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A STATE ESTIMATION METHOD FOR SOUND ENVIRONMENT SYSTEM WITH UNCERTAINTY BY INTRODUCING FUZZY INFERENCE

Akira Ikuta*[1], and Mitsuo Ohta[2]

[1]Department of Management and Information Systems, Prefectural University of Hiroshima
Hiroshima 734-8558, Japan
[2]Hiroshima University, Emeritus, Japan
ikuta@pu-hiroshima.ac.jp

Abstract

The observed phenomena in the actual sound environment often contain uncertainty such as the additional external noise (i.e., background noise) with unknown statistics. For example, there are complex nonlinear relationships between noise evaluation quantities and background noises, and these can not be exactly expressed in a functional form. In these situations, it is one of reasonable analyses to consider that the observed signal is contaminated by an external noise containing uncertainty. In this study, an estimation method for state variables on a specific signal under the existence of a background noise with unknown statistics is proposed by introducing fuzzy inference. The effectiveness of the proposed theoretical method is experimentally confirmed by applying it to the actually observed data for road traffic noise.

INTRODUCTION

The observation data in actual sound environment system exhibit various types of fluctuation characteristics, and these often contain uncertainty. For example, the observed signal is inevitably contaminated by the concurrent external noise (i.e., background noise) of arbitrary distribution type with unknown statistics. In this situation, in order to evaluate the specific signal based on the observed noisy data, it is indispensable to introduce some unified state estimation methods.

Though several state estimation methods have been proposed up to now, these state estimation algorithms have been realized by introducing the additive model of the specific signal and the external noise under an assumption of known statistics of the external noise [1-3]. The observation equation in sound environment can be generally expressed in an additive model of the specific signal and the background noise in

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energy scale by using physically the additive property of acoustic energy. However, there exist complex nonlinear relationships between the noise evaluation quantities of the specific signal and the background noise, and it is difficult in general to find a functional relationship between them. Furthermore, it is actually difficult to know in advance statistical properties of the concurrent background noise.

In this study, a state estimation method for a specific signal under the existence of a background noise with unknown statistical properties is proposed in a usual form without considering the additive model. More specifically, after introducing a membership function for uncertainty of a sound environment system, by applying a fuzzy inference for a relationship between the observed data and the state variable on the specific signal, a state estimation algorithm is derived. The effectiveness of the proposed method is experimentally confirmed by applying it to the estimation of noise evaluation quantities for road traffic noise under existence of a background noise.

FORMULATION OF SOUND ENVIRONMENT SYSTEM WITH UNCERTAINTY

Let x_k and y_k be state variable and observation at a time k for sound environment system with uncertainty. It is assumed that the mutual relationship between x_k and y_k is unknown. For example, the observations in sound environment are inevitably contaminated by the external noise (i.e., background noise) of arbitrary distribution type. In general, by using the additive property of acoustic intensity, the observation intensity under the existence of external noise can be expressed as an additive model of the specific signal and the background noise intensities. However, for the stochastic evaluation quantities: L_{eq} and L_x ((100-x) percentile level) which are required in the evaluation of actual sound and vibration environment, there exist complex nonlinear relationships between the evaluation quantities of the specific signal and the observed evaluation quantities. These evaluation quantities of the specific signal and the observed evaluation quantities have to be regarded as the unknown state variable x_k and the observation y_k respectively. Since it is difficult in general to find a functional relationship between the state variable x_k and the observation y_k, any consideration for the mutual relationship between x_k and y_k as a sound environment system with uncertainty is necessary. In this study, the following IF-THEN rule is introduced for the uncertainty of the sound environment system:

Rule 1: IF x_k is A_1 THEN y_k is B_1

Rule 2: IF x_k is A_2 THEN y_k is B_2

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Rule N: IF x_k is A_N THEN y_k is B_N, (1)

where A_i and B_i (i=1, 2, …, N) denote fuzzy sets corresponding to the divided several state spaces of x_k and y_k, and have membership functions $\mu_{A_i}(x_k)$ and

$\mu_{B_i}(y_k)$ (i =1, 2, …, N). For example, A_1, A_2, …, A_N are fuzzy sets describing the divided state spaces: "very low level", "low level", …, "very high level" for the sound level x_k. Applying fuzzy inference to (1), y_k is given by [4]

$$y_k = \frac{\sum_{i=1}^{N} \bar{y}_i \mu_{A_i}(x_k)}{\sum_{i=1}^{N} \mu_{A_i}(x_k)}, \tag{2}$$

where $\bar{y}_i$ is the value at which $\mu_{B_i}(y_k)$ achieved its maximum value. Furthermore, as the membership function $\mu_{A_i}(x_k)$, the Gaussian type, defined by [4]

$$\mu_{A_i}(x_k) = \exp\{-\frac{1}{2}(\frac{x_k - \bar{x}_i}{\sigma_i})^2\}, \tag{3}$$

where $\bar{x}_i$ and σ_i are parameters, is adopted.

In the next section, an estimation method for the state variable x_k of the specific signal based on the recursive observation y_k is derived. Though the parameter σ_i in (3) can be generally given based on the prior information (or, through trial and error), it can be regarded as unknown constant parameter α ($= \sigma_i$ for every i) and estimated simultaneously with the state variable x_k by introducing the following simple dynamic model:

$$\alpha_{k+1} = \alpha_k. \tag{4}$$

STATE ESTIMATION METHOD FOR THE SOUND ENVIRONMENT SYSTEM

In order to derive an estimation algorithm for a state variable x_k, with an arbitrary distribution, we focus our attention on Bayes' theorem for the conditional probability density function (abbr. pdf). Since the parameter α_k is also unknown, the conditional pdf of x_k and α_k must be considered.

$$P(x_k, \alpha_k \mid Y_k) = \frac{P(x_k, \alpha_k, y_k \mid Y_{k-1})}{P(y_k \mid Y_{k-1})}, \tag{5}$$

where $Y_k (= \{y_1, y_2, ..., y_k\})$ is a set of observation data up to time k. Based on (5), through a similar calculation process to that used in a previously reported estimation method [1], the estimate of an arbitrary polynomial function $f_{N1,N2}(x_k, \alpha_k)$ of x_k and α_k of $N (\equiv (N1, N2))$-th order can be derived in an infinite series expression, as follows:

$$\hat{f}_{N1,N2}(x_k, \alpha_k) \equiv < f_{N1,N2}(x_k, \alpha_k) \mid Y_k >$$

$$= \frac{\sum_{l=0}^{N1} \sum_{m=0}^{N2} \sum_{n=0}^{\infty} C_{lm}^{N1N2} A_{lmn} \varphi_n^{(3)}(y_k)}{\sum_{n=0}^{\infty} A_{00n} \varphi_n^{(3)}(y_k)} \tag{6}$$

with

$$A_{lmn} \equiv < \varphi_l^{(1)}(x_k) \varphi_m^{(2)}(\alpha_k) \varphi_n^{(3)}(y_k) | Y_{k-1} > . \tag{7}$$

The three functions $\varphi_l^{(1)}(x_k)$, $\varphi_m^{(2)}(\alpha_k)$ and $\varphi_n^{(3)}(y_k)$ are the orthonormal polynomials of degrees l, m and n, with weighting functions $P_0(x_k | Y_{k-1})$, $P_0(\alpha_k | Y_{k-1})$ and $P_0(y_k | Y_{k-1})$, which can be artificially chosen as the pdfs describing the above dominant parts of the actual fluctuation, or as well-known standard pdfs such as Gaussian or Gamma distribution functions. All the coefficients $C_{lm}^{N1,N2}$ are appropriate constants in the case when the function $f_{N1,N2}(x_k, \alpha_k)$ is expressed in a series expansion form using $\varphi_l^{(1)}(x_k)$ and $\varphi_m^{(2)}(\alpha_k)$:

$$f_{N1,N2}(x_k, \alpha_k) = \sum_{l=0}^{N1} \sum_{m=0}^{N2} C_{lm}^{N1N2} \varphi_l^{(1)}(x_k) \varphi_m^{(2)}(\alpha_k). \tag{8}$$

As a concrete example of a standard pdf, the well-known Gaussian distribution is adopted:

$$P_0(x_k | Y_{k-1}) = N\left(x_k; x_k^*, \Gamma x_k\right), \tag{9}$$

$$P_0(\alpha_k | Y_{k-1}) = N\left(\alpha_k; \alpha_k^*, \Gamma \alpha_k\right), \tag{10}$$

$$P_0(y_k | Y_{k-1}) = N\left(y_k; y_k^*, \Gamma y_k\right) \tag{11}$$

with

$$N(x; \mu, \sigma^2) \equiv \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(x-\mu)^2}{2\sigma^2}\},$$

$$x_k^* \equiv < x_k | Y_{k-1} >, \qquad \Gamma x_k \equiv < \left(x_k - x_k^*\right)^2 | Y_{k-1} >,$$

$$\alpha_k^* \equiv < \alpha_k | Y_{k-1} >, \qquad \Gamma \alpha_k \equiv < \left(\alpha_k - \alpha_k^*\right)^2 | Y_{k-1} >,$$

$$y_k^* \equiv < y_k | Y_{k-1} >, \qquad \Gamma y_k \equiv < \left(y_k - y_k^*\right)^2 | Y_{k-1} > . \tag{12}$$

Then, the orthonormal polynomials with three weighting pdfs in (9)-(11) can be given in terms of the well-known Hermite polynomials [5]

$$\varphi_l^{(1)}(x_k) = \frac{1}{\sqrt{l!}} H_l\left(\frac{x_k - x_k^*}{\sqrt{\Gamma x_k}}\right), \tag{13}$$

$$\varphi_m^{(2)}(\alpha_k) = \frac{1}{\sqrt{m!}} H_m\left(\frac{\alpha_k - \alpha_k^*}{\sqrt{\Gamma \alpha_k}}\right), \tag{14}$$

$$\varphi_n^{(3)}(y_k) = \frac{1}{\sqrt{n!}} H_n\left(\frac{y_k - y_k^*}{\sqrt{\Gamma y_k}}\right). \tag{15}$$

Using (2) and (3), the two variables y_k^* and Γy_k in (12) can be expressed as:

$$y_k^* = < \sum_{i=1}^{N} \bar{y}_i \exp\{-\frac{1}{2}(\frac{x_k - \bar{x}_i}{\alpha_k})^2\} / \sum_{i=1}^{N} \exp\{-\frac{1}{2}(\frac{x_k - \bar{x}_i}{\alpha_k})^2\} \mid Y_{k-1} > , \tag{16}$$

$$\Gamma_{y_k} = < [\sum_{i=1}^{N} \bar{y}_i \exp\{-\frac{1}{2}(\frac{x_k - \bar{x}_i}{\alpha_k})^2\} / \sum_{i=1}^{N} \exp\{-\frac{1}{2}(\frac{x_k - \bar{x}_i}{\alpha_k})^2\} - y_k^*]^2 \mid Y_{k-1} > . \tag{17}$$

Furthermore, each expansion coefficient A_{lmn} defined by (7) can be expressed as

$$A_{lmn} = < \frac{1}{\sqrt{l!}} H_l\left(\frac{x_k - x_k^*}{\sqrt{\Gamma x_k}}\right) \frac{1}{\sqrt{m!}} H_m\left(\frac{\alpha_k - \alpha_k^*}{\sqrt{\Gamma \alpha_k}}\right)$$

$$\frac{1}{\sqrt{n!}} H_n\left(\frac{\sum_{i=1}^{N} \bar{y}_i \exp\{-\frac{1}{2}(\frac{x_k - \bar{x}_i}{\alpha_k})^2\} / \sum_{i=1}^{N} \exp\{-\frac{1}{2}(\frac{x_k - \bar{x}_i}{\alpha_k})^2\} - y_k^*}{\sqrt{\Gamma y_k}}\right) \mid Y_{k-1} > . \tag{18}$$

The variables y_k^*, Γy_k and the expansion coefficient A_{lmn} in (16), (17) and (18) are given by the predictions of x_k and α_k at a discrete time $k-1$ (i.e., the expectation value of arbitrary functions of x_k and α_k conditioned by Y_{k-1}).

Finally, in order to derive the prediction step necessary to perform the recurrence estimation, the fuzzy inference is introduced again. More specifically, after dividing the state spaces into M fuzzy regions S_i ($i = 1, 2, \ldots, M$) with membership functions $\mu_i(x_k)$ ($i = 1, 2, \ldots, M$), by creating fuzzy rules from given data sets and using the center average defuzzification formula [6], a state transition model can be determined as

$$x_{k+1} = \frac{\sum_{i=1}^{M} \tilde{x}_i \mu_i(x_k)}{\sum_{i=1}^{M} \mu_i(x_k)}, \tag{19}$$

$$\mu_i(x_k) = \exp\{-\frac{1}{2}(\frac{x_k - \tilde{x}_i}{\sigma})^2\}, \tag{20}$$

where $\tilde{x}_i$ is the value at which $\mu_i(x_k)$ achieved its maximum value, and σ is a constant parameter. By considering (4) and (19), the prediction algorithm can be given for an arbitrary polynomial function $g_r(x_{k+1}, \alpha_{k+1})$ of x_{k+1} and α_{k+1} of $r(\equiv (r_1, r_2))$-th order, as follows:

$$g_r^*(x_{k+1}, \alpha_{k+1}) \equiv < g_r(x_{k+1}, \alpha_{k+1}) \mid Y_k >$$

$$=< g_r(\sum_{i=1}^{M}\tilde{x}_i\mu_i(x_k)/\sum_{i=1}^{M}\mu_i(x_k),\alpha_k)\,|\,Y_k > . \qquad (21)$$

The above prediction can be evaluated by the estimates at discrete time $k-1$. Therefore, by combining (6) with (21), the recurrence estimation of x_k and α_k can be achieved.

APPLICATION TO NOISE EVALUATION QUANTITIES FOR ROAD TRAFFIC NOISE

In order to confirm experimentally the validity of the proposed method, it is applied to the actually observed data in sound environment. As the concrete specific signal, the road traffic noise becoming one of the aggravating environmental noise pollution is adopted. Based on the observations on noise evaluation quantity: $L_{A50,T}$ $(T=1$ min) under existence of a background noise, the fluctuation forms of evaluation quantities: $L_{ALeq,T}$ and $L_{A50,T}$ $(T=1$ min) for the specific signal are estimated.

By dividing the state spaces in sound level scale into 3 fuzzy regions, A_1, B_1: “low level”, A_2, B_2: “middle level”, and : A_3, B_3 “high level”, (1) and (2) are expressed as follows:

(Inference 1)

IF-THEN Rule Rule 1: IF x_k is A_1 THEN y_k is B_1

Rule 2: IF x_k is A_2 THEN y_k is B_2

Rule 3: IF x_k is A_3 THEN y_k is B_3, (22)

Defuzzification
$$y_k = \frac{\bar{y}_1\mu_{A_1}(x_k)+\bar{y}_2\mu_{A_2}(x_k)+\bar{y}_3\mu_{A_3}(x_k)}{\mu_{A_1}(x_k)+\mu_{A_2}(x_k)+\mu_{A_3}(x_k)}, \qquad (23)$$

where the three parameters are decided as: $\bar{y}_1$=(mean of y_k)-3$\times$(standard deviation of y_k), $\bar{y}_2$=(mean of y_k), and $\bar{y}_3$=(mean of y_k)+3$\times$(standard deviation of y_k).

Furethermore, the following two IF-THEN rules slightly modified (1) and (2) are introduced:

(Inference 2)

IF-THEN rule Rule 1: IF x_k is A_1 or A_2 THEN y_k is B_1

Rule 2: IF x_k is A_1 or A_2 or A_3 THEN y_k is B_2

Rule 3: IF x_k is A_2 or A_3 THEN y_k is B_3, (24)

Defuzzification
$$y_k = [\bar{y}_1\{\mu_{A_1}(x_k)+\mu_{A_2}(x_k)\}+\bar{y}_2\{\mu_{A_1}(x_k)+\mu_{A_2}(x_k)+\mu_{A_3}(x_k)\}$$
$$+\bar{y}_3\{\mu_{A_2}(x_k)+\mu_{A_3}(x_k)\}]/[2\mu_{A_1}(x_k)+3\mu_{A_2}(x_k)+2\mu_{A_3}(x_k)]. \qquad (25)$$

(Inference 3)

IF-THEN rule Rule 1: IF x_k is A_1 or A_2 THEN y_k is B_1

$$\text{Rule 2: IF } x_k \text{ is } A_2 \text{ THEN } y_k \text{ is } B_2$$

$$\text{Rule 3: IF } x_k \text{ is } A_2 \text{ or } A_3 \text{ THEN } y_k \text{ is } B_3, \tag{26}$$

Defuzzification

$$y_k = \frac{\bar{y}_1\{\mu_{A_1}(x_k) + \mu_{A_2}(x_k)\} + \bar{y}_2\mu_{A_2}(x_k) + \bar{y}_3\{\mu_{A_2}(x_k) + \mu_{A_3}(x_k)\}}{\mu_{A_1}(x_k) + 3\mu_{A_2}(x_k) + \mu_{A_3}(x_k)}. \tag{27}$$

One of the estimation results in the case of adopting $L_{ALq,T}$ and $L_{A50,T}$ as the state variable x_k and the observation y_k contaminated by a background noise and applying the fuzzy inference 1 is shown in Fig.1. The estimation result shows a good agreement with the true values. For comparison, the extended Kalman filter [7] is also applied to the observed data after introducing a linear system model:

$$y_k = \beta_k x_k + \gamma_k v_k, \tag{28}$$

$$x_{k+1} = Fx_k + Gu_k, \quad \beta_{k+1} = \beta_k, \quad \gamma_{k+1} = \gamma_k. \tag{29}$$

In (28) and (29), β_k and γ_k are unknown parameters to be estimated simultaneously with the state variable x_k, and v_k, u_k are random noises with mean 0 and variance 1. Tow parameters F and G are estimated by use of auto-correlation technique [1].

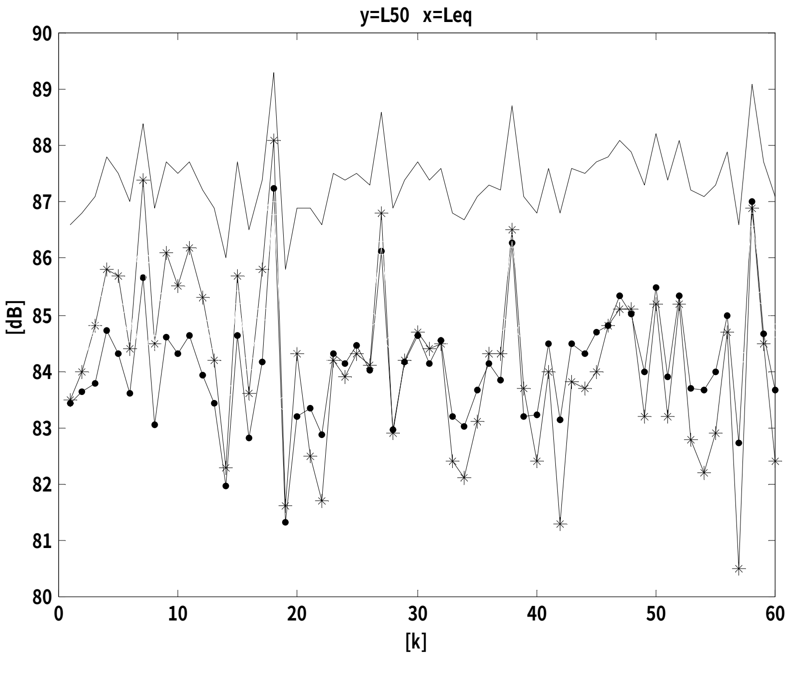

Figure 1 – State estimation results for $L_{AL_{eq},T}$ based on the observation of $L_{A50,T}$ contaminated by a background noise (–; observations, $*$; true values, $\bullet$; estimated results by the proposed method).

Table 1 – Root mean squared error of the estimation in [dBA].

State Variable	Inference 1	Inference 2	Inference 3	Extended Kalman Filter
$L_{ALeq,T}$	0.894	0.977	0.885	1.36
$L_{A50,T}$	1.04	1.50	0.954	1.47

The squared sum of the estimation error is shown in Table 1. It is obvious that the proposed method based on the use of fuzzy theory shows more accurate estimation than the results based on the method introducing usual linear system model like (28) and (29). Especially, the results by the inference 3 show the most precise estimation among the results by the proposed methods based on fuzzy inference and the extended Kalman filter.

CONCLUSIONS

In this study, a state estimation method for sound environment system with uncertainty has been theoretically proposed by introducing the fuzzy inference. More specifically, after considering the relationship between the state variable and observed evaluation quantities as the sound environment system containing uncertainty, a recursive estimation algorithm for noise evaluation quantities of the specific signal based on the observed evaluation quantities under the existence of a background noise has been derived. Furthermore, by applying the proposed method to the actually observed data of road traffic noise in sound environment, the effectiveness of the theory has been confirmed experimentally too.

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