



MUTUALLY CONVERGING ADAPTIVE FEEDBACK ACTIVE NOISE CONTROL WITH ON-LINE SECONDARY-PATH MODELING

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Abstract

In this paper we propose a new method to attenuate predictable components inside desired signal by using adaptive feedback control with on-line secondary-path modeling. The proposed method is based on feedback FxLMS algorithm and the secondary path is on-line estimated by internally adding random noise. For this proposed algorithm, the incoherent disturbances inside each residual error of three adaptive laws are mutually eliminated, and the synthesized desired signal converges to desired signal when attenuation of desired signal and estimation of the secondary path are operating simultaneously. Performances of narrowband and broadband active noise/vibration control (ANC/AVC) in some specific simulated system are presented.

INTRODUCTION

Traditional adaptive feedback ANC/AVC is implemented with off-line secondary-path modeling [1]. For some structures such as a time-varying system path, the secondary-path estimation prior to control is rather inefficient, and then it's not suitable for control of time-varying system path. Here we propose a new algorithm to control predictable components inside desired signal and estimate the secondary path simultaneously, which means on-line secondary-path modeling. The proposed algorithm can be furthermore used in the hybrid ANC systems to control the broadband desired signal including white noise component.

ALGORITHMS REVIEW

Kuo and Vijayan [2] proposed an adaptive feedback ANC system which using a synthesized signal as the control system's input.

Fig. 1 shows the control block diagram of adaptive feedback ANC system using the FxLMS algorithm. The input $x(n)$ is synthesized as the next sample $d(n)$ of the desired signal $d(n-1)$, which is

$$x(n) \equiv \hat{d}(n) = e(n) + y(n)\hat{S}(z) = e(n) + d(n-1)W(z)\hat{S}(z) \quad (1)$$

Eq. 1 means that the input signal $x(n)$ is synthesized to track the desired signal $d(n)$. It also emphasized that the desired signal $d(n)$ must be predictable to be predicted its next sample $d(n+1)$.

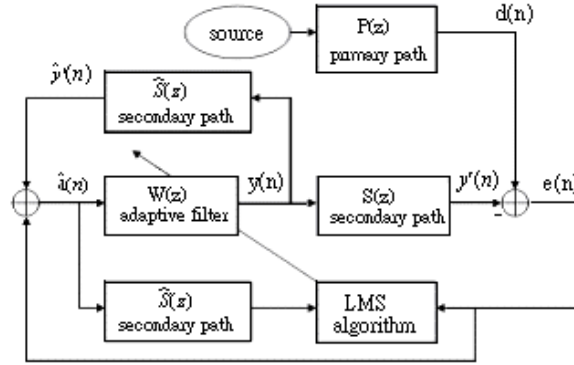


Figure 1 – Adaptive feedback ANC system using the FxLMS algorithm.

For Kuo's algorithm, after the estimated secondary path $\hat{S}(z)$ converges to the secondary path $S(z)$, the synthesized input signal $x(n)$ behaves as

$$x(n) \equiv \hat{d}(n) \text{ converging to } d(n) \quad (2)$$

Then the control system can be viewed as an adaptive feedforward ANC system like Fig. 2.

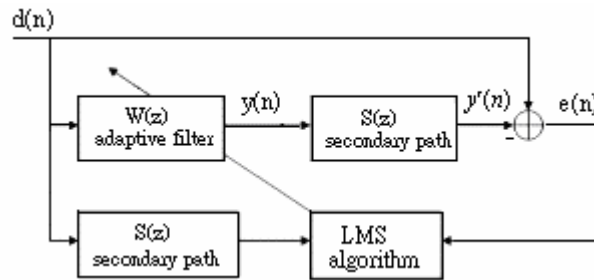


Figure 2 – Equivalent adaptive feedforward ANC system established when $\hat{S}(z) = S(z)$

In the field of adaptive feedforward ANC system with on-line secondary-path modeling, Ming Zhang, Hui Lan, and Wee Ser [3] proposed the algorithm “Cross-Updated Active Noise Control System with Online Secondary Path Modeling”,

whose control block diagram is shown as Fig. 3. In this algorithm, the incoherent perturbations in the residual error of following adaptive laws 3, 4, and 5 are eliminated mutually, which means “Cross-Updated” in Zhang’s method, and then Zhang’s algorithm can be more robust and faster than Bao’s algorithm.

$$W(n+1) = W(n) + \mu_w x'(n)[e(n) - \hat{v}'(n)] \quad (3)$$

$$\hat{S}(n+1) = \hat{S}(n) + \mu_S v(n)[e'(n) - \hat{v}'(n)] \quad (4)$$

$$\hat{H}(n+1) = \hat{H}(n) + \mu_H x(n)[e(n) - \hat{v}'(n) - \hat{e}(n)] \quad (5)$$

The incoherent perturbations existed in each residual error concerning each adaptive law are eliminated, which are stated in the following Eq. 6, 7, and 8.

Error in Eq. 3 is

$$\begin{aligned} e(n) - \hat{v}'(n) &= \{x(n)P(z) - [x(n)W(z) - v(n)]S(z)\} - v(n)\hat{S}(z) \\ &= x(n)[P(z) - W(z)S(z)] + v(n)[S(z) - \hat{S}(z)] \end{aligned} \quad (6)$$

From Eq. 6, if $\hat{S}(n)$ converges to $S(z)$, then $P(z)$ can converge to $W(z)S(z)$.

Error in Eq. 4 is

$$\begin{aligned} e'(n) - \hat{v}'(n) &= \{x(n)P(z) - [x(n)W(z) - v(n)]S(z) - x(n)\hat{H}(z)\} - v(n)\hat{S}(z) \\ &= v(n)[S(z) - \hat{S}(z)] + x(n)[P(z) - W(z)S(z) - \hat{H}(z)] \end{aligned} \quad (7)$$

From Eq. 7, if $\hat{H}(z)$ converges to $P(z) - W(z)S(z)$, then $\hat{S}(z)$ can converge to $S(z)$.

Error in Eq. 5 is

$$\begin{aligned} e(n) - \hat{v}'(n) - \hat{e}(n) &= \{x(n)P(z) - [x(n)W(z) - v(n)]S(z)\} - v(n)\hat{S}(z) - x(n)\hat{H}(z) \\ &= x(n)[P(z) - W(z)S(z) - \hat{H}(z)] + v(n)[S(z) - \hat{S}(z)] \end{aligned} \quad (8)$$

From Eq. 8, if $\hat{S}(z)$ converges to $S(z)$, then $\hat{H}(z)$ can converge to $P(z) - W(z)S(z)$.

From Eq. 6, 7, and 8 the incoherent perturbations existed in each residual error can be eliminated mutually and each adaptive controller is able to update to convergence.

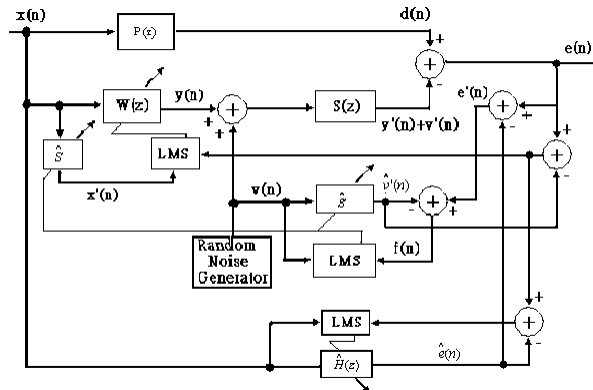


Figure 3 – Cross-Updated Active Noise Control System with Online Secondary Path Modeling.

THE PROPOSED ALGORITHM

From Fig. 3, Eq. 7 and Eq. 8 we can find that the residual error signals $e'(n) - \hat{v}'(n)$ and $e(n) - \hat{v}'(n) - \hat{e}(n)$ are exactly the same. Then Zhang's algorithm can be further improved by reducing two adders, which is shown as Fig. 4.

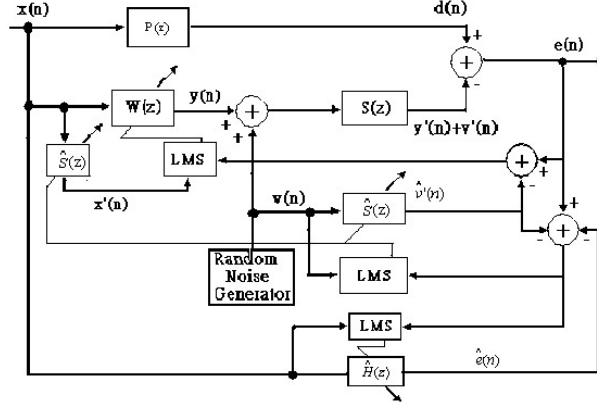


Figure 4 – Further improved method of Zhang's algorithm.

By utilizing the feedback method and the improved method as Fig. 4, the proposed algorithm internally adds random noise to on-line estimate the secondary path and synthesizes the desired signal to be the control system's input.

The control block diagram of the proposed algorithm is shown as Fig. 5, and convergence of each adaptive law and the synthesized desired signal are discussed later.

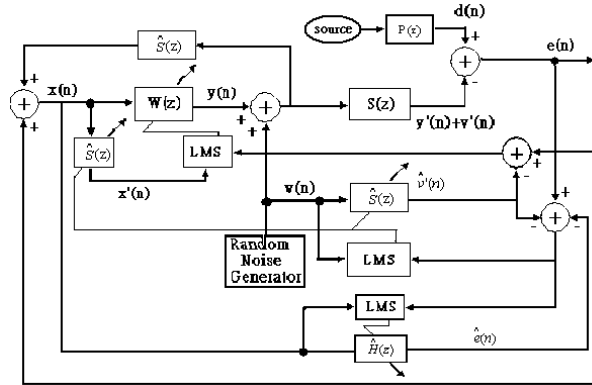


Figure 5 – Proposed method.

From the control block diagram Fig. 5 of the proposed algorithm, the adaptive law of controller $W(z)$ is expressed as

$$W(n+1) = W(n) + \mu_W [x(n)\hat{S}(z)][e(n) - \hat{v}'(n)] \quad (9)$$

where the residual error $e(n) - \hat{v}'(n)$ can be expressed as

$$\begin{aligned} e(n) - \hat{v}'(n) &= \left\{ d(n) - [\hat{d}(n)W(z) - v(n)]S(z) \right\} - v(n)\hat{S}(z) \\ &= [d(n) - \hat{d}(n)W(z)S(z)] + v(n)[S(z) - \hat{S}(z)] \end{aligned} \quad (10)$$

We can find that the perturbation $v(n)[S(z) - \hat{S}(z)]$ is incoherent with $x(n)\hat{S}(z)$. From Eq. 10, if $\hat{d}(n)$ and $\hat{S}(z)$ converge to $d(n)$ and $S(z)$ respectively, the perturbation $v(n)[S(z) - \hat{S}(z)]$ can be eliminated and then $W(z)S(z)$ can converge to 1.

Similarly, the adaptive law of the estimated transfer function $\hat{H}(z)$ is expressed as

$$\hat{H}(n+1) = \hat{H}(n) + \mu_H \hat{d}(n) [e(n) - \hat{v}'(n) - \hat{e}(n)] \quad (11)$$

where the residual error $e(n) - \hat{v}'(n) - \hat{e}(n)$ can be expressed as

$$\begin{aligned} e(n) - \hat{v}'(n) - \hat{e}(n) &= \left\{ d(n) - [\hat{d}(n)W(z) - v(n)]S(z) \right\} - v(n)\hat{S}(z) - \hat{d}(n)\hat{H}(z) \\ &= [d(n) - \hat{d}(n)W(z)S(z) - \hat{d}(n)\hat{H}(z)] + v(n)[S(z) - \hat{S}(z)] \end{aligned} \quad (12)$$

We can find that the perturbation $v(n)[S(z) - \hat{S}(z)]$ is incoherent with $\hat{d}(n)$. From Eq. 12, if $\hat{S}(z)$ and $\hat{d}(n)$ converge to $S(z)$ and $d(n)$ respectively, the perturbation $v(n)[S(z) - \hat{S}(z)]$ can be eliminated and then $\hat{H}(z)$ can converge to $1 - W(z)S(z)$.

Next, the adaptive law of the secondary path $S(z)$ is expressed as

$$\hat{S}(n+1) = \hat{S}(n) + \mu_S v(n) [e(n) - \hat{v}'(n) - \hat{e}(n)] \quad (13)$$

where the residual error $e(n) - \hat{v}'(n) - \hat{e}(n)$ can be expressed the same as Eq. 12. That is

$$e(n) - \hat{v}'(n) - \hat{e}(n) = v(n)[S(z) - \hat{S}(z)] + [d(n) - \hat{d}(n)W(z)S(z) - \hat{d}(n)\hat{H}(z)] \quad (14)$$

We can find that the perturbation $d(n) - \hat{d}(n)W(z)S(z) - \hat{d}(n)\hat{H}(z)$ is incoherent with $v(n)$. From Eq. 14, if $\hat{d}(n)$ and $\hat{H}(z)$ converge to $d(n)$ and $1 - W(z)S(z)$ respectively, the perturbation $d(n) - \hat{d}(n)W(z)S(z) - \hat{d}(n)\hat{H}(z)$ can be eliminated and then $\hat{S}(z)$ can converge to $S(z)$.

Finally, the synthesized input $x(n)$ is expressed as

$$\begin{aligned} x(n) \equiv \hat{d}(n) &= [y(n) - v(n)]\hat{S}(z) + e(n) \\ &= [\hat{d}(n-1)W(z) - v(n)]\hat{S}(z) + \left\{ d(n) - [\hat{d}(n-1)W(z) - v(n)]S(z) \right\} \end{aligned} \quad (15)$$

From Eq. 15, if $\hat{S}(z)$ converges to $S(z)$, then the synthesized desired signal $\hat{d}(n)$ can

converge to the desired signal $d(n)$. Eq. 15 also implies that the desired (unwanted) signal $d(n)$ must be predictable so that the adaptive controller $W(z)$ can predict the next sample $d(n+1)$, which also means that the adaptive feedback algorithm can not attenuate white noise components.

SIMULATION RESULTS

The control performances shown bellow are narrowband and broadband noise control. For narrowband control, two harmonic waves 300Hz and 900Hz are set to be desired signals respectively. For broadband control, the output-whitening method is used to simulate an output signal from a 4th order plant excited by a band-limited white noise. Both the damping ratios of the 4th order model are modeled for the same value, which is $\xi=0.0015$. Both the resonance frequencies are $W_{R1}=300\text{Hz}$ and $W_{R2}=900\text{Hz}$ respectively. The frequency response of the 4th order plant is shown in Fig. 6. The three adaptive Finite-Impulse-Response (FIR) filters used in the proposed algorithm all use Least Mean Square (LMS) algorithm. The secondary path is also modeled for a FIR filter. whose frequency response is shown in Fig. 7.

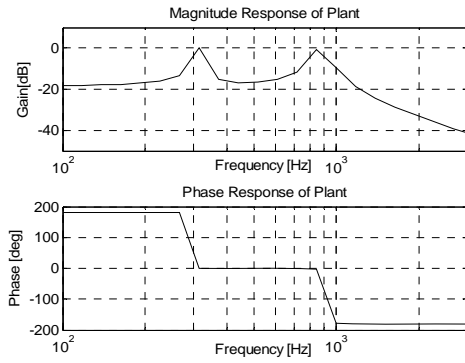


Figure 6 – Frequency response of the modeled 4th plant.

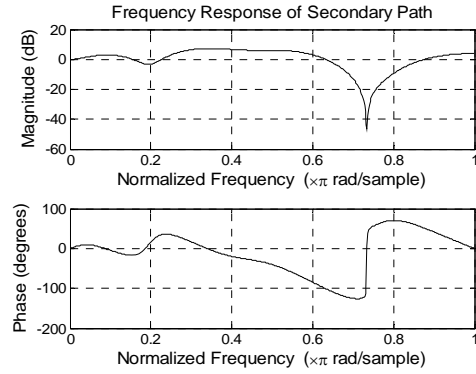


Figure 7 – Frequency response of secondary path.

The mean square error (MSE) and converging performance of the estimated secondary path $\hat{S}(z)$ for two narrowband controls are shown in Fig. 8, 9, and 10 respectively.

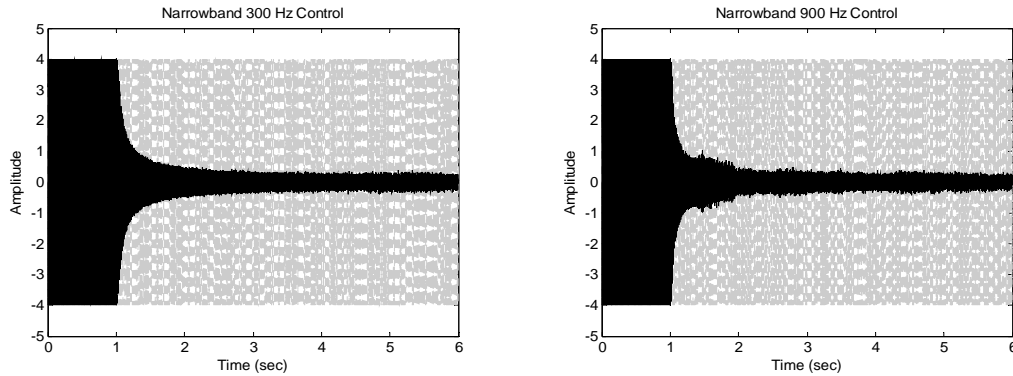


Figure 8 – Time responses of two narrowband control respectively.

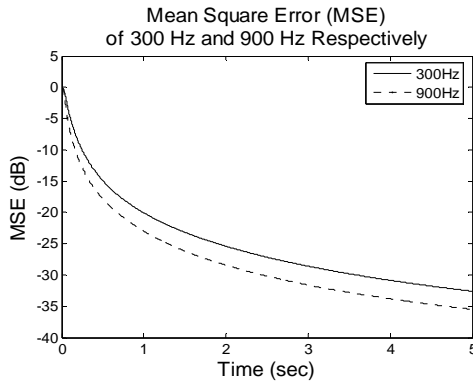


Figure 9 – MSE performance of two narrowband control respectively

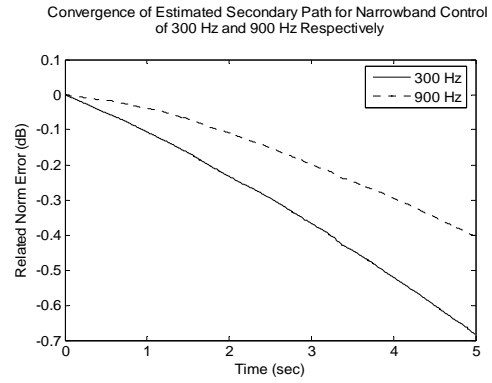


Figure 10 – Convergent performance of the estimated secondary path for two narrowband control respectively.

The broadband control begins operation after the plant's output turns into a steady state response. The time response of the plant's output and its broadband control result are shown in Fig. 11, the MSE performance is shown in Fig. 12, and convergent performance of the estimated secondary path $\hat{S}(z)$ is shown in Fig. 13.

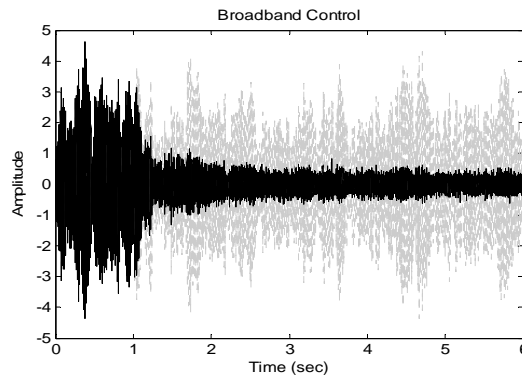


Figure 11 – Time response of the plant's output and its broadband control result respectively.

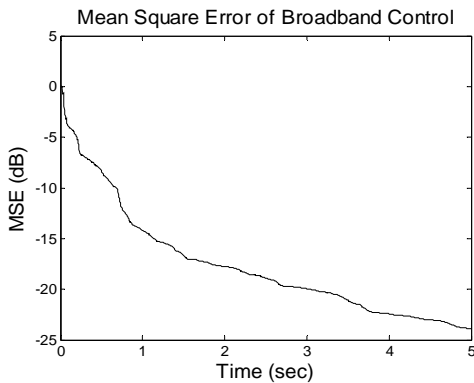


Figure 12 – MSE performance of the broadband control.

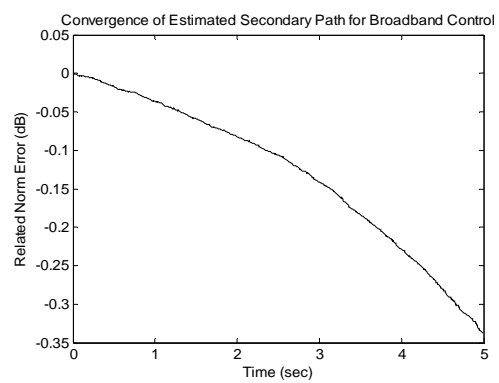


Figure 13 – Convergent performance of the estimated secondary path for broadband control.

Though the random signal used to estimate the secondary path does affect the residual error, it can be set to be a quite small amplitude to avoid enlarging the error amplitude.

CONCLUSIONS

For some ANC/AVC systems having time-varying secondary path, it is impossible or inefficient to estimate the secondary path off-line. For example, the multi-path in acoustic space can not be estimated prior to noise control. Then the proposed method is valuable for these control systems. Furthermore, since adaptive feedback control systems can only predict predictable primary signal, the random component inside the primary signal will remain in the residual error signal. Then the resonant components in broadband primary signal can be attenuated since these components are more detectable than random components. If attenuation of random noise is necessary, the proposed method can be used in the hybrid method, which combines feedforward and feedback control systems.

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