Using Dither Signals for Recovering Periodic Motion of Chaotic Automotive Wiper System

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In this paper, we will verify chaotic motion first and go on to the chaotic control in an automotive wiper system, which consists of two blades driven by a DC motor via one link. The complex nonlinear behaviors may be observed more thoroughly over a range of parameter values by the bifurcation diagram. Next, by using the estimation of the largest Lyapunov exponent, the periodic and chaotic motions can be identified. Finally, a dither signal will be suggested to control a chaotic automotive wiper system. We also present some simulation results to demonstrate the feasibility of the proposed method.

INTRODUCTION

Many vibrations that may be harmful to the driver are often observed by running a wiper on an automotive windshield wiper system. To find an effective way to control vibrations, we attempt to study the dynamic behaviors of the wiper system. Several studies have been carried out to investigate the chatter vibrations in an automotive wiper system (Oya et al.; Suzuki and Yasuda). By applying various numerical analyses results, such as bifurcation diagram, phase portraits, Poincare map, frequency spectra and Lyapunov exponents are presented to observe periodic and chaotic motions. For a broad range of parameters, the Lyapunov exponent is the most effective method to measure the sensitivity to initial conditions of the dynamical system. The algorithms for computing Lyapunov exponents of smooth dynamical systems are well developed (Wolf et al.). Nevertheless, there are non-smooth dynamical systems with discontinuities where this algorithm cannot be directly applied. However, the methods of the calculation of Lyapunov exponents for non-smooth dynamical systems have been proposed only in several papers (Muler; Stefanski). The estimated method of the largest Lyapunov exponent for wiper system proposed by Stefanski is used in this paper. Recently, the control of chaotic stick-slip mechanical system is making great progress and several techniques have been proposed in references (Galvanetto; Feeny and Moon). Feeny and Moon have applied high-frequency excitation, or dither, to quench stick-slip chaos. This technique is also widely used in practice in many nonlinear systems (Tung and Chen; Fuh and Tung). In this paper, we demonstrate that the control of chaos can be realized by injecting another external input, called a dither signal, into the system.

MODEL DESCRIPTION

A front windshield wiper system has two blades which attached to the windshield at the driver's side and the passenger's side. Each blade is supported by an arm, which moves to and fro around the pivot. This motion is given by the rotation of a DC motor via a pantographic link. The schematic diagram of automotive wiper system is shown in Fig. 1. In this figure, the symbols with subscripts *D* and *P* are referred to as driver's and passenger's side, respectively. The lines with notations L_i represent the positions which the wiper arms take when no deflections occur. The symbols θ_i (i=D, P) are the angular deflections with respect to the position L_i while the notations $\dot{\psi}_i$ are the angular velocity of the arms. The symbol l_i represents the length of the wiper arm and $\dot{z}_i$ represents relative velocities of the blades with respect to L_i at the position of the top of the wiper arms. Then,

$$\dot{z}_i = (\dot{\theta}_i + \dot{\psi}_i) l_i \qquad (i = D,\ P) \tag{1}$$

In accordance with Newton's second law, the governing equations for a wiper on the i's (i = D, P) side can be expressed as follows (Suzuki and Yasuda, 1998):

when $\dot{z}_i \neq 0$, $I_i \ddot{\theta}_i = -R_i - D_i - M_i(\dot{z}_i)$,

when $\dot{z}_i = 0$, $|R_i| \geq N_i l_i \mu_0$, $I_i \ddot{\theta}_i = -R_i - D_i$,

when $\dot{z}_i = 0$, $|R_i| < N_i l_i \mu_0$, $I_i \ddot{\theta}_i = 0$ $(\dot{\theta}_i = -\dot{\psi}_i)$, (2)

where the symbols I_i are the moments of inertia and M_i are the moments induced by the friction force between the wiper blades and the windshield.

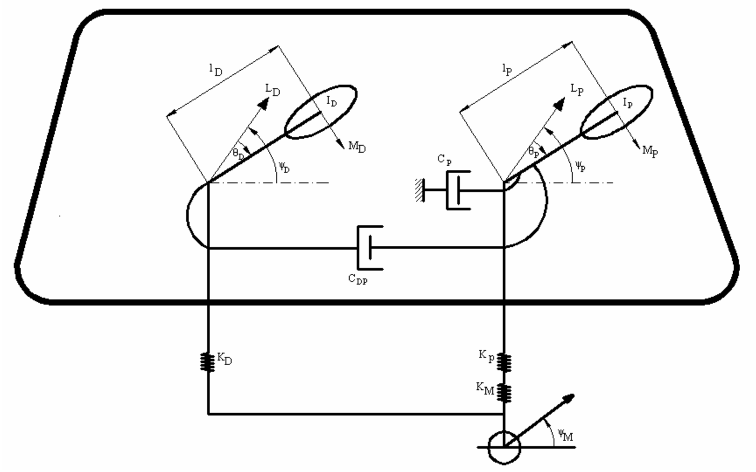

Figure 1 The analyzed automotive windshield wiper system

R_i and D_i are the moments produced by the restoring force and the damping force, respectively. That is given as follows:

$$R_D = k_D\theta_D - k_{DP}\theta_P\,,\ \ R_P = k_P\theta_P - k_{PD}\theta_D\,, \tag{3}$$

$$D_D = c_D\dot{\theta}_D - c_{DP}\dot{\theta}_P\,,\ \ D_P = c_P\dot{\theta}_P - c_{PD}\dot{\theta}_D\,, \tag{4}$$

where,

$$k_D = K_D(K_P + K_M)/(K_D + K_P + K_M)\,,\ \ k_{DP} = k_{PD} = K_D K_P/(K_D + K_P + K_M)\,,$$

$$k_P = K_P(K_D + K_M)/(K_D + K_P + K_M)\,,\ \ c_D = C_{DP}\,,\ \ c_P = C_{DP} + C_P\,,$$

$$c_{DP} = c_{PD} = C_{DP}\,.$$

The moments M_i can be written as:

$$M_i(\dot{z}_i) = N_i l_i \mu(\dot{z}_i)\,, \tag{5}$$

where N_i is the normal force. The coefficient of friction, μ, which can be expressed using the following relationship proposed by Suzuki and Yasuda:

$$\mu(\dot{z}_i) = \mu_0\,\mathrm{sgn}(\dot{z}_i) + \mu_1\dot{z}_i + \mu_2\dot{z}_i^{\,3} \qquad (i = D,\ P)\,. \tag{6}$$

Let $x_1 = \theta_D$, $x_2 = \dot{\theta}_D$, $x_3 = \theta_P$ and $x_4 = \dot{\theta}_P$ be the state variables, the state equations of the wiper system (Eq. (2)) on the driver's side can be written as follows:

when $\dot{z}_D \neq 0$,

$$\dot{x}_1 = x_2\,,$$

$$\dot{x}_2 = (-R_D - D_D - M_D(\dot{z}_D))/I_D\,,$$

when $\dot{z}_D = 0,\ |R_D| \geq N_D l_D \mu_0$,

$$\dot{x}_1 = x_2\,,$$

$$\dot{x}_2 = (-R_D - D_D)/I_D\,,$$

when $\dot{z}_D = 0,\ |R_D| < N_D l_D \mu_0$,

$$x_2 = -\dot{\psi}_D,$$
$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = 0. \tag{7a}$$

The state equations of the wiper system (Eq. (2)) on the passenger's side can be written as follows:

when $\dot{z}_P \neq 0$,

$$\dot{x}_3 = x_4,$$
$$\dot{x}_4 = (-R_P - D_P - M_P(\dot{z}_P))/I_P,$$

when $\dot{z}_P = 0,\ |R_P| \geq N_P l_P \mu_0$,

$$\dot{x}_3 = x_4,$$
$$\dot{x}_4 = (-R_P - D_P)/I_P,$$

when $\dot{z}_P = 0,\ |R_P| < N_P l_P \mu_0$,

$$x_4 = -\dot{\psi}_P,$$
$$\dot{x}_3 = x_4,$$
$$\dot{x}_4 = 0. \tag{7b}$$

The values of the parameters of the above equations are listed in Table 1.

CHAOTIC MOTION AND LYAPUNOV EXPONENT

To clearly understand the dynamics of this system, we carry out a series of numerical simulations from Eqs. (7). To see these behaviors in detail are presented in Chang and Lin. The largest Lyapunov exponent is one of the most useful diagnostics for chaotic system. Algorithms for computing the Lyapunov spectrum of "smooth" dynamical systems are well established (Wolf et al.; Benettin et al.). But there are "non-smooth" dynamical systems with discontinuities where this algorithm cannot be directly applied. Recently, Stefanski has suggested a simple and effective method to estimate the largest Lyapunov exponent, which utilizes the properties of synchronization phenomenon. This method can be explained briefly: the dynamical system is decomposed into two subsystems as follows:

drive system:

$$\dot{x} = f(x), \tag{8}$$

response system:

$$\dot{y} = f(y). \tag{9}$$

Let us consider a dynamical system, which is composed of two identical n-dimensional subsystems, where only the response system (8) is combined with a coupling coefficient d, while the equation of drive remains the same. The first order differential equations describing such a system can be written as:

$$\dot{x} = f(x),$$
$$\dot{y} = f(y) + d(x - y). \tag{10}$$

Now the condition of synchronization (Eq. (10)) is given by the inequality: $d>\lambda_{\max}$. In the synchronization, d_s, the smallest value of the coupling coefficient d, is assumed to be equal to the maximum Lyapunov exponent: $d_s=\lambda_{\max}$. The results of

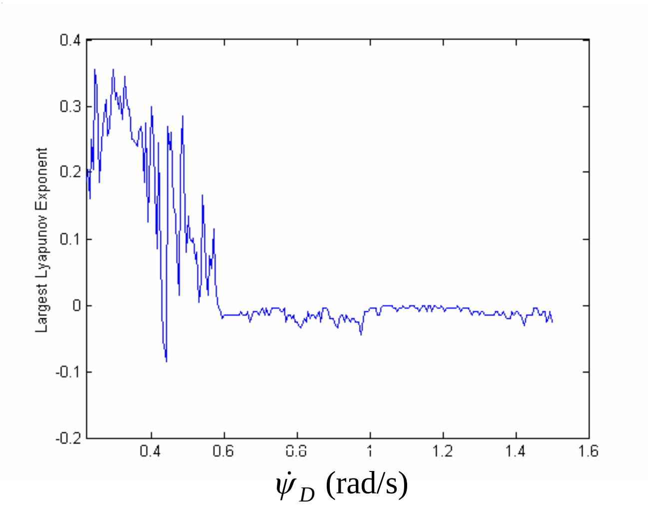

Figure 3 The evolutions of the largest Lyapunov exponent

numerical calculations are listed in Fig. 3, which shows the largest Lyapunov exponents that have been obtained from using the described synchronization method.

CONTROLLING CHAOS

In this section, we will demonstrate that the injection of another external input, called a dither signal, into this chaotic system can control the chaotic motion. Recently, a dither smoothing techniques have been proposed to stabilize the chaotic system. A simple dither signal is a high-frequency sinusoid. In this case, the effective value of n is its average over a complete period of the sinusoidal dither signal oscillation, namely

$$n = \frac{1}{2\pi}\int_0^{2\pi} f(x + W\sin\theta)\mathrm{d}\theta. \tag{11}$$

Now, we add a sinusoidal dither in front of nonlinearity (6). Adding the sinusoidal dither signal to Eq. (7a) (the driver's side) obtains a coupled system as follows:

when $\dot{z}_D \neq 0$,

$\dot{x}_1 = x_2$,

$$\dot{x}_2 = (-R_D - D_D - N_D l_D n_1)/I_D,$$

when $\dot{z}_D = 0,\ |R_D| \ge N_D l_D \mu_0$,

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = (-R_D - D_D)/I_D,$$

when $\dot{z}_D = 0,\ |R_D| < N_D l_D \mu_0$,

$$x_2 = -\dot{\psi}_D,$$
$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = 0, \qquad (12a)$$

where

$$n_1 = \frac{1}{2\pi}\int_0^{2\pi} \mu(\dot{z}_D + W\sin\theta)\mathrm{d}\theta$$
$$= \frac{1}{2\pi}\int_0^{2\pi} [\mu_0 \operatorname{sgn}(\dot{z}_D + W\sin\theta) + \mu_1(\dot{z}_D + W\sin\theta) + \mu_1(\dot{z}_D + W\sin\theta)^3]\mathrm{d}\theta.$$

And, adding the same dither signal to Eq. (7b), the passenger's side can be written as follows:

when $\dot{z}_P \ne 0$,

$$\dot{x}_3 = x_4,$$
$$\dot{x}_4 = (-R_P - D_P - N_P l_P n_2)/I_P,$$

when $\dot{z}_P = 0,\ |R_P| \ge N_P l_P \mu_0$,

$$\dot{x}_3 = x_4,$$
$$\dot{x}_4 = (-R_P - D_P)/I_P,$$

when $\dot{z}_P = 0,\ |R_P| < N_P l_P \mu_0$,

$$x_4 = -\dot{\psi}_P,$$
$$\dot{x}_3 = x_4,$$
$$\dot{x}_4 = 0, \qquad (12b)$$

where

$$n_2 = \frac{1}{2\pi}\int_0^{2\pi} \mu(\dot{z}_P + W\sin\theta)\mathrm{d}\theta$$
$$= \frac{1}{2\pi}\int_0^{2\pi} [\mu_0 \operatorname{sgn}(\dot{z}_P + W\sin\theta) + \mu_1(\dot{z}_P + W\sin\theta) + \mu_1(\dot{z}_P + W\sin\theta)^3]\mathrm{d}\theta.$$

Now, let us choose the system parameter $\dot{\psi}_D = 0.3$ and frequency of the sinusoidal dither is 2000 rad/s. We choose the sinusoidal dither amplitude W = 1.2 and frequency = 2000 rad/s, and add this signal in front of the nonlinearity (6). In simulations, we choose

W = 1.2 and apply dither signal after 4 seconds. The results are shown in Fig. 9. As we can see, the system exhibits a chaotic behavior before the application of dither, whereas it exhibits a periodic motion after the injection of dither.

CONCLUSIONS

This paper is concerned with the complex nonlinear behaviors and chaos control on a wiper system. The Lyapunov exponent will be the most powerful method to examine whether the system is in chaotic motion or not. The method of estimating the largest Lyapunov exponent for wiper system uses the properties of synchronization phenomenon. In order to effectively improve the performance of wiper system o r avoid the chaotic motions, the dither signals in front of the nonlinearity of a chaotic system are applied to suppress chaotic motion. Finally, through sinusoidal type of dither signal, we can also efficiently convert the chaotic system into a periodic orbit by injecting dither signal in front of the nonlinearity of a chaotic system.

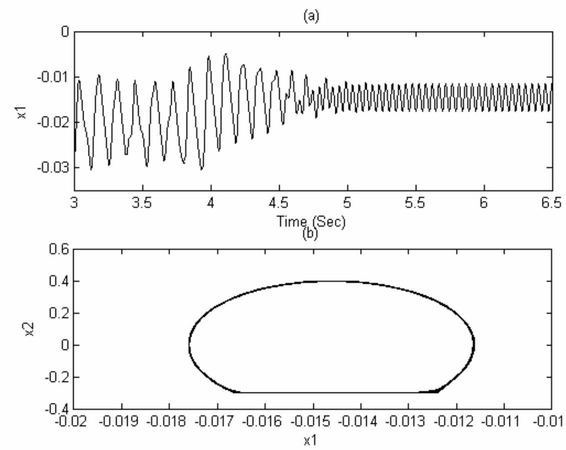

Figure 9 Controlling chaotic motion to a desired period-one orbit for W = 1.2 and $\dot{\psi}_D = 0.3$ (rad/s). (a)Time response; (b)Phase portrait.

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TABLE

Table 1

system parameter	Value	Unit
I_D	0.0407	kgm^2
I_P	0.0367	kgm^2
K_D	720	Nm/rad
K_P	751	Nm/rad
K_M	353	Nm/rad
C_{DP}	0.01	Nms/rad
C_P	0.01	Nms/rad
l_P	0.45	m
l_D	0.47	m
$\dot{\psi}_P$	$1.16 \times \dot{\psi}_D$	rad/s
N_D	7.35	N
N_P	5.98	N
μ_0	1.18	
μ_1	-0.0984	
μ_2	0.474	