ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

VIBRATION ISOLATION STRUCTURE WITH THE IDEA OF PHONONIC CRYSTALS

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Abstract
A periodic binary straight beam with different cross sections is constructed and studied. The band structure and the transmission frequency response function of flexural waves in the structure are calculated with the plane-wave expansion method and the finite element method. The theoretical results are validated with vibration experiments and the results match mainly. Based on the feature of flexural wave band gaps of beam with periodic structure, a vibration isolation structure is designed. The transmission frequency response function of the vibration isolator is calculated with finite element method, where a band gap with attenuation of about 20dB exists. Vibration experiments are also conducted and the vibration attenuation of the vibration isolator is validated.

INTRODUCTION

The propagation of elastic wave in periodic composite called phononic crystals (PCs) has received a great deal of attentions[1-15]. Particular interests are focused on the existence of the so-called phononic band gaps (PBG) where elastic waves are all forbidden. The study on PBG materials and structures is driven partly by potential applications such as elastic wave filters, vibrationless environments for high-precision systems, transducer improvements, as well as pure physical concerns with the Anderson localization of sound and vibration. To our knowledge, a few works about the applications of PCs has been published[13-15], but none of them has been used directly in the applications of vibration isolation or attenuation.

In this paper, we present a structure with the idea of PCs that can be used to isolate vibrations. First, we construct a periodic binary straight beam with different cross sections. The band structures of flexural waves in the structure are calculated with the plane-wave expansion (PWE) method and the transmission frequency response function (FRF) of a finite sample of it is calculated with the finite element method. Both the theoretical results are validated with the vibration experiment, and the results match mainly. Second, a vibration isolator is designed based on the feature of flexural wave band gaps of beam with periodic structure. The transmission FRF of the isolator is calculated with finite element method, which shows the existence of the band gaps and an attenuation of about 20dB in the band gaps. Finally, the vibration attenuation of the vibration isolator is validated by a vibration experiment.

THEORY

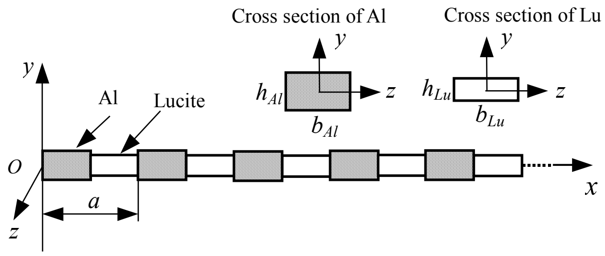


Figure 1 – Periodic binary straight beam with different section for each segment

Figure 1 illustrates a periodic binary straight beam with different cross sections. Aluminum and Lucite segments with different sizes of cross sections are arrayed along x-dimension periodically. The lattice constant is a, and the cross section of the beam is a $b_{Al}\times h_{Al}$ and $b_{Lu}\times h_{Lu}$ rectangles respectively. We consider the flexural wave in x-y plane.

If the thickness h in y-direction and width b in z-direction in each segment of the beam are much smaller than the lattice constant a (often a≥5b and a≥5h), each unit cell can be regarded as Euler-Bernoulli beam, where both the shearing deformation and rotational inertia of the cross sections are negligible. The flexural elastic wave equation along x-direction is [16]

$$\frac{\partial^2}{\partial x^2}\left[E(x)I(x)\frac{\partial^2 y(x,t)}{\partial x^2}\right]+\rho(x)A(x)\frac{\partial^2 y(x,t)}{\partial t^2}=0 \tag{1}$$

where $\rho(x)$ is the mass density, $E(x)$ is the Young's module , $y(x, t)$ is the displacement in y-direction, $I(x)$ is the moment inertia, and $A(x)$ is the cross section area. The I and A are calculated with the size and shape of cross section which are independent with the materials. For example, if the cross section is a $b\times h$ rectangle, they can be calculated with $I=hb^3/12$ and $A=bh$.

For a periodic structure, Bloch's theorem [17] asserts that $y(x, t)$ in Eq. (1) can be written as

$$y(x,t)=\sum_{G_0} e^{i(kx-\omega t)} y_k(x) \tag{2}$$

where k is restricted within the first Brillion zone and $y_k(x)$ is a function with the same period as $1/\rho(x)$, $E(x)$, $A(x)$ and $I(x)$ which can be expanded in Fourier series

$$f(x)=\sum_{G} f(G)e^{iGx} \tag{3}$$

where $f(x)$ stands for $1/\rho(x)$, $E(x)$, $A(x)$ and $I(x)$ respectively, G is the reciprocal lattice vector.

Substituting Eqs. (2) and (3) into Eq. (1), we obtain

$$\omega^2\sum_{G_0}\rho(\frac{G'-G_0}{2})A(\frac{G'-G_0}{2})y_k(G_0)=\sum_{G_0}(G_0+k)^2(G'+k)^2E(\frac{G'-G_0}{2})I(\frac{G'-G_0}{2})y_k(G_0) \tag{4}$$

where $G'=G_0-G$.

Equation (4) is an infinite set of linear equations. In practice, only a finite number of vectors G' and G_0 (plane waves) can be employed in the calculation. We employ 441 plane waves in this paper and the convergence is satisfied. The band structure represents all the stable flexural wave modes propagating in the infinite periodic structure of binary straight beam with different cross sections.

THEORETICAL AND EXPERIMENTAL RESULTS

Using the PWE in this paper, we calculate the flexural wave band structure of the beam with infinite periods illustrated in Figure 1 The structural parameters employed in the calculations are a=0.07m, filling fraction f=2.5(where $f=l_{Al}/l_{Lu}$, l_{Lu} and l_{Al} are the lengths of Al and Lucite segments), $b_{Al}=b_{Lu}=h_{Al}$=0.01m, h_{Lu}=0.005m. The material parameters used are ρ_{Al}=2799 kg·m^{-3}, $E_{Al}=7.2\times10^{10}$ Pa for Aluminium; ρ_{Lu}=1142 kg·m^{-3}, $E_{Lu}=2.01\times10^{9}$ Pa for Lucite. Figure 3(a) shows the calculated flexural wave band structures. With the FEM, we calculated the transmission frequency response function of a finite sample of the beam with 6 and 10 periods, which are showed in Figure 3(b) respectively.

In order to verify the results calculated with the PWE and the FEM, a vibration experiment is performed. The experimental system is shown in Figure 2. Here, a white noise signal with bandwidth from 0 to 3.2kHz is input into the vibration shaker, which transmits vibrations to the left end of the beam through the force transducer. Then the flexural waves propagate through the beam. The acceleration at the right end of the beam is measured with an accelerometer. The measured results are shown in Figure 3 (b) comparing with that calculated with the FEM.

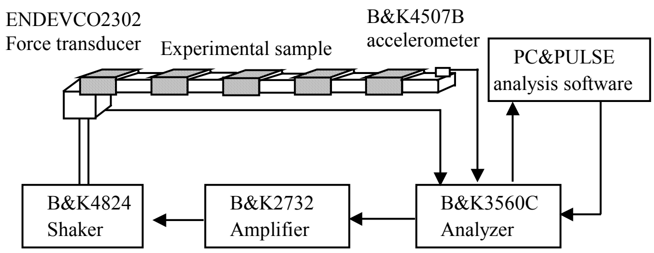


Figure 2 – Experimental setup and experimental sample

In Figure 3(a), the first gap in the dispersion curves [solid lines in Figure 3(a)] of the flexural wave locates between 501 Hz and 1430 Hz. The frequency range of the large attenuation in the calculated transmission FRF curves [dotted line and dash doted line] is from 480 Hz to 1350 Hz. The frequency range of large attenuation in the measured transmission FRF curves [solid line and dashed line] is from 430 Hz to 1250 Hz. All the theoretical and experimental results match in the main. The measured and calculated results also show that the attenuation in the band gap is in the direct ratio with the periodic number.

The influence of the rotational inertia of the cross section and the shear deformation is involved in the FEM and experiment results, while it is not considered in the PWE method. Because both the shear deformation and the rotational inertia tend to lower the value of the frequencies, the start and stop frequencies of sharp drops are slightly smaller than those of the flexural wave band gaps.

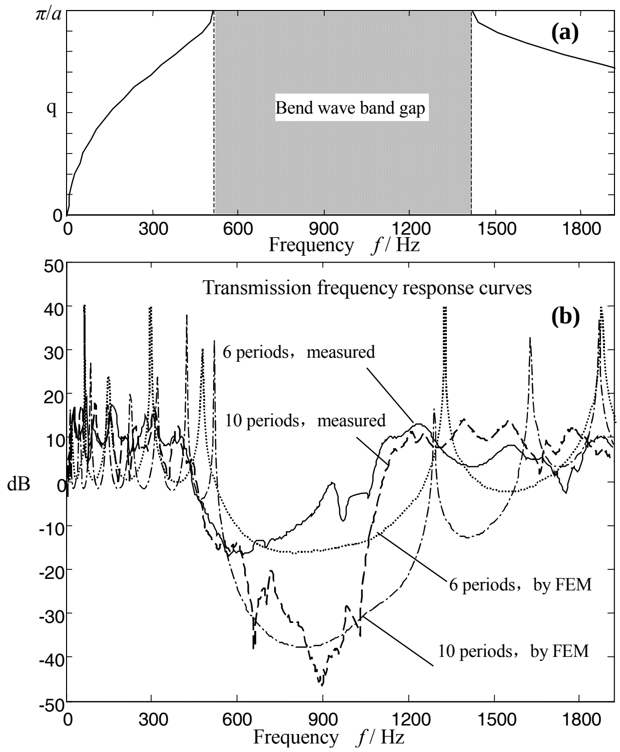


Figure 3 (a) – Calculated band structure of the periodic binary straight beam with different cross sections illustrated in Figure 1. (b) Calculated and measured transmission FRF of finite samples of the periodic beam. The solid line and dashed line represent the measured results corresponding to a sample of 6 periods and 10 periods respectively. The dotted line and dash dotted line represent the calculated results corresponding to samples of 6 and 10 periods respectively

APPLICATION TO A VIBRATION ISOLATION STRUCTURE

A vibration isolation structure illustrated in Figure 4 is designed based on the band gap feature of flexural waves on beam with periodic structure. Aluminium and Lucite bars with different rectangular cross sections are jointed together and arrayed alternatively in a framework of eight layers. Four Al blocks located at the four angles or the mid of four sides of each layer acts as shoring to connect each layer one by one. To strengthen the structure, a Lucite cross is applied in each layer. When vibration occurs, it is transferred to the bottom layer through the shoring located at the angles of the bottom frame and converted into the flexural wave in it. Whereafter, the vertical vibrations on the mid of the sides of the first frame induced by the flexural wave on it is transferred to the second layer. At last, the vibration propagates through the whole structure in mode of flexural wave and transferred to the bearing plate. The lattice constant a is about 0.07m, and the filling fraction f is about 2.5(for the existence of the shoring, we can't get the exact values).

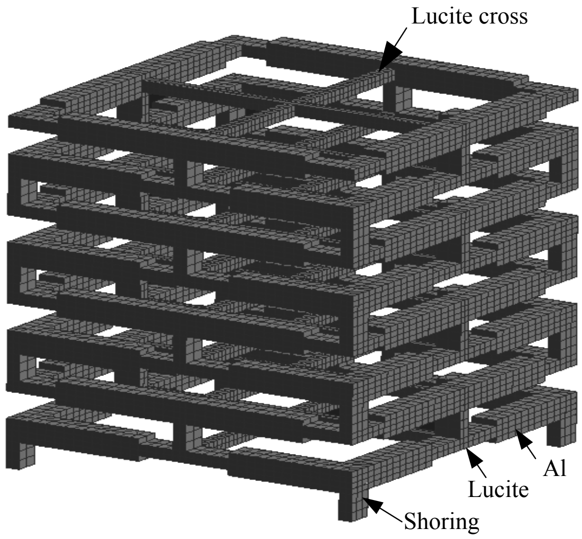
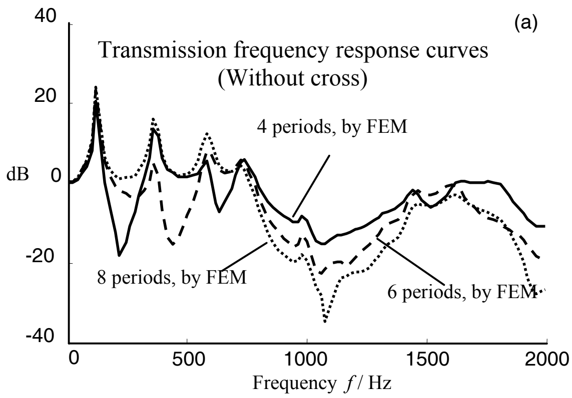


Figure 4 –The finite element model of a vibration isolation structure composed of periodical binary beams

The transmission FRFs of the vibration isolation structures are calculated with the FEM and illustrated in Figure 5. For the result of the isolator without Lucite cross illustrated in Figure 5(a), the frequency range of the large attenuation is from 720 Hz to 1450 Hz which is also known as the band gap. For the isolator with Lucite cross, the calculated result illustrated in Figure 5(b) shows that the band gap is between 750Hz and 1400Hz. For both cases, the attenuations in the band gaps are all in direct ratio with the period number. Two results match well, which indicts that the Lucite cross introduced to the vibration isolation structure has little influence on the transmission FRF. The calculated upper frequency of the band gap of the vibration isolator and the calculated and measured result of the periodic beam matches in the main. Although the vibration isolation structure is composed of same units as in the periodic beam, they are different structures with different boundary conditions. Therefore, the lower frequency of the band gap of the vibration isolation structure is a little higher than that calculated with the periodic beam. However, the difference is small enough for us to design the vibration isolator using the band gap results of periodic beam glancingly.



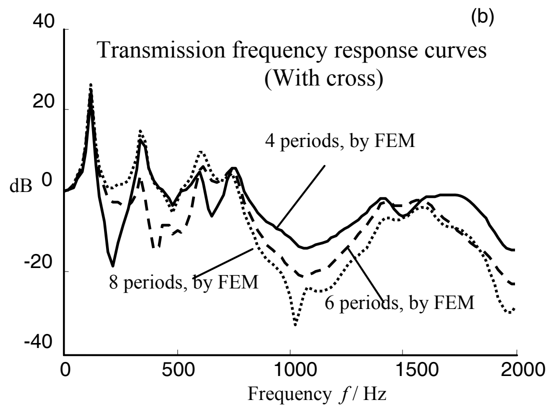


Figure 5 (a) – The transmission FRF curves of the vibration isolation structure without the Lucite cross. (b) The transmission FRF curves of the vibration isolation structure with the Lucite cross. The solid line, dashed line and dotted line represent the calculated results corresponding 4 periods, 6 periods and 8 periods.

EXPERIMENTAL MODEL AND RESULTS

As the Lucite and the Aluminium is difficult to joint together in order to construct the vibration isolator illustrated in Figure 4, we employ another isolator with similar structure shown in Figure 6 in the vibration experiment. In the new structure, Aluminium is replaced with a sandwich structure consists of Lucite and Copper. Our intent is just to validate the vibration attenuation ability of such structures instead of the exact theoretical results that calculated before with this different structure.

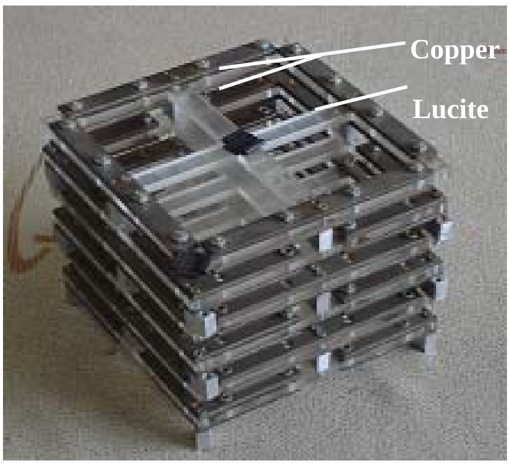


Figure 6 The experimental model of the vibration isolation structure

The frequency range of large attenuation in the measured acceleration FRF curves of this structure shown in Figure 7 is from 280 Hz to 1200 Hz. The measured results also show that the attenuations in the band gap are in direct ratio with the period number. The existence of band gap of the structure verifies that the vibration isolation structure can be designed with the idea of the phononic crystals.

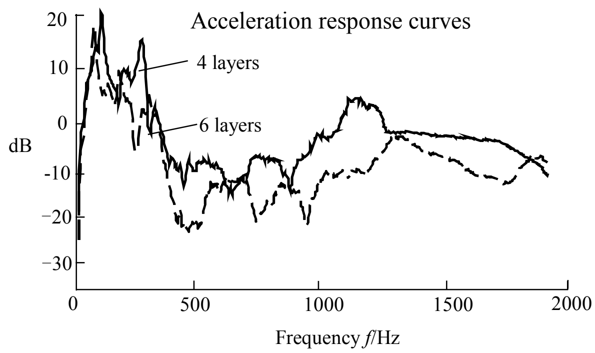


Figure 7 – Measured acceleration FRF of the vibration isolation structure. The solid line and dashed line represent the measured results corresponding 4 periods and 6 periods

CONCLUSIONS

In conclusion, both the theoretical and the experimental studies show the existence of flexural wave band gap in the periodic binary straight beam with different cross sections, and the calculated and the measured results match mainly. Based on these results, a vibration isolation structure is designed. A band gap with attenuation of about 20dB is found with both the finite element method and the vibration experiments, which validates the vibration attenuation capability of the vibration isolator based on the idea of phononic crystals.

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