,3) mode and (3,3) mode due to control spillover. Here it is worth exploiting the nodal lines of the (3,3) mode for actuator placement so as not to excite the (3, 3) mode, which is unwanted; that is,

$$x_1 = \frac{l_x}{3}, \qquad 20$$

$$x_2 = \frac{2l_x}{3}. \qquad 21$$

When placing two smart actuators along these nodal lines, the excitation of the (3,3) mode is avoided, the residual mode being the (2,3) mode

D. Activation of modal amplitude

Distributed smart actuators placed at x_1 and x_2 enables one to conduct modal actuation on the target structural mode. In order to solve the matrix equation in Eq. 17, necessary matrices are described as follow:

$$\mathbf{e} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}, \qquad 22$$

$$\mathbf{b} = \begin{pmatrix} \eta_{13} \\ 0 \end{pmatrix}, \qquad 23$$

$$\hat{L} = \frac{h_0 \kappa a_{13} l_y}{2} \begin{pmatrix} \dfrac{1}{\omega_{11}^2 - \omega^2} & 0 \\ 0 & \dfrac{1}{\omega_{21}^2 - \omega^2} \end{pmatrix}, \qquad 24$$

$$\mathbf{A} = \begin{pmatrix} \dfrac{b_{23}}{\omega_{21}^2 - \omega^2} \sin\alpha_1 x_1 \sin\alpha_2 x_1 & \dfrac{b_{13}}{\omega_{11}^2 - \omega^2} \sin^2\alpha_1 x_1 \\ \dfrac{b_{23}}{\omega_{21}^2 - \omega^2} \sin\alpha_1 x_1 \sin\alpha_2 x_1 & \dfrac{-b_{13}}{\omega_{11}^2 - \omega^2} \sin^2\alpha_1 x_1 \end{pmatrix}. \qquad 25$$

Then Eq. 17 may be written as

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = e_0 \begin{pmatrix} \dfrac{b_{23}}{\omega_{21}^2 - \omega^2} \sin\alpha_1 x_1 \sin\alpha_2 x_1 & \dfrac{b_{13}}{\omega_{11}^2 - \omega^2} \sin^2\alpha_1 x_1 \\ \dfrac{b_{23}}{\omega_{21}^2 - \omega^2} \sin\alpha_1 x_1 \sin\alpha_2 x_1 & \dfrac{-b_{13}}{\omega_{11}^2 - \omega^2} \sin^2\alpha_1 x_1 \end{pmatrix} \begin{pmatrix} \eta_{13} \\ 0 \end{pmatrix} \qquad 25$$

where

$$e_0 = \frac{1}{h_0 \kappa a_{13} b_{13} b_{23} l_y \sin^3 \alpha_1 x_1 \sin \alpha_2 x_1}, \tag{26}$$

so that

$$e_1 = e_2 = \frac{e_0 b_{23} \eta_{13}}{\omega_{21}^2 - \omega^2} \sin \alpha_1 x_1 \sin \alpha_2 x_1 . \tag{27}$$

Apparently from Eq. 27, the input voltages, e_1 and e_2 are the same so that driving the two actuators with the same signal enables the smart actuators to excite only the target structural mode.

EXPERIMENT

In order to verify the validity of modal control using PVDF smart sensors / actuators, an experiment is conducted. The specification utilized in the experiment is as follows: the plate with the dimension of $l_x = 175mm$, $l_y = 330mm$ and $h = 0.3mm$ is uniform, thin, duralumin and simply supported; thickness h_0 of the PVDF film is $25mm$ for sensors and $250mm$ for actuators; Piezo-electric directional coefficients are $e_{31} = 25\text{x}10^6 c/N$ and $e_{32} = 2\text{x}10^6 c/N$ for both sensors and actuators.

Figure 2 depicts a simply supported panel of the test rig used for experiment; on the panel surface a pair of shaped smart actuators are bonded. As with the measurement of a dynamic mobility of the panel, a load effect resulting from the attachment of a sensor and a disturbance exciter must be eschewed; otherwise the experimental data obtained will be insignificant since the fundamental characteristics of such a thin and flexible panel are vulnerable to the measurement influence. For this purpose, a considerably light and tiny permanent magnet is installed at $\mathbf{r}_d = (135mm, 275mm)$, whereby the magnet is provided with a magnetic field via an electrical magnetic exciter, hence non-contact disturbance excitation. Regarding the measurement of a dynamical behavior of the panel response, a laser vibrometer is employed, measuring the velocity at $\mathbf{r}_m = (52mm, 54mm)$ of the panel, hence non-contact measurement.

Figure 3 shows the experimental result of mobility of the panel measured at $\mathbf{r}_m$ obtained by using the non-contact excitation and measurement method. According to Table 1, there should be 20 structural modes appearing in the figure; however there are some degenerate modes in the frequency range of interest, and hence the number of structural modes appearing in the figure is not necessarily the same with that in analysis.

In order to estimate the potential of the two-dimensional smart sensors for extracting the targeted (1, 3) structural mode, PVDF films with the thickness of , shaped with the basic function $\psi(y) = \sin \pi y / l_x$ are respectively bonded at $x_1 = l_x / 3$ and $x_2 = 2l_x / 3$ of the panel along the y direction

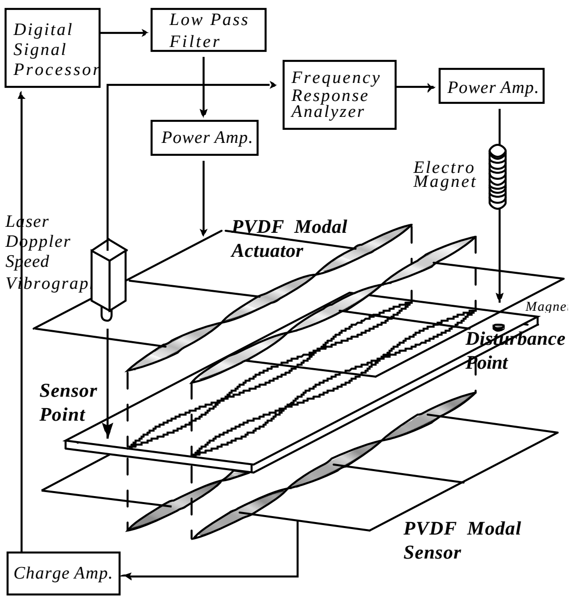

Fig. 2 Schematic diagram of experimental setup of modal control using both smart sensors and smart actuators

so as to avoid the excitation of the unwanted (3,3) mode. Regarding the load effect as a result of attaching PVDF films on the plate, it is negligible due to the weight and flexibility of a PVDF film. Each two-dimensional modal sensor consists of three segments with a half period of the sine function, these segments being connected by conductive foils. Polarity of the film sensor is changed by swapping the top and bottom surface of a sensor. Combining these two sensor outputs with the same polarity enables the sensors to filter out the (1,3) modal amplitude, while summing with the opposite polarity allows one to extract the (2,3) modal amplitude. Here it should be noted that the smart sensor output is the desired control signal *per se*, so that no further signal processing is needed.

Figure 4 illustrates the experimental result of the frequency spectrum obtained by summing up the two smart sensor outputs. It is evident that the two-dimensional smart sensors extract the targeted (1,3) mode out of 20 structural modes that are all excited as seen in the figure. In experiment, PVDF film sensors were shaped by hand using a utility cutter, and this handiwork seems to have caused some deterioration in modal filtering. In fact, this effect can be identified as spikes in the gain characteristics; however, the maximum amplitude of the targeted (1,3) mode is still large enough compared with those of the spikes. Improving the shaping technology of film sensors would lead to more accurate extraction of modal amplitudes. As a whole, the frequency characteristics are similar to a single degree of freedom vibratory system, hence modal filtering.

For the purpose of estimating the capability of smart modal actuation, a pair of PVDF laminae with the thickness of shaped with the same basic function as was used for modal filtering were glued on the back side of the panel at the same location of the smart sensors. The two-dimensional smart actuators were then driven simultaneously by a white noise so as to activate the targeted (1,3) structural mode. Figure 6 shows the frequency characteristics of the panel measured at . Clearly from the figure, even when the smart actuators are driven by

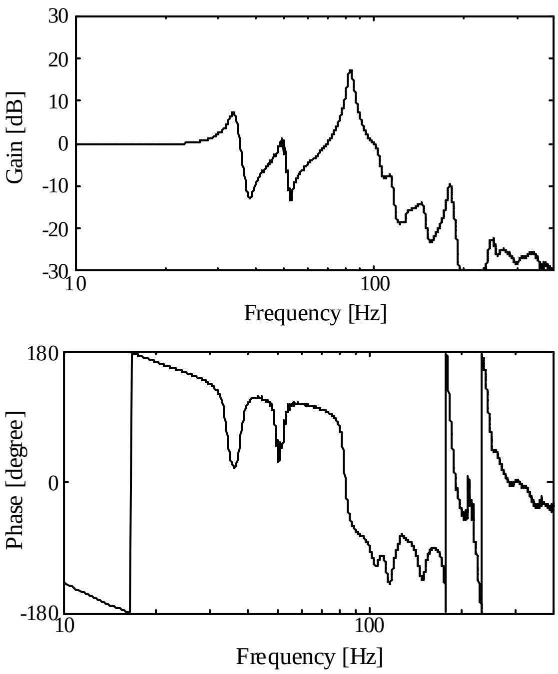

Fig. 3 Frequency characteristics of smart sensor outputs for extracting the (1,3) structural mode

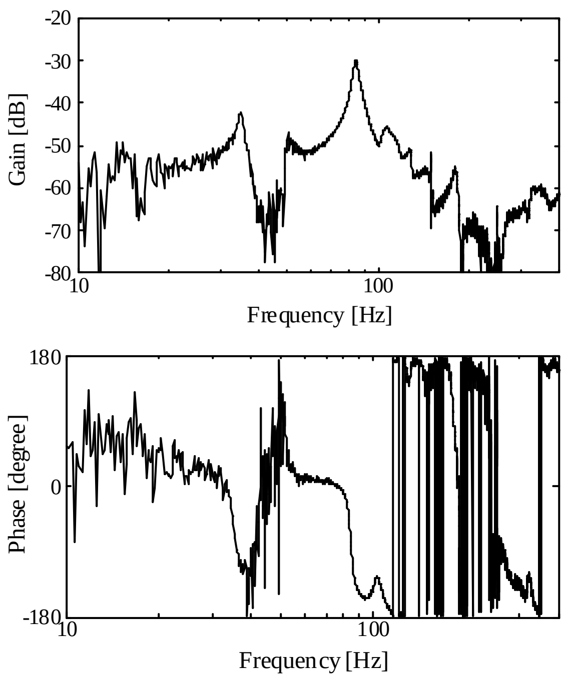

Fig. 4 Frequency characteristics of a panel when driven by the smart actuators

a random signal, they activate only the targeted (1,3) mode among 20 modes of the panel. Again, the precision of the shaping function applied to the smart actuators was not perfect, however the frequency characteristics in the figure shows a single degree of freedom vibratory system, hence modal actuation.

Figure 5 shows the experimental result of the mobility characteristics of the panel at before and after modal control. Clearly from the figure, the targeted (1, 3) mode peak is reduced by 20 dB, whereas the other modes remain intact. The velocity feedback gain was then applied by $59x10^{-3} V / pC$ to acquire the control effect. Because of the control system comprising both distributed smart sensors and distributed smart actuators, the observation spillover and control spillover leading to a destabilization of the control system was significantly alleviated. Throughout the experiment, it was possible to increase the velocity feedback gain to a considerably high level, although the excess amount of feedback gain caused instability of the system due to the saturation of a power amplifier.

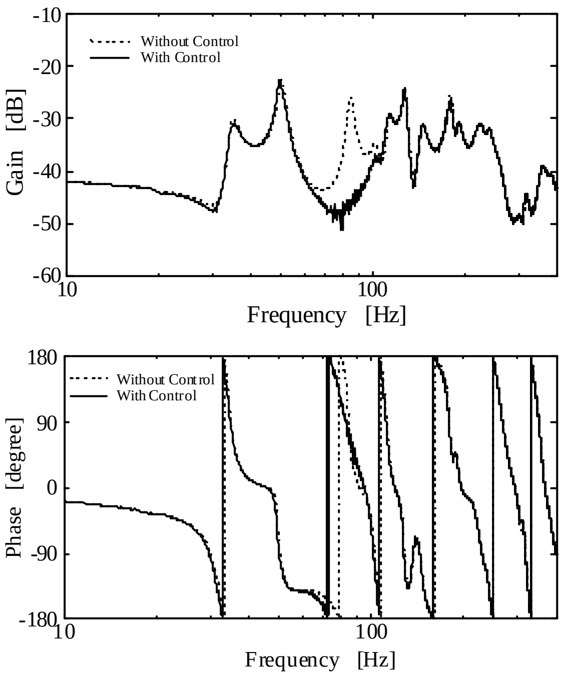

Fig. 5 Mobility characteristics of a panel before and after modal control of the (1,3) mod

CONCLUSIONS

In order to perform modal control, distributed smart sensors and actuators have been presented, the validity of the modal controllability being verified. First from a viewpoint of theoretical perspective, modal control of a distributed panel was discussed. Then, using both three-dimensional smart sensors and actuators, an ideal modal control system was investigated, the underlying properties being discussed. Taking into consideration the practicability, two-dimensional distributed smart sensors and actuators were then presented, the design procedures being presented. It was found that the two-dimensional smart sensor / actuator extracts / excites the targeted structural mode together with unwanted modes with the modal index in the perpendicular direction of the sensor / actuator placement. To cope with the problem, several smart sensors / actuators are needed, the design procedures being presented. Finally, an experiment was conducted using both PVDF shaped distributed sensors and actuators in an aim to conduct the modal control on the (1,3) structural mode of a panel, verifying the validity for suppressing the target structural mode without spillover.

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