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FLEXURAL VIBRATION BAND GAPS IN PERIODIC PLATES WITH FLUID LOADING

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Abstract

In this paper, the flexural vibration in periodic plates with fluid loading is studied theoretically. The effective wavenumber of the flexural wave through the plate with heavy fluid loading is obtained and the approximate transfer matrix method can be used to calculate the complex band structure to investigate the gap frequency range and the vibration attenuation. Furthermore, the effect of the fluid on the band gaps is considered. The existence of flexural vibration gaps in thin plate with fluid loading gives a new idea in vibration control of plate.

INTRODUCTION

In the last decade, the propagation of elastic or acoustic waves in periodic composite materials that are called phononic crystals (PCs) has received much attention [1-3, 7, 8]. The emphasis of these studies was laid on the existence of complete elastic band gaps within which sound and vibration are both forbidden. This is of interest for applications such as frequency filters, vibrationless environments for high-precision mechanical systems, design of new transducers, and so on.

The vibration propagation in periodic structures was researched some time ago [4-6]. The theory to predicting the vibration response of periodic structures has been applied primarily to analysis the periodic structures as pass band and stop band. Recently, with the theory of PCs, the vibration band gaps including longitudinal vibration, flexural vibration and so on, in periodic beams have been studied both theoretically and experimentally [7, 8].

In practice, a periodic structure composed of continuous elastic segments may be loaded by a uniform acoustic medium and it is necessary to control wave propagation in such a strongly coupled structural-acoustic wave guide (an elastic structure with heavy fluid loading) [9]. This information can be useful in designing marine vehicles that exhibit reduced acoustic signatures and self-noise characteristics, and for reducing interior noise levels in passenger and crew compartments [10,11].

Recently, an infinitely long plate and an infinitely long cylindrical shell comprising sets of alternating continuous elements both without and with heavy fluid loading are considered [9]. But the approximate calculation method they used is very complex. An energy finite element method is developed for predicting the high-frequency vibration response of fluid-loaded cylindrical shells and plates under heavy fluid loading [10,11]. Also, the applicability of the periodic characteristics of wave stop and wave propagation bands are investigated for piping systems conveying fluid by employing the wave approach and are proved through experiments [12]. They found the periodic support design is effective in vibration reduction in a piping system.

In this paper, we studied the flexural vibration band gaps in periodic plates with fluid loading. The effective wavenumber of the flexural wave through the plate with heavy fluid loading is obtained. So we can calculate the complex band structure to investigate the gap frequency range and the vibration attenuation in band gap by transfer matrix method easily. Furthermore, the effect of the fluid on the band gaps is researched.

TANSFER MATRIX THEORY

Figure 1 shows an infinitely long plate exposed to heavy fluid loading from one side. The depth of the fluid is infinite. The system consists of an infinite repetition of alternating plate A with length a_1 and plate B with length a_2. Thus the PCs plate's lattice constant is $a=a_1+a_2$. Plate A and plate B can be made of different material parameters illustrated as figure 1. The thickness of the plate A and B are both h.

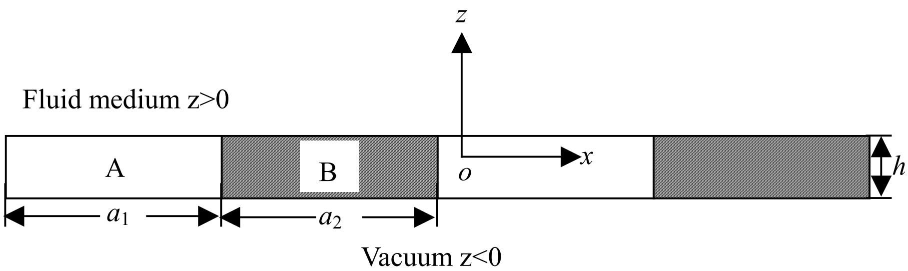

Figure 1 – The sketch map of periodic binary thin plate exposed to heavy fluid loading from one side.

The equations associated with the vibration of a thin plate under heavy fluid loading are summarized first. The governing equation of motion for the flexural displacement is

$$D\nabla^4 w + \rho_s h \frac{\partial^2 w}{\partial t^2} = -P_{z=0} \tag{1}$$

where D is the bending stiffness of the plate and, ρ_s is the mass density, h is the plate thickness, and $P_{z=0}$ is the pressure exerted by the fluid on the vibrating plate.

At high frequencies, the Eq.(1) can be degenerated as[10,11]

$$D\gamma^4 - \rho_s h\omega^2 - \frac{\omega^2 \rho_0}{i\sqrt{k^2 - k_f^2}} = 0 \tag{2}$$

where ρ_0 is the fluid mass density, k is the acoustical wavenumber of the fluid medium, k_f is the flexural wavenumber of the plate in vacuum and $k_f = \sqrt[4]{(\rho_s h / D)\omega^2}$. And the flexural wavenumber γ of the plate under heavy fluid loading depends on the relation value between k and k_f. [10,11]

If $k<k_f$, the solution is

$$\gamma = \sqrt[4]{\frac{\rho_s h}{D}\omega^2\left(1 + \frac{\rho_0}{\rho_s h\sqrt{k_f^2 - k^2}}\right)} \tag{3}$$

If $k>k_f$, the solution for γ is

$$\gamma = \sqrt[4]{\frac{\rho_s h}{D}\omega^2\left(1 - i\frac{\rho_0}{\rho_s h\sqrt{k^2 - k_f^2}}\right)} \tag{4}$$

We consider the normal-mode condition $z(x,t) = X(x)\exp(i\omega t)$, where ω is the circular natural frequency. For a thin plate, the amplitude $X(x)$ reads as

$$X(x) = A\cos(\gamma x) + B\sin(\gamma x) + C\cosh(\gamma x) + D\sinh(\gamma x) \tag{5}$$

For material A in the (n+1)th cell shown in Figure1(a), the amplitude is

$$X_{n1}(x') = A_{n1}\cos(\gamma_1 x') + B_{n1}\sin(\gamma_1 x') + C_{n1}\cosh(\gamma_1 x') + D_{n1}\sinh(\gamma_1 x') \tag{6}$$

where $x' = x - na$, $na \le x \le na + a_1$, and γ_1 getting from equation (1) for material A.

For material B in the (n+1)th cell, the amplitude can be written as

$$X_{n2}(x'') = A_{n2}\cos(\gamma_2 x'') + B_{n2}\sin(\gamma_2 x'') + C_{n2}\cosh(\gamma_2 x'') + D_{n2}\sinh(\gamma_2 x'') \tag{7}$$

where $x'' = x - na$, $na + a_1 \le x \le (n+1)a$, and γ_2 from equation (1) for material B.

The continuities of displacement, slop, bending moment and shear force at the interfaces between cell n and n+1, i.e. $x = na$ give

$$X_{n2}(a) = X_{(n+1)1}(0) \tag{8a}$$

$$X'_{n2}(a) = X'_{(n+1)1}(0) \tag{8b}$$

$$D_1 X''_{n2}(a) = D_2 X''_{(n+1)1}(0) \tag{8c}$$

$$D_1 X'''_{n2}(a) = D_2 X'''_{(n+1)1}(0) \tag{8d}$$

One can obtain the matrix form of equation (8)

$$\mathbf{K}\boldsymbol{\Psi}_{n2} = \mathbf{H}\boldsymbol{\Psi}_{(n+1)1} \tag{9}$$

where $\boldsymbol{\Psi}_{ni} = \begin{bmatrix} A_{ni} & B_{ni} & C_{ni} & D_{ni} \end{bmatrix}^T$, i=1,2 represent material A and material B. Similarly, The continuities of displacement, slop, bending moment and shear force at the interfaces between material A and material B in cell n+1, i.e. $x = na + a_1$ give

$$\mathbf{K}\boldsymbol{\Psi}_{(n+1)1} = \mathbf{H}\boldsymbol{\Psi}_{(n+1)2} \tag{10}$$

Basing equations (9) and (10), the relation between the nth cell and (n+1)th cell is given

$$\mathbf{A}_{(n+1)2} = \mathbf{T}\mathbf{A}_{n2} \tag{11}$$

where $\mathbf{T} = \mathbf{H}_1^{-1}\mathbf{K}_1\mathbf{H}^{-1}\mathbf{K}$ is the transfer matrix.

Due to the periodicity of the infinite structure in the x direction, the vector $\mathbf{A}_n$ must satisfy the Bloch theorem [13]

$$\mathbf{A}_n = e^{iqa}\mathbf{A}_{n-1} \tag{12}$$

where q is the wave vector in the x direction. For convenience, we write all the one-dimension vectors as scalar form in this paper.

It follows that the eigenvalues of the infinite periodic structures are the roots of the determinant

$$\left| \mathbf{T} - e^{iqa}\mathbf{I} \right| = 0 \tag{13}$$

where $\mathbf{I}$ is the 4×4 unit matrix. For given ω, equation (13) gives the values of q. Depending on whether q is real or has an imaginary part, the corresponding wave propagates through the plate (pass band) or is damped (band gap).

CALCULATED RESULTS AND DISCUSSION

We choose a thin plate to calculate the band structure. The geometric properties of the plate are a_1=a_2=0.035m, h=0.005m. As an example, we calculated the band structure of the thin plate with material A being epoxy and material B being Al, the material properties are given in table I.

Table I. Physical parameters of the materials

Materials	Density ρ (Kg/m^3)	Young's modulus E (Pa)	Poisson ratio σ
Pb	11600	4.08×10^{10}	0.37
Cu	8960	12.71×10^{10}	0.35
Steel	7780	21.06×10^{10}	0.30
Al_2O_3	3986	40.27×10^{10}	0.23
Al	2730	7.76×10^{10}	0.35
C	1750	23.01×10^{10}	0.30
Epoxy	1180	4.35×10^{9}	0.37

The complex band structure can be used to describe the band gaps and attenuation in the gaps [14]. Figure 2 illustrate the complex band structure of the infinite periodic structure epoxy/steel plate with water loading. The density and velocity of water is ρ_{water} =1000 Kg/m^3 and c_{water}=1500m/s, respectively. The real wave vector is illustrated in figure 2(a), and the absolute value of the imaginary part of complex wave vector is illustrated in figure 2(b). We can find two complete band gaps within 5000Hz, settled as the shadowed regions in figure 2(a). The first gap extends from the frequency of 374Hz up to 676Hz, the second gap occurs between 2473Hz and 4582Hz. The first gap absolute width is $\Delta f = 302$Hz, and the normalized gap width $\Delta f / f_g = 0.575$, where f_g is the midgap frequency of the first gap.

From figure 2(b), one can note that there are two imaginary wave vector associated within gaps. One imaginary wave vector corresponds to flexural propagation wave illustrated as continue line in figure 2(b). And the other imaginary wave vector is the near-field wave that has an imaginary component for all frequencies [15] illustrated as dashed line in figure 2(b). As for the first gap, the maximum attenuation is 0.11.

As a comparison, we also calculate the complex band structure of the same epoxy/steel plate without fluid loading shown as in figure 3. There are two complete band gaps within 10000Hz, settled as the shadowed regions in figure 3(a). The frequency ranges of the gaps are 957-1424Hz and 4302-7426Hz respectively. The first gap absolute width is $\Delta f = 467$Hz, and the normalized gap width $\Delta f / f_g = 0.392$. The maximum attenuation of the first gap is 0.09.

As is seen from figure 2 and figure 3, heavy fluid produces a very significant influence on the location and the width of band gaps. The band gaps are shifted towards lower frequencies due to the fluid loading. Also the absolute width of band gaps is markedly narrowed. These conclusions had been discovered early [9]. But the first normalized gap width of the plate with fluid loading become wider than that without fluid loading. And the maximum attenuation of the first gap becomes stronger for the plate with heavy fluid loading. The last two findings will be significant for the gaps application to the vibration control of the plates.

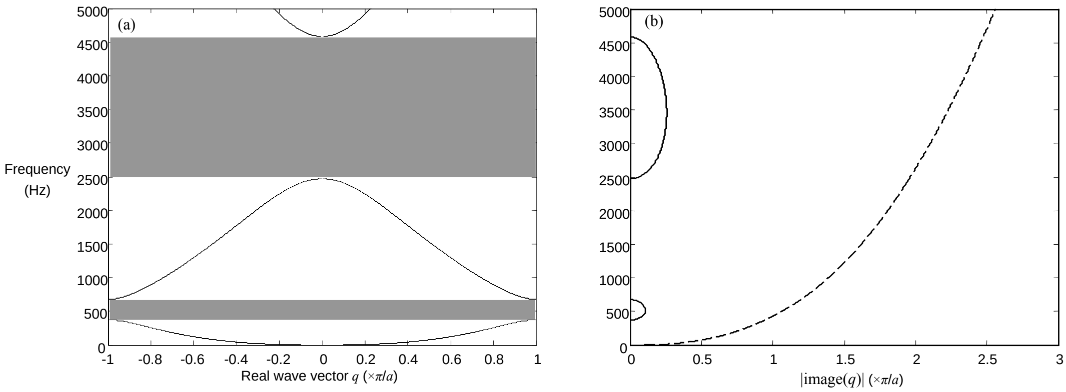

Figure 2 – The complex band structure of the flexural vibration with infinite periodic structure epoxy/Al plate with heavy fluid loading, the lattice constant a=0.07m *and* a_1:a_2=1:1. *(a) real wave vector, (b) the absolute value of the imaginary part of complex wave vector.*

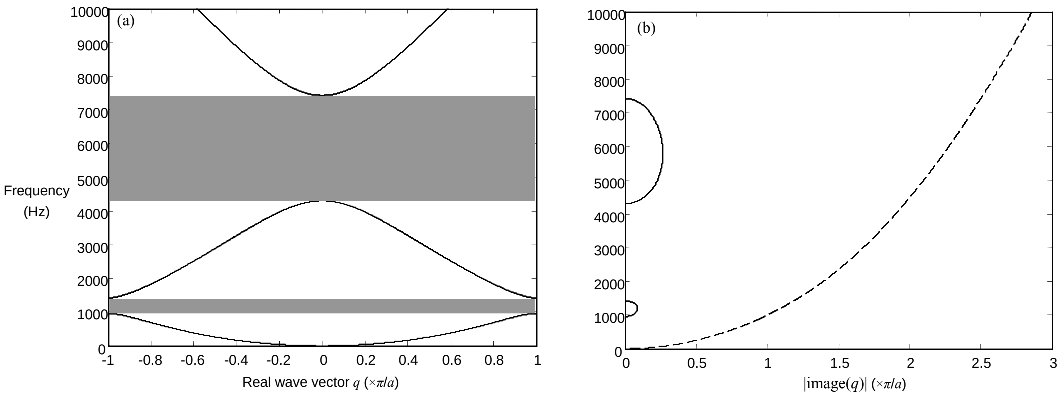

Figure 3 – The complex band structure of the flexural vibration for the same epoxy/steel plate without fluid loading

As the next step, we consider the influence of material properties on the first band gap of the plate. Keeping material A being epoxy, we only substituted material B with different material in Table I. In the calculation, the geometric parameters keep same as those of figure 2. In Figure 3(a), we plot the size of the first gap for the plate with and without fluid loading as a function of the density of the material *B*. The continuous lines mark the upper and lower edge of the first gap of the plate with fluid loading. And the dashed lines express the upper and lower edge of the first gap of the plate without fluid loading. The material B corresponding with its density is labeled as square cross section (with fluid loading) or diamond cross section (without fluid loading). And figure 3(b) illustrates the normalized gap width as changing of material B. From figure 3(b), one can find the normalized gap width of the plate with fluid loading become wider than that without fluid loading. We note that the band gap frequency ranges change evidently as density of material B for the plates without fluid loading. But band gap frequency ranges change gently as density of material B for the plates with fluid loading. Namely, the effect of material parameters on the band gaps becomes weaker due to the fluid loading.

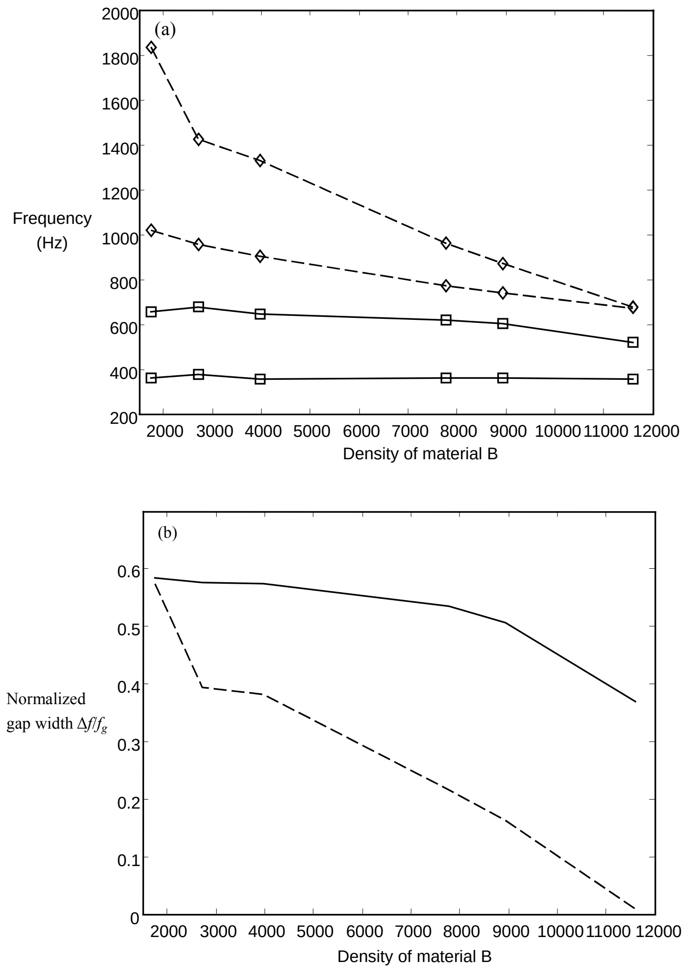

Figure 4 – (a) The dependency of the first gap frequency range related to the changing of material B. (b) The dependency of the normalized gap width as changing of material B. The continuous (dashed) lines mark the upper and lower edge of the first gap of the plate with (without) fluid loading.

CONCLUSIONS

In conclusion, the flexural vibration for a periodic binary thin plate with fluid loading is studied theoretically in this paper. The approximate TM method is provided by the effective wavenumber.

By comparing the calculated results of the plate with fluid loading to those without fluid loading, we find that the frequency ranges of the band gaps become lower.

Also the absolute width of band gaps is narrowed. But the first normalized gap width of the plate with fluid loading become wider than that without fluid loading. The maximum attenuation of the first gap becomes stronger. Also, the effect of material parameters on the band gaps becomes weaker due to the fluid loading.

The existence of flexural vibration gaps in thin plate with fluid loading gives a new idea in vibration control of plate. The findings will be significant in the application of PCs.

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