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DESIGN OF LOW FREQUENCY SOUND SHIELD BASED ON RIGID BODY RESONANCE

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Abstract

In this paper, we present numerical investigations on the Mie scattering, and show the rigid body resonance of single sphere. It is shown that the bottom of the lowest complete gap and the low frequency transmission dip of one-layer slab correspond with the rigid body resonance. The attenuation efficiency of the finite phononic crystal slab depends on the interaction between the rigid body resonance and matrix. By varying the size and geometry of the structural unit, we can tune the resonant frequency, i.e., the dip frequency of the slab. Base on this idea, a stacked four-layer slab, each layer containing different scatterer, is introduced as an effective broadband sound barrier. The improvement of the sound insulation in comparison with the periodic layered plate is also verified.

INTRODUCTION

As an analogy to electromagnetic wave attenuated by "photonic band gap", sound attenuation for a certain frequency range can be achieved by phononic band gap, which comes from a strong periodic modulation in density and/or elastic coefficients. Recently, several articles try to bring the gap of the phononic crystal (PC) with a reasonable size to low frequency range, around 500Hz, as a more useful sound shield. The investigations focus mainly on the locally resonant sonic materials (LRSM). It is well known that the sound attenuation effect is predicted conventionally by the mass density law within this low frequency range. The improvement of the gap insulation in comparison with the mass law was demonstrated theoretically and experimentally [1,2]. It is known that the mechanism of the gap formation for two-component PC is mainly induced by the rigid body resonance (RBR) [3,4], but it dose not show the RBR explicitly and accordingly the relation of the gap of infinite system, transmission dip of finite slab and the RBR is somewhat unclear.

This paper tries to deal with these inexplicit questions. Then keep in mind that the mechanism for the attenuation of acoustic wave, we can improve the attenuation of the PC slab with finite thickness under the condition of the required rigidity for practical applications.

THE MODEL AND THE MULTIPLE-SCATTERING THEORY

We consider a simple cubic lattice of nonoverlapping spheres of radius *R*, arranged periodically in a continuous matrix. Figure 1 shows the model of the three-dimensional PC considered in this paper under the Cartesian coordinates system. The spheres of a plane (xy plane) are arranged infinitely on a two-dimensional lattice defined by the primitive vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ [see Figure 1(a) and Figure 1(c) shows the corresponding first Brillouin zone]. The finite crystal slab is viewed as a sequence of planes of spheres perpendicular to the z axis [see Figure 1(b)]. For simplicity, the slab extends infinitely on both sides in order to avoid all the interfacial phenomena that could alter the results.

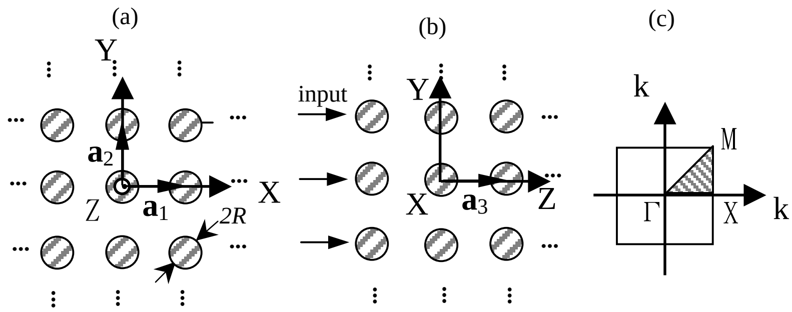

Figure 1 –The primitive vectors $\mathbf{a}_1$*,* $\mathbf{a}_2$ *and* $\mathbf{a}_3$ *in three dimensional lattice. (a) and (b) denode the lattice in xy-plane and out of xy-plane respectively. (c) The first unreduced Brillouin zone of the array in xy-plane.*

The displacement in a homogeneous elastic medium represents the following time-independent equation

$$(\lambda + 2\mu)\nabla(\nabla \cdot \mathbf{u}) - \mu\nabla \times \nabla \times \mathbf{u} + \rho\omega^2\mathbf{u} = 0\,. \qquad (1)$$

In a spherical system, the solution can be decomposed into a longitudinal and two transverse solutions, that is

$$\mathbf{u} = \mathbf{L} + \mathbf{M} + \mathbf{N}\,. \qquad (2)$$

The spherical-wave solutions of the wave equation can be written in generally form

$$\mathbf{u}(\mathbf{r}) = \sum_{lm\sigma}\left[a_{lm\sigma}\mathbf{J}_{lm\sigma}(\mathbf{r}) + b_{lm\sigma}\mathbf{H}_{lm\sigma}(\mathbf{r})\right], \qquad (3)$$

where $\sigma = 1, 2, 3$ correspond to the $\mathbf{L}, \mathbf{M}, \mathbf{N}$ modes respectively. The definitions of $\mathbf{J}_{lm\sigma}(\mathbf{r})$ and $\mathbf{H}_{lm\sigma}(\mathbf{r})$ can be found in Ref. [5,6]; $b_{lm\sigma}, a_{lm\sigma}$ represent the expansion coefficients of scattered and input waves. The relation between the coefficients $B = \{b_{lm\sigma}\}$ and $A = \{a_{lm\sigma}\}$ can be acquired by the solution of the elastic Mie scattering, i. e.,

$$B = TA. \tag{4}$$

The scattering matrix T of an incident longitudinal wave presents the following form [5],

$$T = \begin{bmatrix} LL & 0 & LN \\ 0 & MM & 0 \\ NL & 0 & NN \end{bmatrix} \tag{5}$$

The meanings of the five nonzero elements are pellucid, for example, LL stands for the conversion from L to L mode during the scattering procedure. It shows that the L mode and the N mode are coupled to each other, while the M mode is decoupled.

Table I. Physical parameters of the materials

Material	ρ (Kgm^{-3})	c_l (ms^{-1})	c_t (ms^{-1})
steel	7890	5780	3220
Pb	11600	2493	1133
W	19300	5090	2800
Epoxy	1180	2490	1180
Silicon rubber	1300	22.8	5.5

CALCULATED RESULTS AND DISCUSSION

To begin with, we consider the single steel sphere (R_1 = 5.25mm), coated with silicon rubber (R_2 = 6.75mm), immersed in infinite epoxy. When a plane wave is incident on the sphere, it is scattered by it, so the wavefield outside the sphere consists of the incident wave and a scattered wave. Figure 2 shows the absolute values of the T-matrix elements as a function of frequency, defined by Eq.(5) in the previous section. For the frequencies near the first complete band gap [see figure 3(a), which represents the band structure of the coated steel spheres arranged in epoxy with simple cubic lattice], we find that only the first order spherical-wave expansion (l =1) for T-matrix has significant amplitude. The T-matrix elements are showed in Figure 2(a). Figure 2(b) shows the corresponding elements for l = 2. One can readily see that the resonance amplitude is almost two orders lower than that of l =1and is out the frequency range of the first gap of the corresponding PC. From figure 2(a) we can readily see that the strong LL, NL, LN and NN resonance exists around 504 Hz and 1591 Hz, as showed by the peaks of the *T* elements, which gives rise to the relatively flat bands at the bottom of the first and second gaps. Here we pay more attention to the low frequency resonance for *l* =1, known as RBR [7]. It is generally known that the coated sphere resonates as a whole at this frequency. The physical insight of the RBR can be explained by a mass-spring model, i.e., the scatterer offers the mass and the coating offers the spring. Away from these resonances, the almost linear dispersion lines denote that the three-component system behaves like an effective medium, the slopes of the dispersion lines correspond to the speeds of the elastic wave for each polarization. When the frequency reaches the resonance, the coupling between the linear dispersion and the resonance of the individual spheres opens a gap, which accounts for the first gap existed between 504Hz and 802Hz in figure 3(a). So the mechanism of the lowest gap formation emerges as the interaction of the RBR and the effective medium. Figure 3(b) shows the transmittance of an incident longitudinal wave crossing one-layer slab with steel spheres coated silicon rubber immersed in epoxy. We can see that the first dip is induced by the RBR (and the second dip is induced by the high-frequency resonance of *l* =1). In viewpoint of wave, part of elastic wave travels with resonant mode accompanying with the 180° phase jump, and the rest propagates in a nonresonant way across the structure. As a consequence, interferences between both wave components may occur, which results in the transmission dip for finite slab and the lowest complete gap for infinite PC.

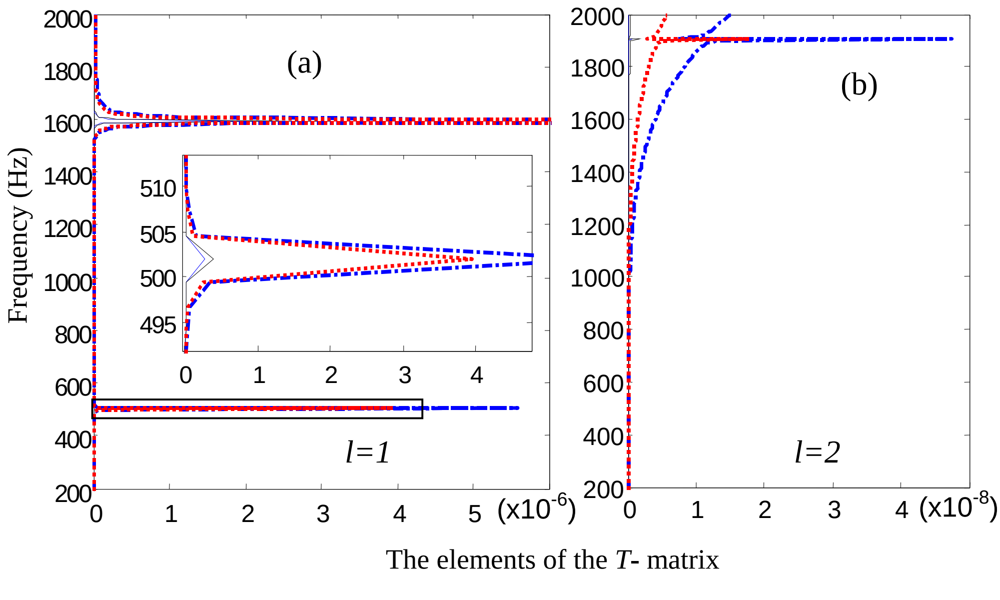

Figure 2 – The elements of the Mie scattering matrix for steel spheres (R_1 = 5.25mm) coated with soft rubber (R_2 = 6.75mm) embedded in epoxy as a function of the frequency. Here (a) and (b) represent the results for l=1 and l=2 respectively. The inset in (a) presents the zoom area of the first resonance. L mode to L (N) mode is presented by dashed line with blue colour (solid line with black colour), and N mode to N (L) mode is presented by blue thick dash-dot (red dot) line.

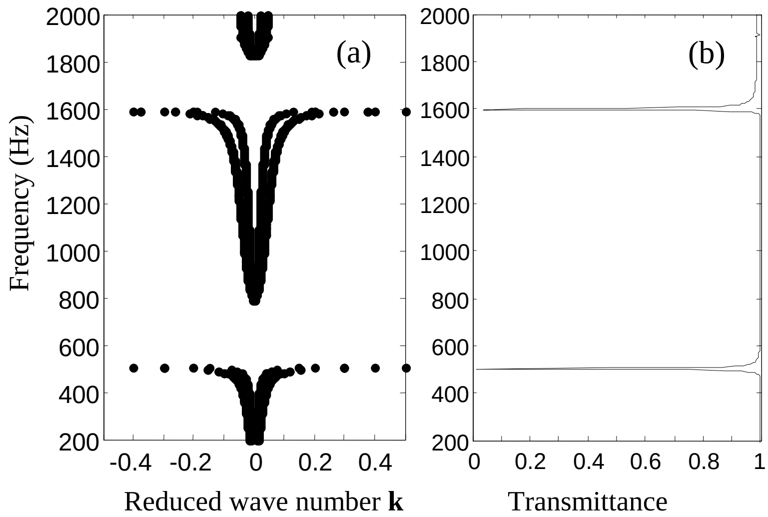

Figure 3 –(a) Band structure of simple cubic lattice of steel spheres (R_1 = 5.25mm) coated by soft rubber (R_2 = 6.75mm) embedded in epoxy. (b) The transmittance of an incident longitudinal wave crossing one-layer slab with steel spheres coated silicon rubber immersed in epoxy in a square array. The lattice constant a is 15mm.

Now we replace the rigid matrix with the soft silicon rubber, i.e., forming the two-component PC with the same lattice as the PC discussed above. Great changes appear in the resonance of the *T*-matrix. For example, the sharp peak, accordingly the flat band, disappears [see figure 4 and figure 5(a)]. But we can see that the first complete gap is induced by the RBR too. As a verification, the peaks of the *NL* and *NN* resonance for *l* = 1 in figure 4(a) correspond to the gap bottom and the transmittance dip of the one-layer slab in figure 5(b).

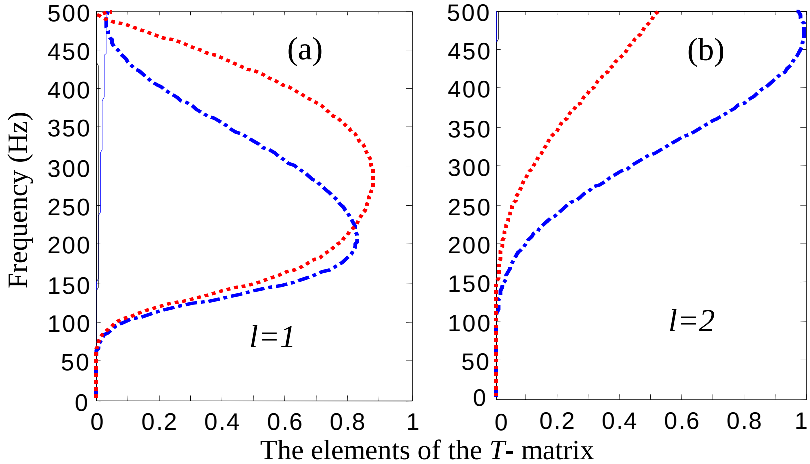

Figure 4 –The elements of the Mie scattering matrix for steel sphere in silicon rubber. The line styles used here possess the same attribute as that in Figure 2.

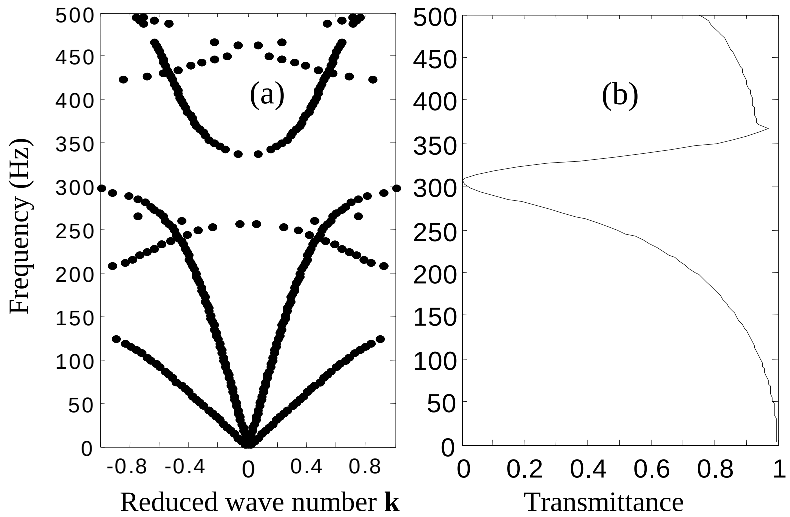

Figure 5 –(a) Band structure of steel spheres embedded in silicon rubber. (b) The transmittance of the one layer slab. The lattice used here as the same as that in figure 3.

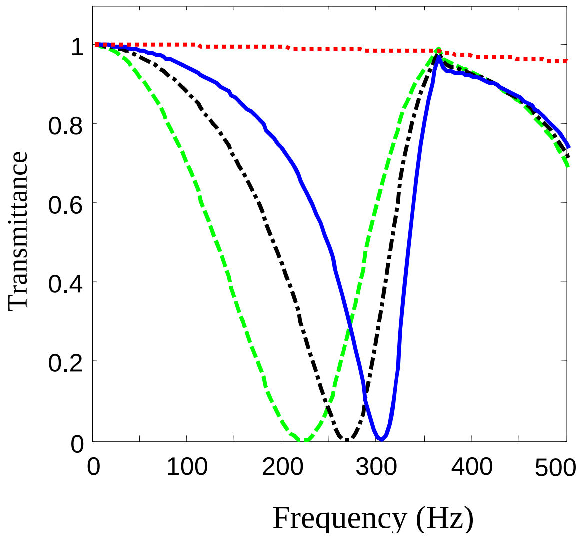

Figure 6 –Transmittance of longitudinal elastic wave incident normally on a single layer of W, Pb, steel and D spheres (R = 6.75mm) with a square lattice in silicon rubber, indicated by dashed (green), dash-dot (black) solid (blue) and dot (red) lines respectively.

As verification, we investigate the transmittance of a plane elastic longitudinal wave incident normally on one-layer slabs made of various scatterers embedded in silicon rubber. What can be seen from figure 6 is that the dips exist at 220Hz, 270Hz and 306Hz, representing W, Pb and steel spheres respectively. The dip is dependent on the mass of the scatterer, i.e., heavier scatterer inducing lower-frequency dip. We believe that the observed dip in the transmission spectra is induced by the RBR of the sphere. The mechanism can also be explained by a mass-spring model as the case of the LRSM, i.e., the scatterer offers the mass and the matrix offers spring. But here no simple formula can be given for more interaction among the spheres. As the density of the scatterer increases, the stronger interaction between the spheres and the matrix, which, in turn, leads to wider dips in the transmission spectra (see figure 6). As another verification, the transmittance of the slab composed of one-layer spheres D (got by reducing the density of steel sphere as the same density as the matrix but remaining other parameters artificially) cannot present any dip, shown as thick dot line (red color). The reason is that the RBR disappears in the present case. Note here that the large impedance mismatch induced by the velocity cannot induce dip in the transmittance, accordingly the band structure of the infinite system shows no complete gap (the band structure is omitted for paper length). So the first complete gap and the corresponding attenuation are mainly induced by the RBR because of the density of the scatterer.

With a detail comparison between the figure 4(a) and figure 2(a), one can see that the amplitudes of the *T* elements of the bi-component are much lager than that of the tri-component system. The peaks of *NL* and *NN* resonance exceed 0.8, compared with the corresponding peaks of tri-component in figure 2(a), justly about 6×10^{-6}, which induce the good attenuation efficiency in bi-component PC.

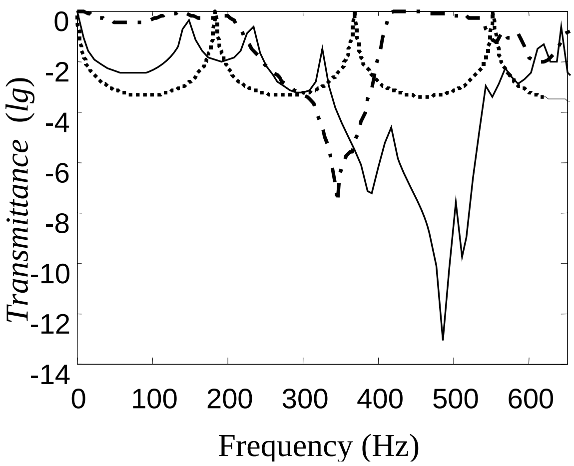

Figure 7 –Dash-dot line, the layers in turn include W ball, steel ball and hollow steel ball with inner radius 1.5mm and 4.5mm respectively, and all the scatterers with outer radius 6.75mm, the overall thickness 60mm; solid line, the array maintained but each layer is separated by rigid epoxy plate with thick 1mm, so the overall thickness is 65mm; dot line, the epoxy/silicon rubber layered structure, with the thickness 1mm and 15mm.

The discussion above demonstrates that the relative intense interaction between the sphere and the matrix is a key factor to develop the good attenuation in the finite slab. And we can tune the attenuation frequency by adjusting the frequency of the RBR. The soft two-component PC can produce good attenuation, but the whole structure might be so soft that can’t be used in technological applications. Can we improve the rigidity of the two-component PC, but do not reduce (or reduce little) the attenuation? Firstly, we stack four layers with different scatterer, i.e., with different resonant frequency. A broadband sound attenuation ranges from 210 Hz to 410 Hz (with centre frequency 346Hz), is showed in figure 7 by dash-dot line. In order to give the detail of the attenuation, the transmittance shows in denary logarithm form. Note that the respective dip of the various scatterer vanishes, which implies that the inter-layer effects make the structure plays a whole role. Then, we separate different layer by rigid plate as a simple support. Obviously, the inter-layer effects between various scatterer is weakened. One can see that there are three obvious dips in obvious attenuation domain (the solid steel sphere behaves as almost the same resonant frequency as the hollow steel sphere with inner radius 1.5mm, so the central dip around 480 Hz is more obvious), as showed in figure 7 by solid line, which denotes various resonant frequency associated with different scatterer. The stacked slab displays a well attenuation range over 330–540 Hz, which is little higher than that of the system without epoxy reinforced. The reason is that the epoxy plates enhance the rigid of the effective medium, so the resonant frequency gets high. If we look the tri-component PC as another type of support, the resonant frequency will get higher too. The reader may doubt that the attenuation may be induced by silicon/epoxy interfacial phenomena. Here, we give the transmittance of four-layer slab stacked by epoxy and silicon rubber plate alternately, with thickness 1mm and 15mm respectively. This system presents gap attenuation with small width exists around 720Hz, which is out of our consideration. The dot line shows the Fabry-Perot-type oscillations induced by the finite layer slab.

CONCLUSIONS

In summary, we have shown that the first complete band gap is induced by the RBR in two- and tri-component PC. The physical reason is, part of an elastic wave traveling with resonant mode and the rest propagating in a nonresonant way across the structure. As a consequence, interferences between both wave components may occur, which induces the dip in the finite PC slab. The reason for better attenuation in two-component PC is answered by the Mie scattering of the single scatterer. Finally, A broadband sound shield based on layers of RBRs was developed. These findings open up opportunities for the structural optimization for low-frequency sound attenuation.

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